17 - 15 Breakdown of Synchronization in Three Coupled Oscillators

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1 Introduction

Synchronization in nonlinear systems is one of typical nonlinear phenomena observed in the field of natural science. Breakdown of synchronization is a kind of cooperative phenomenon for dissipated assembly oscillators and is important to clarify its mechanism for better understanding of higher-dimensional complicated phenomena. In this study, we investigate the breakdown of synchronization observed from three coupled oscillators consisting of two types of oscillators; the first one is chaotic oscillator and the second one is van der Pol oscillators with noise. We compare the breakdown of the synchronization observed from two types of coupled oscillators.

2 Circuit Model

Figure 1 shows the circuit model. In the circuit, three identical chaotic circuits are coupled by one resistor.

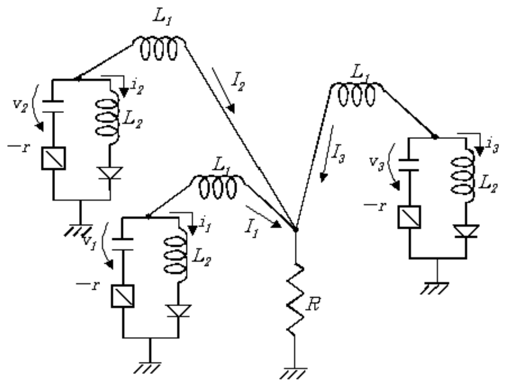

Figure 1: Circuit model.

The normalized circuit equations of the circuit are described as

$$\begin{cases} \dfrac{dx_k}{d\tau} = \beta(x_k + y_k) - z_k - \gamma \displaystyle\sum_{j=1}^{3} x_j \\ \dfrac{dy_k}{d\tau} = \alpha\{\beta(x_k + y_k) - z_k - f(y_k)\} \\ \dfrac{dz_k}{d\tau} = (1 + \Delta\omega_k)(x_k + y_k) \qquad (k = 1, 2, 3) \end{cases} \tag{1}$$

where

$$f(y_k) = 0.5(\delta y_k + 1 - |\delta y_k - 1|), \tag{2}$$

and $\Delta\omega_k$ corresponds to the difference between the natural oscillating frequency of the reference oscillator and those of other oscillators.

Next, we consider three coupled van der Pol oscillators with noise. In this study we assume that noise with uniform distribution is added to each voltage of the oscillator. The normalized equations are described as follows.

$$\begin{cases} \dfrac{dx_k}{d\tau} = -y_k + \varepsilon \left\{ (1 + \sigma(\tau))x_k - \dfrac{((1 + \sigma(\tau))x_k)^3}{3} \right\} \\ \dfrac{dy_k}{d\tau} = (1 + \Delta\omega_k)(1 + \sigma(\tau))x_k - \gamma \displaystyle\sum_{j=1}^{3} y_j \\ (k = 1, 2, 3) \end{cases} \tag{3}$$

where $\sigma(\tau)$ is the added noise.

3 Computer Calculated Results

Computer simulated results are shown in Fig 2. The horizontal axis shows the strength of the added noise and the vertical axis shows the coupling parameter at which the synchronization breaks down. The circuit parameters are fixed as $\alpha = 10.0$, $\beta = 0.06$, $\delta = 100.0$ and $\Delta\omega_k = 0.002(k - 1)$, ($k$=1,2,3) for computer calculations.

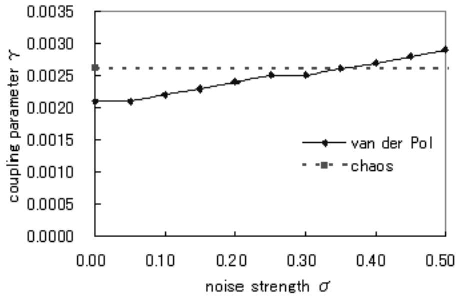

Figure 2: Coupling parameter at which synchronization breaks down (computer calculated results). $\varepsilon = 0.1$ and $\Delta\omega_k = 0.002(k - 1)$, ($k$=1,2,3).

For van der Pol oscillators without noise, the breakdown of the synchronization is observed for $\gamma \leq 0.0021$. While, for chaotic oscillators, the breakdown appear for $\gamma \leq 0.0026$. This means that the coupled van der Pol oscillators are more stable than the chaotic system. When we add noise to van der Pol oscillators, the parameter for the breakdown increases as shown in Fig. 2. Around σ=0.35, the coupled van der Pol oscillators become to show similar breakdown to the chaotic system.

4 Conclusions

In this study, we investigated the breakdown of the synchronization observed from both three chaotic circuits and three van der Pol oscillators with noise.

References

[1] Seiichiro MORO, Yoshifumi NISHIO and Shinsaku MORI, "Synchronization Phenomena in Oscillators Coupled by One Resistor," IEICE Transactions on Fundamentals, vol. E78-A, no. 2, pp. 244-253, Feb. 1995.

[2] Yoshifumi NISHIO and Akio USHIDA, "Chaotic Wandering and its Analysis in Simple Coupled Chaotic Circuits," IEICE Transactions on Fundamentals, vol. E85-A, no. 1, pp. 248-255, Jan. 2002.