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Volterra Series Method versus Harmonic Balance Method in Frequency-Domain Analysis

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1. Introduction

Distortion analysis of nonlinear circuits is very important for designing analog integrated circuits and/or communication systems. In this paper, Volterra method is compared with the harmonic balance method. The Volterra series is based on the bilinear theorem, so that the higher order solutions can be found in a sequential manner, and the harmonic distortions (HDs) are given in the analytical forms. However, it can be only applied to the weakly nonlinear systems, and, furthermore, the nonlinear characteristics should be described by approximate polynomial functions at the DC operation points. Our harmonic balance method can get the Fourier coefficients in symbolic forms using MATLAB, and whole the harmonic components are considered as the variables in the analysis, all together. Thus, the harmonic balance method can be applied to the strong nonlinear systems. We will compare the results of the above two methods with a simple example.

2. Volterra series method

To understand the properties of Volterra method, we consider a simple example shown in Fig.1, where nonlinear resistor is described by the following polynomial function:

$$i_G = k_1 v_G + k_2 v_G^2 + k_3 v_G^3 \quad (1)$$

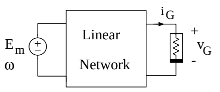

Fig.1 Nonlinear resistive circuit.

The first term in (1) is included in the linear network, so that the circuit for computing the first order-kernel is given by the transfer function $K(j\omega)$ as follows:

$$V_1(j\omega) = H_1(j\omega), \quad \text{for} \ \ H_1(j\omega) = K_1(j\omega) \quad (2.1)$$

. Next, the current for computing the 2nd-order kernel is given by [1]

$$I_2(j\omega, j\omega) = -k_2(H_1(j\omega))^2 \quad (2.2)$$

and we have the 2nd-kernel as follows:

$$V_2(j\omega, j\omega) = H_2(j\omega, j\omega), \quad (2.3)$$

$$\text{for} \ \ H_2(j\omega, j\omega) = K_2(j\omega, j\omega) I_2(j\omega, j\omega)$$

. Finally, the current for computing the 3rd-kernel is given by

$$I_3(j\omega, j\omega, j\omega) = k_3(H_1(j\omega))^3 + 2k_2 H_1(j\omega) H_2(j\omega, j\omega) \quad (2.4)$$

and we have the 3rd-kernel as follows:

$$V_3(j\omega, j\omega, j\omega) = H_3(j\omega, j\omega, j\omega) \quad (2.5)$$

$$\text{for} \ \ H_3(j\omega, j\omega, j\omega) = K_3(j\omega, j\omega, j\omega) I_3(j\omega, j\omega, j\omega)$$

Thus, the combining complex solutions up to 3rd-order kernel is given by

$$V(j\omega, j\omega, j\omega) = H_1(j\omega) X_{in}(j\omega) + H_2(j\omega, j\omega) X_{in}^2(j\omega) + H_3(j\omega, j\omega, j\omega) X_{in}^3(j\omega) \quad (3)$$

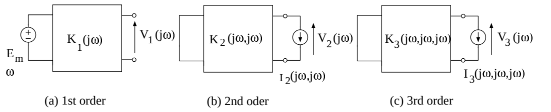

Fig.2 Computation of Volterra kernels of Fig.1.

Example2.1 Now, consider an example of nonlinear circuit shown in Fig.3. The DC operating point of diode is $v_0 = 0.2796$, and the current is expanded by the Taylor expansion as follows:

$$i_d = 7.19 \times 10^{-4} + 0.0288 v_d + 0.576 v_d^2 + 7.68 v_d^3 \quad (4)$$

Therefor, we have $g_1 = 0.0288$, $g_2 = 0.576$, $g_3 = 7.68$

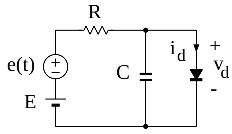

Fig.3 An example
$i_d = 10^{-8}\exp(40 v_d)$,
$R = 1000[\Omega]$, $C = 1[\mu F]$,
$E = 1[V]$, $e(t) = Em\cos\omega t$

The Volterra kernels are decided by Fig.4 as follows:

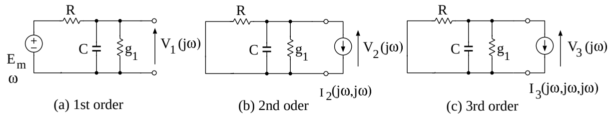

Fig.4 Computations of Volterra kernels.

Then, the harmonic distortions are given by

$$HD2 = \frac{g_2}{2}|K_1(j\omega)||K_2(j\omega, j\omega)|E_m \quad (5.1)$$

$$HD3 = \frac{1}{4}|K_1(j\omega)|^2 |2g_2 K_2(j\omega, j\omega) + g_3 K_3(j\omega, j\omega, j\omega)| E_m^2 \quad (5.2)$$

where

$$K_1(j\omega) = \frac{1}{1 + g_1 R + j\omega CR}, \quad K_2(j\omega, j\omega) = \frac{R}{1 + g_1 R + j2\omega CR}$$

$$K_3(j\omega, j\omega, j\omega) = \frac{R}{1 + g_1 R + j3\omega CR}$$

Note that the computations of HD2 and HD3 are obtained by Fig. 4(b) and (c), respectively.

3. Our harmonic balance method

In our harmonic balance method, the nonlinear characteristics are described by polynomial forms in symbolic manners. Namely, the coefficients are functions of the unknown operating DC point v_0s as follows:

$$i_G = k_0(v_0) + k_1(v_0) v_G + k_2(v_0) v_G^2 + k_3(v_0) v_G^3 \quad (6)$$

Therefore, the Fourier coefficients are also described in the following forms with MATLAB :

$$v_G(t) = V_{G,0} + \sum_{k=0}^{M} (V_{G,2k-1}\cos k\omega t + V_{G,2k}\sin k\omega t) \quad (7.1)$$

$$i_G(t) = I_{G,0} + \sum_{k=0}^{M} (I_{G,2k-1}\cos k\omega t + I_{G,2k}\sin k\omega t) \quad (7.2)$$

Using these relations, the Sine-Cosine circuits [2] can be constructed in the schematic forms using ABMs (Analog Behavior Models) of Spice, which are corresponding to the *determining equations* of the harmonic balance method. Thus, it can be solved by the DC analysis of Spice. In this case, we can get the frequency response curve if we choose the frequency as an additional variable.

4. Conclusions and remarks

In this paper, we discussed the comparisons between the Volterra series method and our harmonic balance method, and found that latter can be applied to the strong nonlinear system, safely. We will show the numerical examples at our presentation of the conference.

References

[1] P.Wambacq and W.Sansen, *Distortion Analysis of Analog Integrated Circuits*, Kluwer Academic Pub., 1998.

[2] A. Ushida, Y. Yamagami and Y. Nishio, "Frequency responses of nonlinear networks using curve tracing algorithm," Proc. of ISCAS'02, vol.I, pp.641-644, 2002.