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Study on van der Pol Oscillator under Noisy Parametric Excitation

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1. Introduction

Parametric excitation circuit is one of resonant circuits, and it is important to investigate phenomena of the parametric excitation circuits of future engineering applications. Because, they have a possibility to generate various kinds of complex phenomena. In this study, we investigate the phenomena in van der Pol oscillator under noisy parametric excitation.

2. Circuit Model

The circuit model used in this study is shown in Fig 1.

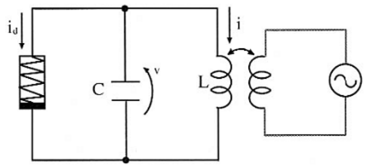


Figure 1: van der Pol oscillator under parametric excitation.

In our system, the circuit includes a time-varying inductor L whose characteristics are given as the following equation.

$$L = L_0\gamma(t), \tag{1}$$

$\gamma(\tau)$ is expressed in a rectangular wave as shown in Fig 2, and its duty ratio is perturbed by noise of amplitude ζ. The noise used in this study is assumed to be uniform distribution noise.

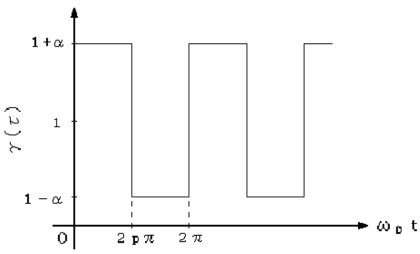


Figure 2: rectangular wave.

First, the $v-i$ characteristics of the nonlinear resistor are approximated by the following equation.

$$i_d = -g_1 v + g_3 v, \tag{2}$$

By changing the variables and the parameters,

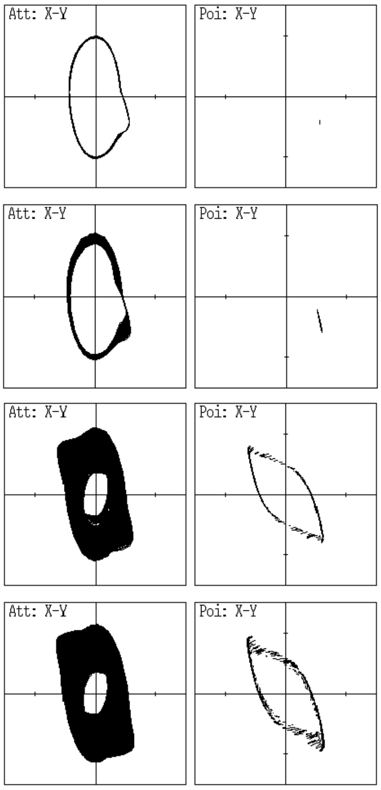
$$t = \sqrt{L_0 C}\tau, \quad v = \sqrt{\frac{g_1}{g_3}}x, \quad \omega = \omega_0\sqrt{L_0 C},$$
$$i = \sqrt{\frac{g_1}{g_3}}\sqrt{\frac{C}{L_0}}y, \quad \epsilon = g_1\sqrt{\frac{L_0}{C}}, \tag{3}$$

the normalized circuit equations are given by the following equations.

$$\begin{cases} \dfrac{dx}{d\tau} = \epsilon(x - x^3) - y \\ \dfrac{dy}{d\tau} = \dfrac{1}{\gamma(\tau)}x. \end{cases} \tag{4}$$

3. Simulation Results

We calculated Eq. (4) by using the Runge-Kutta method. The attractors obtained by the simulations are shown in Fig. 3(a). We can confirm the generation of periodic and chaotic attractors. In order to investigate the attractors in van der Pol oscillator under noisy parametric excitation in detail, we consider the Poincaré map. The result of the Poincaré map are shown in Fig. 3(b). We can see that the attractor changes by noise amplitude.



(1) (2) (3) (4)

(a) (b)

Figure 3: Simulation results. $\epsilon = 0.8$, $\alpha = 0.8$, $\omega = 1.25$. (a) Attractor. (b) Poincaré map. (1) $\zeta = 0$. (2) $\zeta = 0.1$. (3) $\zeta = 0.2$. (4) $\zeta = 1$.

4. Conclusions

In this study, we investigated the phenomena in van der Pol oscillator under noisy parametric excitation. We confirmed that attractor changes to like chaos by noise amplitude. Furthermore, we will research the phenomena, when the duty ratio is perturbed by Gaussian noise.

Reference

[1] M. Inoue,"A Method of Analysis for the Bifurcation of the Almost Periodic Oscillation and the Generation of Chaos in a Parametric Excitation Circuit," Trans. of IEICE, vol. J68-A, no. 7, pp. 621-626, 1985.