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Synchronizations of Four van der Pol Oscillators with Linear Memristor Coupling as Ring Structure

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Abstract

In this study, we proposed four van der Pol oscillators coupled in a ring structure using linear memristors. It was found that this network exhibits various stable states depending on the initial values of the oscillators and memristor parameters, and that the synchronous state and spatiotemporal patterns change because of the linear memristors. These results suggest the potential for using memristors in the physical implementation of brain-inspired networks that mimic the mechanisms of the human brain, such as in physical reservoir computing and brain-inspired computing.

1. Introduction

In recent years, research on "physical reservoir computing" and "brain-inspired computing," which implement neural networks as physical circuits, has garnered significant attention [1]. Among these approaches, utilizing the synchronization phenomena of nonlinear oscillators for information processing is particularly promising as it employs different calculation methods from digital computation. Specifically, the generation of diverse spatiotemporal patterns based on differences in coupling states and initial values of oscillators provides the advantage of naturally forming high-dimensional state spaces.

A memristor is a passive device whose resistance changes in accordance with the history of charge flow or magnetic flux through the element [2]. Using it as a coupling element can impart plasticity and learning behavior to an oscillator network. The memristor also exhibits a characteristic called bidirectionality, where current–voltage characteristics of memristor depend on the direction of the current flow. This is a property similar to the functioning of synapses in the human brain [3]. Therefore, it is highly useful for the physical implementation of nonlinear networks with dynamically changing connection strengths, particularly neural networks.

The van der Pol oscillator is a representative nonlinear oscillator that combines nonlinearity and self-excitation characteristics. It is known to exhibit various synchronization phenomena in coupled systems, such as perfect synchronization, phase-shifted synchronization, and asynchronous states. In physical implementations that utilize the internal state of circuits as computational resources, such as brain-inspired computing, a crossbar array structure with oscillators as nodes is employed. A crossbar array structure is a grid-like circuit structure where row and column wirings are orthogonal, with circuit elements placed at their intersections. Not only can it densely integrate connection weights, but it can also be utilized for in-memory computation [4].

In the future, combining the memristor into a crossbar array structure will enable the large-scale, integrated implementation of oscillator networks [5]. Therefore, this research is an analysis of the operation of the smallest unit of a crossbar array structure using the memristor.

In a previous study, we proposed three van der Pol oscillatorscoupled by linear memristors. The linear memristor means that the memductance increases monotonically in proportion to the magnetic flux. We here investigate the change of synchronization states depending on the linear memristor parameter ζ. Memductance is proportional to the magnetic flux, and ζ is the normalized proportionality constant. In the results, by changing ζ, we obtained three synchronization states: in-phase and anti phase synchronization states, almost in phase synchronization state, and three-phase synchronization state. This indicates that, compared with conventional circuits, the memristor can expand the diversity of synchronous patterns.

In this research, we focus on a minimal network configuration consisting of four van der Pol oscillators coupled in a ring structure using linear memristors. We investigate the effects of initial values and memristor parameters on the formation of synchronized states.

2. Proposed Model

Figure 1 shows the four van der Pol oscillators coupled in a ring structure using linear memristors. This ring circuit constitutes the smallest unit of a crossbar array structure for an oscillator network using memristors.

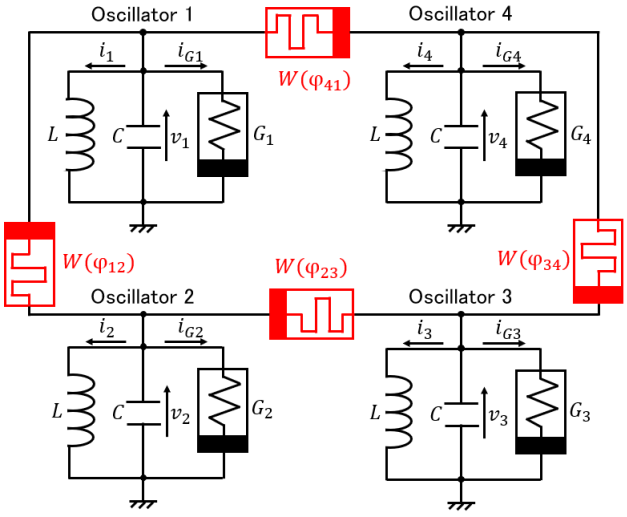

Figure 1: Four van der Pol oscillators coupled in a ring structure using linear memristors

With the currents of the nonlinear resistor on each circuit being denoted by i_{G1}, i_{G2}, i_{G3}, and i_{G4}, and the i–v characteristics are described in Eq. (1).

$$i_{Gn} = -g_1 v_n + g_3 v_n^3 \quad (g_1, g_3 > 0,\ n = 1 \sim 4) \tag{1}$$

The elements shown in red in the circuit are linear memristors. Figure 1 shows the circuit symbol of a memristor. The current and voltage of the memristor are denoted by i and v, and Ohm's law is described in Eq. (2).

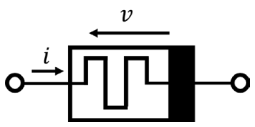

Figure 2: Memristor model

$$\begin{gathered} i = W(\varphi_{pq})v \\ ((p,q) = (1,2),(2,3),(3,4),(4,1)) \end{gathered} \tag{2}$$

The conductance of the memristor is called memductance $W(\varphi)$. $W(\varphi)$ is defined in Eq. (3) with α as the proportionality constant.

$$\begin{gathered} W(\varphi_{pq}) = \alpha\varphi_{pq} \\ ((p,q) = (1,2),(2,3),(3,4),(4,1)) \end{gathered} \tag{3}$$

It is called a linear memristor in this resarch because the memductance increases monotonically in proportion to the magnetic flux. The characteristics of the magnetic flux through the memristor are described as Eq. (4).

$$\begin{gathered} \varphi_{pq} = L(i_p - i_q) \\ ((p,q) = (1,2),(2,3),(3,4),(4,1)) \end{gathered} \tag{4}$$

i_p and i_q are the currents flowing through the inductor of the van der Pol oscillator. From Eq. (4), the magnetic flux through the memristor is the difference between the magnetic flux generated by the inductor in oscillator p and the magnetic flux generated by the inductor in oscillator q.

The circuit equations are given as Eq. (5) from Kirchhoff's circuit laws.

$$\begin{cases} C\dfrac{dv_1}{dt} = -i_1 - i_{G1} - W(\varphi_{12})(v_1 - v_2) \\ \qquad\qquad +W(\varphi_{41})(v_4 - v_1) \\ C\dfrac{dv_2}{dt} = -i_2 - i_{G2} - W(\varphi_{23})(v_2 - v_3) \\ \qquad\qquad +W(\varphi_{12})(v_1 - v_2) \\ C\dfrac{dv_3}{dt} = -i_3 - i_{G3} - W(\varphi_{34})(v_3 - v_4) \\ \qquad\qquad +W(\varphi_{23})(v_2 - v_3) \\ C\dfrac{dv_4}{dt} = -i_4 - i_{G4} - W(\varphi_{41})(v_4 - v_1) \\ \qquad\qquad +W(\varphi_{34})(v_3 - v_4) \\ L\dfrac{di_1}{dt} = v_1 \quad L\dfrac{di_2}{dt} = v_2 \\ L\dfrac{di_3}{dt} = v_3 \quad L\dfrac{di_4}{dt} = v_4 \end{cases} \tag{5}$$

Next, we normalize the circuit equation. (5) and analyze the dynamics using the Runge–Kutta method. We change the variables as follows respectively,

$$v_n = \sqrt{\frac{g_1}{g_3}} V x_n, \quad i_n = \sqrt{\frac{g_1 C}{g_3 L}} V y_n \quad (n = 1 \sim 4)$$

$$\varepsilon = g_1\sqrt{\frac{L}{C}},\ \zeta = \alpha L\sqrt{\frac{g_1}{g_3}},\ t = \sqrt{LC}\tau;\ ' \cdot ' = \frac{d}{d\tau}$$

The normalized circuit equations are shown in Eq. (6).

$$\begin{cases} \dot{x_1} = \varepsilon x_1(1 - x_1^2) + \zeta\{x_4(y_4 - y_1) \\ \qquad -x_1(y_4 - y_2) + x_2(y_1 - y_2)\} - y_1 \\ \dot{x_2} = \varepsilon x_2(1 - x_2^2) + \zeta\{x_1(y_1 - y_2) \\ \qquad -x_2(y_1 - y_3) + x_3(y_2 - y_3)\} - y_2 \\ \dot{x_3} = \varepsilon x_3(1 - x_3^2) + \zeta\{x_2(y_2 - y_3) \\ \qquad -x_3(y_2 - y_4) + x_4(y_3 - y_4)\} - y_3 \\ \dot{x_4} = \varepsilon x_4(1 - x_4^2) + \zeta\{x_3(y_3 - y_4) \\ \qquad -x_4(y_3 - y_1) + x_1(y_4 - y_1)\} - y_4 \\ \dot{y_1} = x_1 \quad \dot{y_2} = x_2 \\ \dot{y_3} = x_3 \quad \dot{y_4} = x_4 \end{cases} \tag{6}$$

Here, τ is the scaling time, ε is a nonlinearity parameter of the van der Pol oscillator, and ζ is the normalized proportionality constant of the linear memristor.

3. Simulation results

We investigate how the initial values of the oscillators x_1–x_4 and the memristor parameter ζ affect the steady state and the selection of spatiotemporal patterns in the proposed circuit. The current through the memristor between oscillators p and q is i_{pq}, and the voltage is v_{pq}. The i–v characteristic of the memristor between oscillators p and q is i_{pq}–v_{pq}.

3.1 Diverse synchronization patterns

As $y_1 = y_2 = y_3 = y_4 = 1$, $\varepsilon = 0.1$, and $\zeta = 0.1$, we observed the time-series waveform and the memristor i–v characteristics when x_1–x_4 are changed. Figure 3 shows representative examples of diverse synchronization patterns.

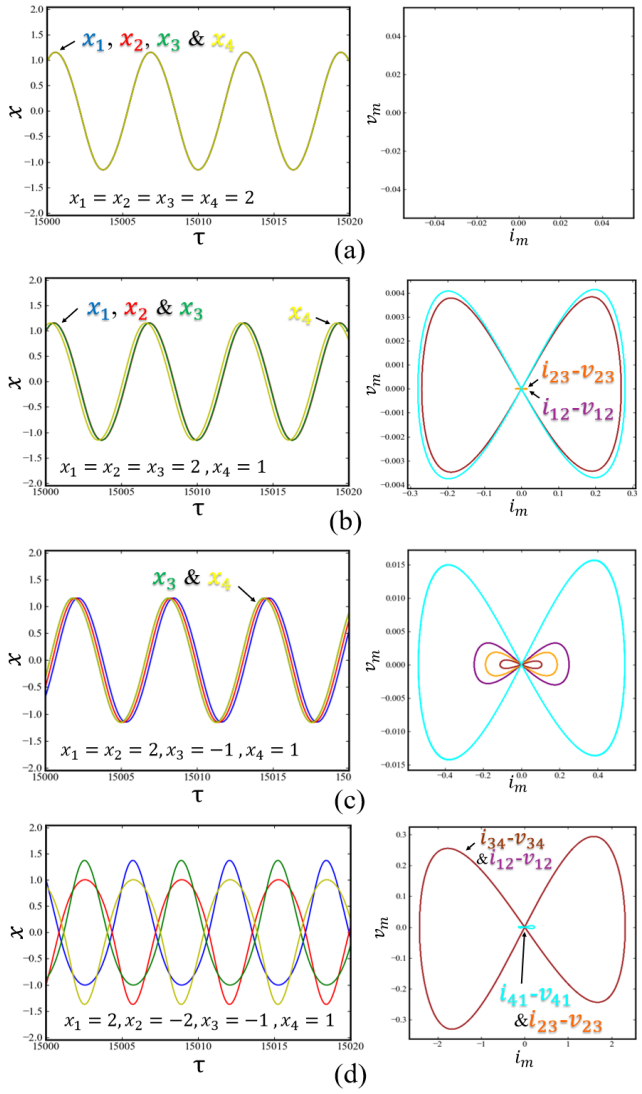

Figure 3: Time series of x_1 (blue), x_2 (red), x_3 (green), and x_4 (yellow), and memristor i–v characteristics of i_{12}–v_{12} (purple), i_{23}–v_{23} (orange), i_{34}–v_{34} (brown), and i_{41}–v_{41} (cyan)

According to Fig. 3(a), x_1–x_4 have the same initial values. The time series waveforms completely overlapped and there is no change in the memristor's current or voltage. This is called four-variable full-amplitude synchronization. As no current flows between the oscillators, the memristor i–v characteristics do not appear. According to Fig. 3(b), we only change the initial value of x_4. x_4 is not amplitude-synchronized with the other variables. This is called three-variable amplitude synchronization. As shown in Fig. 1, oscillator 4 is coupled to oscillators 1 and 3. The voltage difference between oscillators coupled to oscillator 4 becomes large, resulting in large i–v characteristics of i_{34}–v_{34} (brown) and i_{41}–v_{41} (cyan).

According to Fig. 3(c), initial values of x_1 and x_2 are the same, and x_3 and x_4 have reversed initial values. Under these settings, we can observe an interesting phenomenon: oscillators with reversed initial values synchronize, while the others remain asynchronous. This is called two-variable amplitude synchronization. According to Fig. 3(d), x_1 and x_2 have reversed initial values, while x_3 and x_4 have different types of initial value. Under these settings, the amplitudes become asynchronous. This is called amplitude asynchronism. Furthermore, owing to the asymmetric periodic waveform, the memristor i–v characteristics also became asymmetric.

3.2 Focusing on the memristor i–v characteristics

We focus on how the characteristics of each linear memristor change when the memristor parameter is altered. We observe the memristor i–v characteristics when increasing ζ from 0.10 until results diverge. Figure 4 shows the results for the memristor i–v characteristics for each ζ. According to Fig. 4(a), the i–v characteristics are observed when varying ζ in three-variable amplitude synchronization. At $\zeta = 0.10$, the i–v characteristics of i_{34}–v_{34} (brown) and i_{41}–v_{41} (cyan) exhibit significant behavior. At $\zeta = 0.30$, each memristor operates independently. With further increase in ζ to 0.70, the memristors operating at φ_{12} and φ_{23} exhibit identical characteristics, as do those operating at φ_{34} and φ_{41}. When ζ is higher than 0.70, the current diverges, and i–v characteristics are not output. According to Fig. 4(b), the i–v characteristics are observed when varying ζ in two-variable amplitude synchronization. At $\zeta = 0.10$, each memristor operates independently. Increasing ζ further to 0.35, the memristors operating at φ_{41} and φ_{12} exhibit identical characteristics, as do those operating at φ_{23} and φ_{34}. When ζ is higher than 0.35, the current diverges, and i–v characteristics are not output. According to Fig. 4(c), the i–v characteristics are observed when varying ζ in amplitude asynchronism. The memristors operating at φ_{12} and φ_{34} exhibit identical characteristics, as do those operating at φ_{41} and φ_{23}. An interesting phenomenon is observed in which nonadjacent memristors exhibit identical characteristics. When ζ is higher than 0.25, the current diverges, and i–v characteristics are not output.

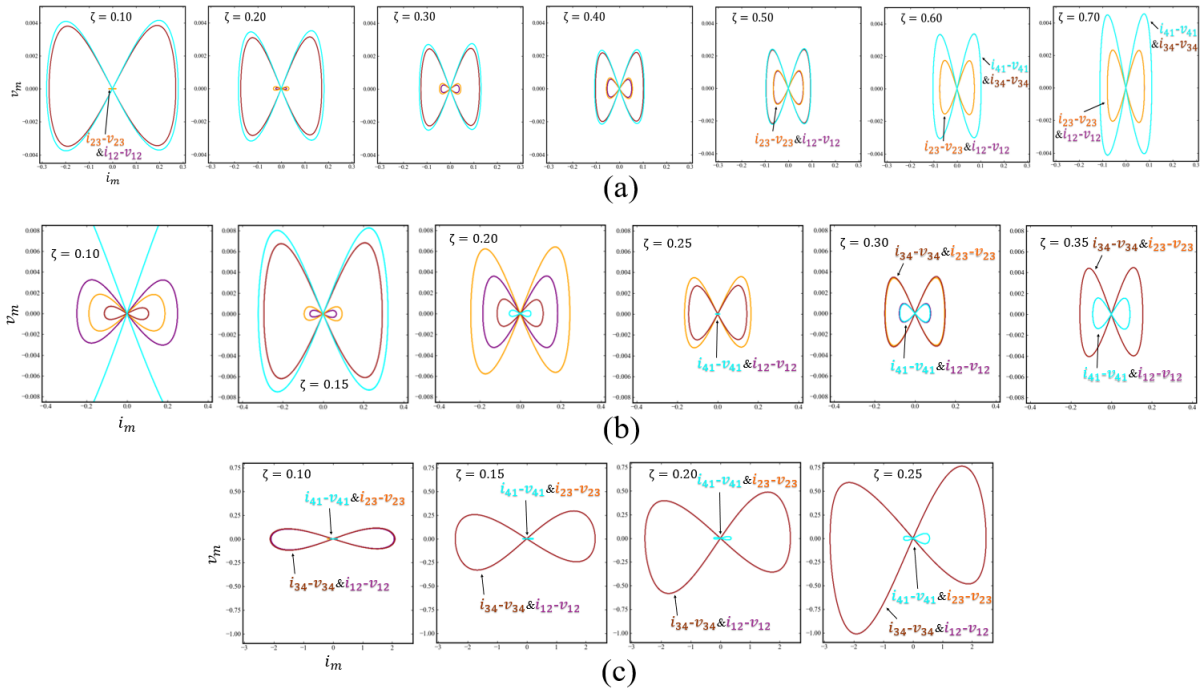

Figure 4: Memristor i–v characteristics for each memristor parameter: i_{12}–v_{12} (purple), i_{23}–v_{23} (orange), i_{34}–v_{34} (brown), and i_{41}–v_{41} (cyan). (a) $x_1 = x_2 = x_3 = 2$, $x_4 = 1$, $\varepsilon = 0.1$, (b) $x_1 = x_2 = 2$, $x_3 = -1$, $x_4 = 1$, $\varepsilon = 0.1$, and (c) $x_1 = 2$, $x_2 = -2$, $x_3 = -1$, $x_4 = 1$, $\varepsilon = 0.1$

From the above, the manner of change differs for each memristor, and the synchronization pattern changes with ζ. In other words, the memristors can be understood as learning the relationship between the initial values and thereby altering their own characteristics and the synchronization patterns.

4. Conclusions

In this study, we proposed a circuit that connects four van der Pol oscillators, the smallest building blocks of a crossbar array, in a ring structure using linear memristors. By modifying the initial values of the oscillators, x_1–x_4, and the memristor parameter ζ, we observed diverse synchronization patterns and the memristor i-v characteristics.

As a result, the proposed circuit exhibits multiple stable states depending on the combination of initial values. This outcome cannot be achieved in conventional circuits without memristors, indicating that memristors expand the diversity of synchronous patterns within the circuit. Additionally, we focused on how the characteristics of each linear memristor change when its parameters are altered. We found that the memristor's parameters modify the temporal evolution of the memristors state, thereby controlling the final synchronized state and spatiotemporal patterns.

In future work, we plan to systematically explore the potential applications of linear memristors in brain-inspired computing.

Acknowledgment

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