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Phase Difference Patterns in Two Coupled Chaotic Circuits Including Memristors

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Abstract

In this study, we design and simulate two coupled chaotic circuits with memristors. As a result, we obtain nonlinear phenomena of the phase differences from coupled chaotic circuits with fixed coupling strength. These are new phenomena in which the phase differences change periodically or chaotically with time.

1. Introduction

A neumann-type computer, which is indispensable to the modern information society, converts all continuous information into 01 discrete information. On the other hand, neurons or neural networks in the human brain are said to be nonlinear oscillators that process continuous information [1]. To realize more complex, flexible, and adaptive information processing technologies in the future, it is useful to gain insight from our brain function. In other words, a better understanding of nonlinear oscillators and coupled nonlinear oscillators, one of the models of brain function, is useful for both scientific and engineering purposes. Furthermore, the abundant dynamics of the human brain is said to have chaotic properties [2].

Chaos has a nonlinear nature in which small differences in initial values expand exponentially into the future, making long-term predictions difficult. Chaos is attracting considerable attention because this nonlinearity is expected to deepen our understanding of nonlinear science and to have engineering applications. One of the most famous applications is the chaotic neural network. The chaotic property of neurons has been reported to make it possible to extract features and form associative memories using various spatiotemporal patterns [3]. However, these neural networks are implemented by digital computers owing to their complex structure. Digital computers cannot handle real numbers with infinite digits, and there are limitations to reproducing brain functions. Furthermore, complex structures have the problem of making the processing in the intermediate layer a black box. For this reason, it is expected that chaotic information processing will be realized with analog devices. Therefore, chaotic circuits are one of the effective means. Furthermore, analog memory elements that can handle infinite digits are also attracting attention. In this study, we focused on a memristor.

A memristor has attracted considerable attention because of its excellent resistance change characteristics depending on the history of the charge or flux flow through it [4].

In this study, we design and simulate a coupled circuit model of two chaotic circuits with memristors. The purpose of this research is to investigate the fundamental dynamics of the proposed model. To achieve our goal, we analyze the time series waveform for each variable, attractor, and Lissajous diagram.

2. Circuit Model [5]

2.1 Memristor model

We use the *Hewlett-Packard* memristor model. This memristor has v-i characteristics showing a pinched hysteresis loop. Figure 1 shows a schematic and the v-i characteristics when a sine wave is input.

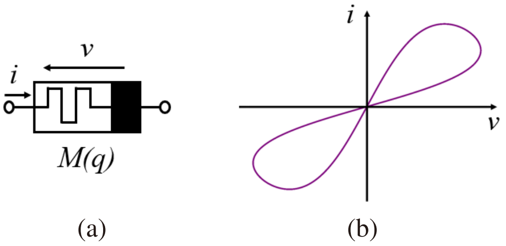


Figure 1: Memristor model: (a) Schematic of memristor and (b) v-i characteristics when a sine wave is input

The resistance of the memristor is called memristance $M(q)$. $M(q)$ is defined in (1) as a function of the charge $q(t)$ that passed through the memristor.

$$M(q) = \mu_v \frac{R^2{}_{\text{on}}}{D^2} q(t) + R_{\text{off}} \left(1 - \mu_v \frac{R_{\text{on}}}{D^2} q(t) \right) \qquad (1)$$

R_{on} is the minimum resistance, R_{off} is the maximum resistance, μ_v is the average drift mobility of the charges, and D is the length of doped and undoped parts combined.

2.2 Chaotic circuit with the memristor

In this study, we use the chaotic circuit with the memristor presented at ISOCC'24 [5], shown in Fig. 2. It is a self-exciting oscillator. The original chaotic circuit consists of one negative resistor r, one capacitor C, two inductors L_1 and L_2, and one dual-directional diode v_d [6]. In this original chaotic circuit, i_1 is usually larger than i_2. To take advantage of this property, the memristor is added between the inductor L_1 and the negative resistor $-r$ to increase the influence of the memristor on the chaotic circuit.

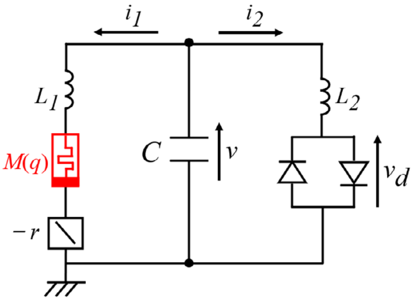


Figure 2: Chaotic circuit with the memristor

The dual-directional diode acts as a nonlinear resistor and is responsible for maintaining stable oscillation. The i-v characteristics of the dual-directional diodes are approximated as the three-segment piecewise-linear function defined in (2). The parameter r_d is the resistance when the diodes are off.

$$v_d(i_2) = \frac{r_d}{2}\left(\left|i_2 + \frac{V}{r_d}\right| - \left|i_2 - \frac{V}{r_d}\right|\right) \tag{2}$$

Then, the circuit dynamics in Fig. 2 are described by the circuit equation from Kirchihoff's circuit lows as

$$\begin{cases} L_1\dfrac{di_1}{dt} = v + ri_1 - M(q)i_1 \\ L_2\dfrac{di_2}{dt} = v - v_d(i_2) \\ C\dfrac{dv}{dt} = -i_1 - i_2 \\ \dfrac{dq}{dt} = i_1 \end{cases} \tag{3}$$

The important part in (3) is the $M(q)i_1$ term because the most significant difference between this model and the original chaotic circuit is that this model has a variable product term, whereas the original chaotic circuit has no variable product term. $M(q)i_1$ is important in the sense that it gives the chaotic circuit with the memristor new nonlinear characteristics.

Next, the circuit equations in (3) are normalized to investigate the dynamics by calculation with the Runge-Kutta method. We change the variables as follows:

$$i_{1_n} = \sqrt{\frac{C}{L_1}}Vx_n\,, \quad i_{2_n} = \frac{\sqrt{L_1C}}{L_2}Vy_n\,, \quad v_n = Vz_n\,,$$
$$q_n = CVw_n\,, \quad t = \sqrt{L_1C}\tau\,, \quad '\cdot' = \frac{d}{d\tau}\,, \quad r\sqrt{\frac{C}{L_1}} = \alpha\,,$$
$$\frac{L_1}{L_2} = \beta\,, \quad r_d\frac{\sqrt{L_1C}}{L_2} = \gamma\,, \quad R_{\mathrm{off}}\sqrt{\frac{C}{L_1}} = \eta\,, \quad \frac{R_{\mathrm{on}}}{R_{\mathrm{off}}} = \zeta\,,$$
$$\mu_v\frac{R_{\mathrm{on}}}{D^2}CV = \xi\,, \quad \frac{1}{R}\sqrt{\frac{L_1}{C}} = \delta$$

When this circuit is coupled with another circuit via a resistor R, the normalized circuit equations are described as (4). To consider a single circuit, we substitute N = 1 into (4).

$$\begin{cases} \dot{x}_n = z_n + \alpha x_n - \eta x_n(\zeta\xi w_n + 1 - \xi w_n) \\ \dot{y}_n = z_n - \dfrac{\gamma}{2}\left(\left|y_n + \dfrac{1}{\gamma}\right| - \left|y_n - \dfrac{1}{\gamma}\right|\right) \\ \dot{z}_n = -x_n - \beta y_n - \sum_{k=1}^{N}\delta(z_n - z_k) \\ \dot{w}_n = x_n \qquad\qquad (n = 1, 2, \cdots, N) \end{cases} \tag{4}$$

Here, τ is the scaling time, α means the negative resistance, β means the inductance, γ means the resistance of the diode when it is off, η means the maximum resistance of the memristor, ξ means the minimum resistance of the memristor, and ζ means the ratio of R_{on} to R_{off}

In the computer calculation, the step size of the Runge-Kutta method is set to h = 0.002. This numerical calculation is performed for τ from 0 to 10,000. Some of the parameters are fixed to β = 2.92, γ = 456, and ξ = 0.00276. The initial values of the variables are shifted by 0.02 for each variable.

3. Oscillation Switching Phenomena [5]

In this section, we investigate in more detail the phenomenon observed in [5].

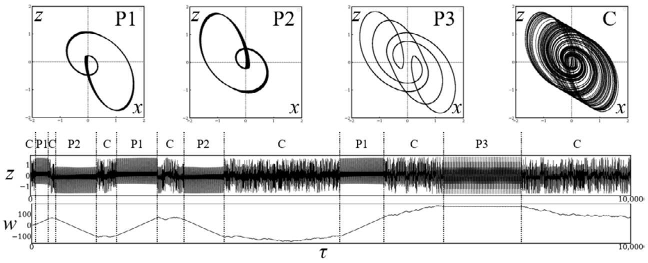


Figure 3: 2-periodic, 5-periodic, and chaotic oscillations (α = 0.588, η = 0.0856, and ξ = 0.125)

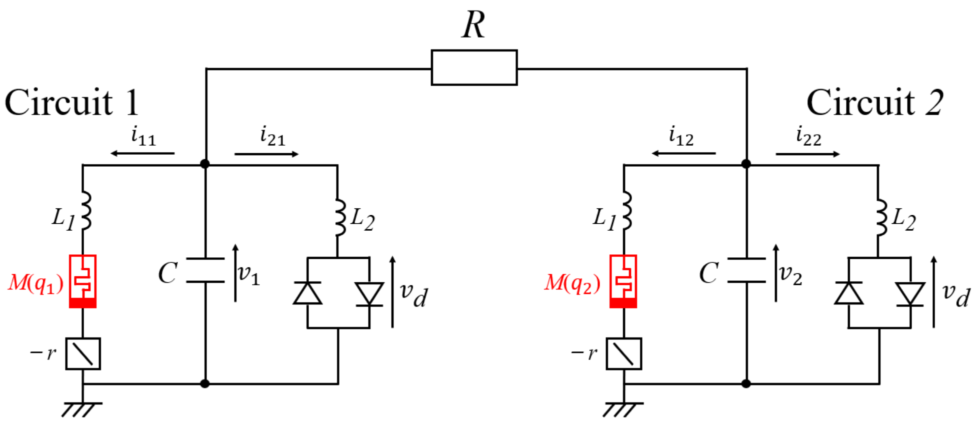


Figure 4: Coupled circuit model

As a result, we confirm the oscillation switching phenomenon between periodic and chaotic oscillations over time. In a conventional chaotic circuit, once parameters are set, either periodic or chaotic oscillations only can be observed. Therefore, the natural occurrence of oscillation switching phenomena as dynamics is a new phenomenon in the chaotic circuit.

Figure 3 shows an example of oscillation switching. This oscillation switching is between 2-periodic, 5-periodic, and chaotic oscillations when the parameters are α = 0.588, η = 0.0856, and ξ = 0.125. The number of periods of periodic oscillation is determined by the number of orbits that make up the closed orbit. With these parameter settings, two types of periodic oscillation appear with different orbital numbers, 2-period and 5-period. In addition, the same 2-periodic oscillation has a point symmetric attractor. The ratios of periodic to chaotic oscillations are 40.5% for periodic oscillations and 59.5% for chaotic oscillations.

4. Two Coupled Circuit Model

In this section, we investigate the dynamics of the coupled circuit model that switches between periodic and chaotic oscillations over time and their interaction with each other.

Figure 4 shows the two coupled circuit model connected by a resistor R. As shown in Fig. 4, Circuit 1 and Circuit 2 are chaotic circuits with memristors, as shown in Fig. 2.

In the coupled circuit model, the parameter settings for each circuit are the same, and the initial values of the variables are inverted for Circuit 1 and Circuit 2 in Fig. 4. Hence, the behavior of the coupled circuit model is obtained by calculating (4). Figure 5 shows attractors on the x-z plane and time series waveforms of z for each circuit in Fig. 4.

As shown in Fig. 5, we discover that in the two coupled circuit model, oscillation switching occurs in each circuit with periodic and chaotic oscillations over time, as in a single circuit. However, the timing of switching is different on each circuit. Therefore, the behavior of the coupled circuit model is also different at each point in time. Hence, we also investigated the phase difference between circuits. Figure 6 shows the phase differences for each type of oscillation.

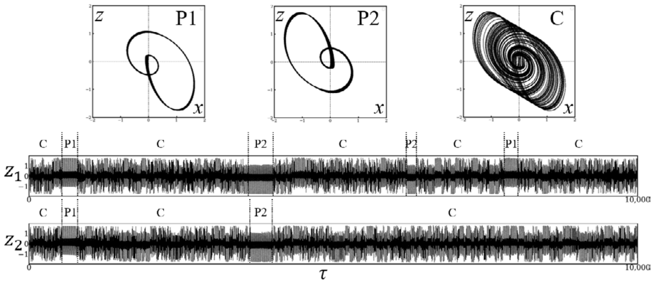
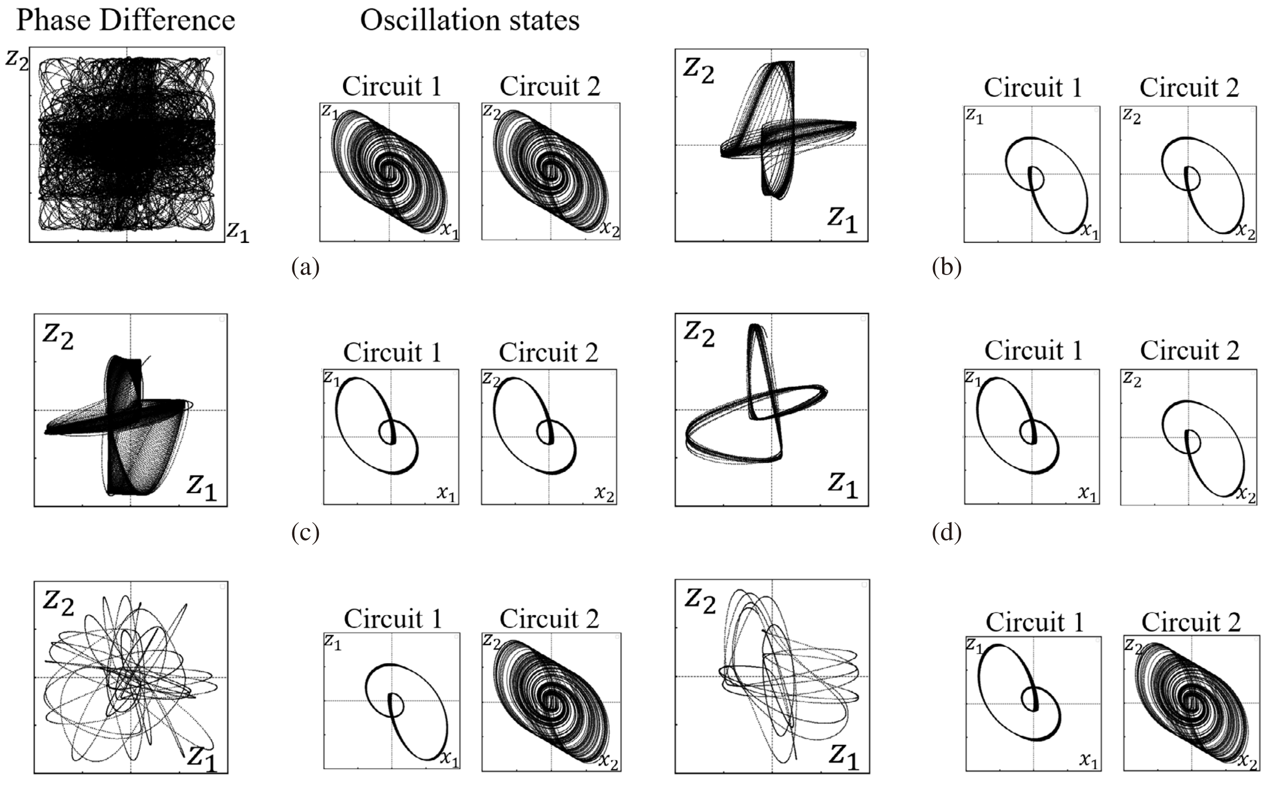


Figure 5: Oscillation switching between Circuit 1 and Circuit 2 in Fig. 4 (δ = 0.005)

As shown in Fig. 6(a), when both circuits are oscillating chaotically, the phase difference varies in a large range and is chaotic. When both circuits undergo 2-periodic oscillations with the same attractor shape, as shown in Figs. 6(b) and 6(c), the phase difference varies within a certain range and is periodic. When both circuits have periodic oscillations with point-symmetric attractors, as shown in Fig. 6(d), the phase difference is almost fixed. When one circuit oscillates periodically and the other circuit oscillates chaotically, as shown in Figs. 6(e) and 6(f), the phase difference varies chaotically within a more limited range than when both circuits oscillate chaotically. Therefore, in the two chaotic circuit coupled model, when Circuit 1 and Circuit 2 switch between periodic and chaotic oscillations, the phase difference also changes dynamically, either periodically or chaotically. These results suggest that it is possible to utilize not only the diverse spatiotemporal patterns of the chaotic circuits but also the diverse phase difference patterns of the coupled model.



(a) (b) (c) (d) (e) (f)

Figure 6: Phase differences for each type of oscillation: (a) Chaos and chaos, (b) 2-period of the same shape, (c) Another 2-period of the same shape, (d) Point symmetry, (e) 2-period and chaos, and (f) Another 2-period and chaos

5. Conclusions

We proposed a coupled model in which two chaotic circuits with memristors are coupled by a pure resistor, and observed the dynamics by computer numerical calculation. As a result, we observed oscillation switching phenomena between periodic and chaotic oscillations over time. Furthermore, we obtained phase differences changing as the oscillation type changes, and the characteristic of the phase differences changing depends on the combination of oscillation types.

Acknowledgment

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