Competitive networks using chaotic circuits with hierarchical structure

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ABSTRACT
Coupled oscillatory systems are good models that are able to describe a variety of higher dimensional nonlinear phenomena. Coupled chaotic circuits produce many kinds of interesting synchronization phenomena. In recent years, research studies on complex networks related to the synchronization of coupled oscillators have attracted much attention. In the real world, there are a variety of different network structures. We focus on the competitive interaction network that includes conflict between two networks. Here, we propose a new paradigm for this competitive interaction network using coupled chaotic circuits.

I. INTRODUCTION
Complex networks have attracted a great deal of attention from various fields since the discovery of small-world and scale-free networks. In particular, understanding the relationship between topological structure and functional behavior is considered a significant topic of interest in respect to practical applications across many disciplines. Of the various dynamical behaviors seen in networks, synchronization is one of the most typical phenomena. Therefore, many researchers have investigated the relationship between synchronization states and network structures and confirmed that the network structure plays an important role with regard to synchronization states.

Coupled oscillatory circuits provide simple system models for describing high-dimensional nonlinear phenomena occurring in our everyday world. Synchronization, in particular, is one of the most important features that can be described as oscillators and explored, because, through their coupling, strongly correlated rhythms among the oscillators emerge, called synchronization states. Therefore, many researchers have proposed different coupled oscillatory networks and have discovered many interesting synchronization phenomena.

Recently, synchronization in complex networks with different types of interactions has been extensively investigated to understand the important role played by the interaction. This is because interactions in networks lead to the emergence of key synchronization phenomena; competitive coupling especially can be observed in real-world networks. In our research group, we have investigated synchronization phenomena in complex networks using electrical oscillators as analogs.

Giron et al. investigated synchronization phenomena in plant communities where usually both facilitation and competition coexist, playing a key role in the structure and organization of these communities. They showed that synchronization provides an efficient way to unveil how species sharing facilitative interactions group themselves in a plant community. Furthermore, with regard to hierarchical leadership, the leadership structure of the collective motion in pigeon flocks was investigated using a mathematical model. Leadership mechanisms in complex networks help in contributing to the design of control strategies and algorithms for emerging distributed technologies.
As mentioned in the above two references, in many cases concerning synchronization in complex networks, the nodes in the complex network are expressed by a mathematical model that incorporates some form of competitive interaction. Although it is very important having a mathematical model of complex networks to understand the synchronization states, we also need to consider physical models for future engineering applications such as cyber-physical systems. A cyber-physical system aims to realize a more advanced society by analyzing huge data sets in a physical world with a cyber system. While huge, the data are also being continually added to and updated. However, collecting data does not make sense on its own. Analytical tools are required to analyze and interpret the vast amounts of data and to extract meaning and value. We have proposed the coupled chaotic circuits networks for modeling social networks. We believe that this approach can be applied to efficient data analysis in the cyber world.

In this study, we focus on synchronization states observed in two networks of chaotic circuits that are coupled hierarchically in multiple directions. We analyze the role of synchronization by changing the competitive coupling strategies. A schematic competitive network is shown in Fig. 1.

As a first simulation, the dependence on the relationship of the coupling strength in each network is investigated. As a second simulation, we also investigate the dependence on network structures. Using computer simulations, we confirm what kinds of coupling strength patterns or network structures are effective when competing with other networks.

There are several engineering applications of the coupled hierarchical networks using chaotic circuits, for example, synchronization problems of complex networks including the Internet, the World Wide Web, and the power grid network. In particular, the results of this study may be useful for controlling sensor networks and collective robots. Developing bioinspired technology such as the strategic reasoning algorithms employed in mobile animal groups with a hierarchical social structure is also one of the applications targeted following this study.

The paper is organized as follows. In Sec. II, we explain the competitive network model of two coupled hierarchical networks. In Sec. III, we investigate synchronization in this network using computer simulations and circuit experiments. Finally, a summary and future work are presented in Sec. IV.

II. NETWORK MODEL

Figure 2 shows the hierarchical network model selected for this study. The chaotic circuits located in each layer are named after a chess piece.

Figure 3 shows the proposed network model called a competitive interaction network. There are two hierarchical networks (labeled Group-A and Group-B), each having connections with multiple directions. Each network consists of 11 chaotic circuits (nodes) with adjacent circuits being coupled by resistors. The coupling strengths in the group and between the two groups are summarized in Table I.

We next define the competitive coupling strategy between the two hierarchical networks. The lowest five chaotic circuits (A7–A11) in Group-A and (B7–B11) in Group-B attack those they confront directly. We call this competitive coupling the "distributed attack."

In the proposed network, each node represents a chaotic circuit. Figure 4(a) shows a diagram of a chaotic circuit, which describes the three-dimensional autonomous circuit proposed by Mori and Shinriki. This circuit is composed of an inductor, a negative...
TABLE I. Definition of coupling strength.

<table>
<thead>
<tr>
<th>Group</th>
<th>Place</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>In Group-A</td>
<td>From higher to lower layer</td>
<td>$\delta_{a\rightarrow up}$</td>
</tr>
<tr>
<td></td>
<td>From lower to higher layer</td>
<td>$\delta_{a\rightarrow down}$</td>
</tr>
<tr>
<td></td>
<td>Same layer</td>
<td>$\delta_{a\rightarrow equal}$</td>
</tr>
<tr>
<td>In Group-B</td>
<td>From higher to lower layer</td>
<td>$\delta_{b\rightarrow up}$</td>
</tr>
<tr>
<td></td>
<td>From lower to higher layer</td>
<td>$\delta_{b\rightarrow down}$</td>
</tr>
<tr>
<td></td>
<td>Same layer</td>
<td>$\delta_{b\rightarrow equal}$</td>
</tr>
<tr>
<td>Between</td>
<td>From Group-A to Group-B</td>
<td>$\delta_{ab}$</td>
</tr>
<tr>
<td>Group-A and Group-B</td>
<td>From Group-B to Group-A</td>
<td>$\delta_{ba}$</td>
</tr>
</tbody>
</table>

resistance, two condensers, and dual-directional diodes. In Fig. 4(b), two chaotic circuits are coupled through a bidirectional connector, each involving a resistor $R$ and a voltage follower.

Figures 5 and 6 show the chaotic attractor that is generated by the circuit; the former was obtained from a computer simulation and the latter obtained in experimental measurements of the equivalent electrical circuit. In the computer simulations, we set the parameters of the normalized circuit equations [see Eq. (2) below] to be $\alpha = 0.4$, $\beta = 20.0$, and $\gamma = 0.5$. In the circuit experiment, the parameters were fixed with $L = 20 \text{ mH}$, $C_1 = 100 \text{ nF}$, $C_2 = 40 \text{ nF}$, and $g = 1 \text{ mS}$. These parameters are used in the computer simulations and circuit experiments to be described below.

These two different attractors (upper/lower attractors) can be produced by setting different initial conditions. Specifically, when the value of $v_1$ is positive, the upper chaotic attractor is generated, whereas, when the value of $v_2$ is negative, the lower chaotic attractor is obtained. For the following simulations, the attractor of Fig. 5(a) is used for Group-A, and the attractor of Fig. 5(b) is used for Group-B.

Next, we develop the expression for the circuit equations of the circuit model (Fig. 4). The $I$–$V$ characteristics of the nonlinear resistor are approximated by the following three-segment piecewise-linear functions:

$$i_n = \begin{cases} 
G_d(v_1 - v_2 - V) & (v_1 - v_2 > V), \\
0 & (|v_1 - v_2| \leq V), \\
G_d(v_1 - v_2 + V) & (v_1 - v_2 < -V).
\end{cases}$$  \hspace{1cm} (1)

The normalized circuit equations governing the circuit take the form

$$\begin{align*}
\frac{dx_n}{d\tau} &= z_n, \\
\frac{dy_n}{d\tau} &= \alpha y_n - \alpha f(y_n - z_n) - \alpha \delta \sum_{k \in S_n} (y_n - y_k), \\
\frac{dz_n}{d\tau} &= f(y_n - z_n) - x_n,
\end{align*}$$  \hspace{1cm} (2)

where

$$t = \sqrt{LC_2}\tau, \quad i_n = \sqrt{\frac{C_2}{L}}Vx_n,$$

$$v_{1n} = Vv_n, \quad v_{2n} = Vz_n.$$

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G_d(v_1 - v_2 + V) & (v_1 - v_2 < -V).
\end{cases}$$  \hspace{1cm} (1)

The normalized circuit equations governing the circuit take the form

$$\begin{align*}
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\frac{dy_n}{d\tau} &= \alpha y_n - \alpha f(y_n - z_n) - \alpha \delta \sum_{k \in S_n} (y_n - y_k), \\
\frac{dz_n}{d\tau} &= f(y_n - z_n) - x_n,
\end{align*}$$  \hspace{1cm} (2)

where

$$t = \sqrt{LC_2}\tau, \quad i_n = \sqrt{\frac{C_2}{L}}Vx_n,$$

$$v_{1n} = Vv_n, \quad v_{2n} = Vz_n.$$
TABLE II. Settings of the coupling strengths.

<table>
<thead>
<tr>
<th>Group</th>
<th>Coupling strength</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>In Group-A</td>
<td>$\delta_{a-up}$</td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td>$\delta_{a-down}$</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>$\delta_{a-equal}$</td>
<td>0.01</td>
</tr>
<tr>
<td>In Group-B</td>
<td>$\delta_{b-up}$</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>$\delta_{b-down}$</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>$\delta_{b-equal}$</td>
<td>0.03</td>
</tr>
<tr>
<td>Between</td>
<td>$\delta_{ab}$</td>
<td>Variable</td>
</tr>
<tr>
<td>Group-A and Group-B</td>
<td>$\delta_{ba}$</td>
<td>Variable</td>
</tr>
</tbody>
</table>

and written as

$$f(y_n - z_n) = \begin{cases} 
\beta(y_n - z_n - 1) & (y_n - z_n > 1), \\
0 & (|y_n - z_n| \leq 1), \\
\beta(y_n - z_n + 1) & (y_n - z_n < -1). 
\end{cases} \tag{3}$$

For the computer simulations, we solve Eq. (2) using the fourth-order Runge–Kutta method with a step size of $h = 0.01$. The software used in this simulation is the standard C-program using the GCC 4.8.4 compiler in Ubuntu 14.04.

III. SYNCHRONIZATION PHENOMENA

A. Dependence on the relationship of the coupling strengths

As a first simulation, we focus on the relationship of the various coupling strengths using the same network structure. The definition of winning the battle is that the attractor of the top (first) layer changes. Table II summarizes the setting of the coupling strength values. In Group-A, the coupling strength from the higher layer to the lower layer has a larger value than the coupling strength in the other direction. In Group-B, however, the coupling strengths in both directions have the same value. We investigate the synchronization state by changing the coupling strengths between the two groups.

Figure 7 shows the obtained attractors from the computer simulations when the coupling strengths with $\delta_{ab} = \delta_{ba}$ are successively set to 0.1 and 0.3. The amplitude of the chaotic attractor located in the lowest layer becomes smaller than the original attractor. When $\delta_{ab} = \delta_{ba} = 0.3$, the five chaotic circuits located in the lowest layer

![Figure 7](image_url)

FIG. 7. Attractors when synchronization of inner group is weak: (a) $\delta_{ab} = \delta_{ba} = 0.1$ and (b) $\delta_{ab} = \delta_{ba} = 0.3$.

![Figure 8](image_url)

FIG. 8. Phase differences when the synchronization of the inner group is weak ($\delta_{ab} = \delta_{ba} = 0.3$).
transition to the upper attractor [Fig. 7(b)]. Essentially, Group-A cannot cover the whole network of Group-B. The observed phenomenon does not depend on the initial conditions of the coupled circuits.

Figure 8 shows the phase difference in each network when the coupling strengths with $\delta_{ab} = \delta_{ba}$ are set to 0.3. We find that the chaotic circuits of Group-A do not synchronize with the others, whereas in Group-B, the five chaotic circuits located in the first, second, and third layers are synchronized. We consider that the synchronization state of the network plays an important role in winning the battle.

In the next simulation, we set the coupling strength in the networks to large values. Table III summarizes the settings of coupling strengths. In Group-A, the coupling strength from a higher layer to a lower layer is larger in value than coupling strength in the other direction. In Group-B, the couplings in both directions have the same value. We investigate the synchronization state by changing the coupling strengths between the two groups.

Figure 9 shows the attractors obtained from the computer simulations. When the coupling strengths with $\delta_{ab} = \delta_{ba}$ are set to 0.1, the amplitude of the chaotic attractor located in the lowest layer becomes smaller than the original attractor. With $\delta_{ab} = \delta_{ba} = 0.3$, all chaotic

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**Table III.** Setting of coupling strength.

<table>
<thead>
<tr>
<th>Group</th>
<th>Coupling strength</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>In Group-A</td>
<td>$\delta_{a\text{-}up}$</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>$\delta_{a\text{-}down}$</td>
<td>0.1</td>
</tr>
<tr>
<td></td>
<td>$\delta_{a\text{-}equal}$</td>
<td>0.3</td>
</tr>
<tr>
<td>In Group-B</td>
<td>$\delta_{b\text{-}up}$</td>
<td>0.3</td>
</tr>
<tr>
<td></td>
<td>$\delta_{b\text{-}down}$</td>
<td>0.3</td>
</tr>
<tr>
<td></td>
<td>$\delta_{b\text{-}equal}$</td>
<td>0.3</td>
</tr>
<tr>
<td>Between Group-A and Group-B</td>
<td>$\delta_{ab}$</td>
<td>Variability</td>
</tr>
<tr>
<td></td>
<td>$\delta_{ba}$</td>
<td>Variability</td>
</tr>
</tbody>
</table>
circuits transition to the upper attractor [Fig. 9(b)]; that is, Group-A wins against Group-B. We confirm that if the network structures of the two groups are the same, the coupling strengths from higher to lower layers need to have larger values than those in the other direction to win the battle.

Figure 10 shows the phase differences in each network when the coupling strengths with \( \delta_{ab} = \delta_{ba} \) are set to 0.3. In this instance, all chaotic circuits are synchronized in each group. If we compare the synchronization of the five chaotic circuits located in the lowest layer, Group-B is stronger than Group-A.

Next, the coupling strengths between the two hierarchical networks are changed systematically. A new parameter \( \beta \) is introduced to calculate the coupling strengths of \( \delta_{a\text{-up}}, \delta_{b\text{-down}}, \) and \( \delta_{b\text{-up}}. \) For the calculations, first, the value of the coupling strength \( \delta_{a\text{-down}} \) is fixed manually; thereafter, the other coupling strengths are determined from

\[
\begin{align*}
\delta_{a\text{-up}} &= \delta_{a\text{-down}} / \beta, \\
\delta_{b\text{-up}} &= \delta_{b\text{-down}} = \frac{1}{2} (\delta_{a\text{-up}} + \delta_{a\text{-down}}), \\
\delta_{b\text{-up}} &= \delta_{b\text{-equal}} = \delta_{b\text{-equal}},
\end{align*}
\]

where \( \beta \) is varied from 2.0 to 10.0 in steps of 1.0. The simulation result is shown in Fig. 11; the horizontal axis corresponds to \( \beta \) and the vertical axis gives the coupling strength between the two groups when Group-A wins. Note that Group-A wins with a small coupling strength between the two networks when \( \beta \) takes a large value for any instance.

Next, we set \( \tau = 10000 \) and with step intervals of 0.01, calculate the synchronization ratio of each connection using the inequality condition

\[
|y_n - y_k| < 0.1 (k \in S_n), \quad (4)
\]

After calculating the synchronization ratio of all connections, the average synchronization of each network is determined.

Figure 12 shows the synchronization ratio in each group by changing the coupling strengths \( \delta_{ab} \) and \( \delta_{ba}. \) Clearly, when the value of \( \delta_{ab} \) is small, the synchronization of Group-A is higher than that of Group-B. However, after Group-A wins, the synchronization of Group-B is higher than that of Group-A.

FIG. 15. Changing attractors (circuit experiment).

FIG. 16. Winning or losing results (Group-B: 1-layer skip connection).
B. Dependence on the network structure

Next, we consider the dependence on the network structure. In this simulation, the coupling strengths are the same in the two groups. The network structures of the two groups have different connection patterns. Specifically, one connection is rewired in Group-B. We introduce two different rewiring connections.

Figure 13 illustrates a competitive interaction network for which Group-B has a connection between the top layer and third layer (a 1-layer skip connection). A typical result of the simulations is shown in Fig. 14. When the coupling strength is small, the circuits in Group-A display the upper chaotic attractor, whereas the circuits in Group-B display the lower chaotic attractor. By increasing the coupling strength, the circuits located in the lower layers are damaged by the effect of the other network. In the end, all the circuits generate the lower chaotic attractor, which is to say, Group-B with the 1-layer skip connection wins the battle.

The attractors in the circuit experiment also change (Fig. 15). The circuit experiment parameters were set to the same values as in the earlier experiment described in Sec. II (Fig. 6). The coupling resistance in the network was fixed at $R = 12.0 \, \text{k}\Omega$, and the coupling resistance $R_{AB}$ between the networks was changed. From this figure, we confirm that the obtained results of the circuits experiment match well with the simulation results at least qualitatively.

A summary of the simulation results for the competitive interaction network, Fig. 13, is shown in Fig. 16. When the coupling strength from a higher layer to a lower layer is larger in value than that in the other direction, neither network cannot win. For $\delta_{\text{up}} = 0.34$ and $0.32$, the winning group depends on the coupling strengths between the two groups. When the coupling strengths $\delta_{ab}$ and $\delta_{ba}$ are small, we obtain a draw. By increasing $\delta_{ab}$ and $\delta_{ba}$, we confirm that in a certain range Group-A wins, but thereafter Group-B wins with large values of $\delta_{ab}$ and $\delta_{ba}$.

Next, Fig. 17 shows the competitive interaction network for which Group-B has a connection between the top layer and bottom layer (a 2-layer skip connection). For the simulations, the coupling strength in each group and between groups is changed. We focus on which network wins.

Similarly, a summary of the simulation results of the competitive interaction network (Fig. 17) is shown in Fig. 18. In this instance, we obtained only two results: a “draw” and a “Group-B win.” We did not observe instances in which Group-A wins. From these results, we confirm that the 1-layer skip connection in the network structure plays an important role in winning the battle.

IV. CONCLUSIONS

We studied the competitive network model using chaotic circuits. Each network is composed of 11 chaotic circuits coupled using resistors into four layers in a hierarchical structure. First, the dependence on the relative coupling strengths in each network was investigated. We observed that the network with the higher coupling strength from the upper to the bottom layers dominate another network. Second, we investigated the influence of the network structure when a 1-layer and a 2-layer skip connection were inserted. We confirmed that these skip connections in the network structure play an important role in winning the battle. We also confirmed using equivalent circuit experiments similar synchronization states appearing in the 1-layer skip connection model.

In future work, we shall investigate the effect of competitive strategies and apply this model to more complex networks such as...
smart grid networks and social networks. Applying various theoretical/mathematical analyses to the proposed models to understand the mechanism underlying the obtained phenomena is also an important problem for future work.

REFERENCES