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Synchronization in Two Rings of Three Coupled van der Pol Oscillators

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Abstract

In this study, we investigate the synchronization phenomena in two rings of van der Pol oscillators coupled by resistors. We propose a novel coupled oscillatory system comprising two rings of van der Pol oscillators with different coupling schema. We focus on the coupling strengths of the coupled van der Pol oscillators. By computer simulation, we investigate how the synchronization phenomena change by changing the coupling strengths. In these results, we observe various synchronization phenomena.

1. Introduction

The synchronization phenomena of coupled oscillators are familiar. Synchronization phenomena have been studied in various fields for many years, such as in electrical systems, mechanical systems and biological systems. Among them, the synchronization phenomena of van der Pol oscillators are similar to natural phenomena when the frequency is changed. A coupled system of van der Pol oscillators is simple and easy to handle. Many researchers have proposed various coupled oscillatory networks of van der Pol oscillators [1]- [3]. We focus on the coupling strengths of coupled oscillatory networks consisting of two van der Pol oscillators.

The van der Pol oscillator is a simple circuit. It consists of a resistor, inductor, capacitor and nonlinear resistor. It was invented by the electrical engineer Balthasar van der Pol. The equation of a van der Pol oscillator is a second-order differential equation.

In this study, we propose a novel coupled oscillatory system comprising two rings of van der Pol oscillators coupled by resistors. The first ring consists of three van der Pol oscillators connected by resistors. The second ring consists of three van der Pol oscillators connected by inductors and resistors. By computer simulation, we investigate the synchronization phenomena observed in the proposed circuit system by changing the coupling strengths.

2. System Model

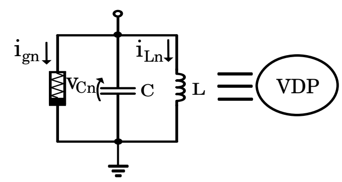

Figure 1: Circuit of van der Pol oscillator

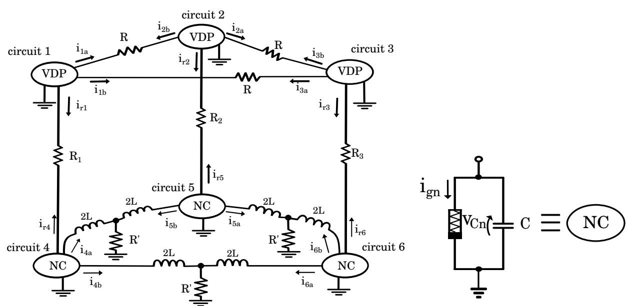

Figure 2: Circuit model

Figure 1 shows the circuit of a van der Pol oscillator. We call this circuit VDP. We use six van der Pol oscillators in this study. Figure 2 shows a model of the system with six van der Pol oscillators. We use two ring circuits of van der Pol oscillators. The three VDP of the first ring are connected by resistors. The three NC of the second ring are connected by inductors and resistors. When the two rings are not connected, the oscillators in the first ring exhibit in-phase synchronization and oscillators of the second ring exhibit three-phase synchronization. The first and second rings are connected by resistors (R_1, R_2, R_3). We observe the synchronization phenomena of adjacent oscillators. We investigate how the synchronization phenomena change upon changing the values of R_1, R_2 and R_3.

The circuit equations of the first ring are given as follows:

$$\begin{cases} -i_{gn} - i_{cn} - i_{Ln} &= i_{an} + i_{bn} + i_{rn} \\ L\frac{di_{Ln}}{dt} &= v_n \\ C\frac{dv_{Cn}}{dt} &= I_{Cn} \\ v_n - v_i &= (i_{an} - i_{b(i)})R \\ v_n - v_j &= (i_{bn} - i_{a(j)})R \\ v_n - v_{n+3} &= (i_{rn} - i_{r(n+3)})R_3 \end{cases} \quad (1)$$

The circuit equations of the second ring are given as follows:

$$\begin{cases} -i_{gn} - C\frac{dv_{Cn}}{dt} &= i_{an} + i_{bn} + i_{rn} \\ v_n - 2L\frac{di_{an}}{dt} &= -(i_{an} + i_{b(i)})R' \\ v_n - 2L\frac{di_{bn}}{dt} &= -(i_{bn} + i_{a(j)})R' \\ v_n - v_{n-3} &= (i_{rn} - i_{r(n-3)})R_3 \end{cases} \quad (2)$$

where n, i and j is denotes the number of the circuit ($n = 1,2,3,4,5,6$, i $i = 2,3,1,5,6,4$ and j $j = 3,1,2,6,4,5$.)

A nonlinear resistor defined as follows:

$$i_{gn} = -g_1 v_n + g_3 v_n^3 \quad (3)$$

By changing the variables and parameters as follows.

$$\begin{aligned} & t = \sqrt{LC}\tau, v_n = \sqrt{\frac{g_1}{3g_3}}x_n, i_n = \sqrt{\frac{g_1 C}{3g_3 L}}y_n, \\ & \varepsilon = g_1\sqrt{\frac{C}{L}}, \alpha = \frac{1}{R}\sqrt{\frac{L}{C}}, \\ & \beta = R'\sqrt{\frac{C}{L}}, \gamma_n = \frac{1}{R_n}\sqrt{\frac{L}{C}} \end{aligned} \quad (4)$$

the normalized equations of the first ring are given as:

$$\begin{cases} \dot{x}_n &= \varepsilon(x_n - x_n^3) - y_n - \gamma_n(x_n - x_{n+3}) \\ & \quad +\alpha(-x_n + x_i + x_j) \\ \dot{y}_n &= x_n. \\ & \qquad (n = 1,2,3) \end{cases} \quad (5)$$

and the normalized equations of the second ring are given as:

$$\begin{cases} \dot{x}_n &= \varepsilon(x_n - x_n^3) - y_{an} \\ & \quad -y_{bn} + \gamma_{n-3}(x_n - x_{n-3}) \\ \dot{y}_{an} &= x_n - \beta(y_{an} + y_{b(i)}) \\ \dot{y}_{bn} &= x_n - \beta(y_{bn} + y_{a(j)}) \\ & \qquad (n = 4,5,6) \end{cases} \quad (6)$$

where n denotes the number of the VDP and NC ($n = 1,2,3,4,5,6$). The parameters ε is non-linear strength. The parameters α, β and γ denote the coupling strengths of the resistor R, resistor R' and resistor R_n, respectively.

3. Simulation Results

The simulation results of the system model are shown from Figs.3-6. The parameters are set to $\varepsilon = 0.05$, $\alpha = 0.05$ and $\beta = 0.05$.

In the case of $\gamma_1 = \gamma_2 = \gamma_3 = 0.02$, we conducted the simulation using different initial values. In Fig.3, synchronization phenomenon is observed in circuits 1-4. However, in Fig.4, synchronization phenomenon is observed in circuit 2-5.

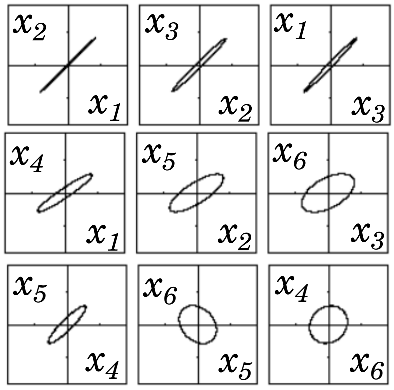

Figure 3: Phase difference ($\gamma_1 = \gamma_2 = \gamma_3 = 0.02$)

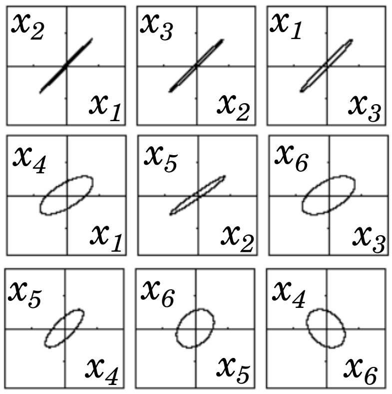

Figure 4: Phase difference ($\gamma_1 = \gamma_2 = \gamma_3 = 0.02$)

By changing γ_1, γ_2 and γ_3, we can control the synchronization phenomena regardless of initial value.

In the case of $\gamma_1 = 0.001, \gamma_2 = 0.001$ and $\gamma_3 = 0.02$, in-phase synchronization phenomenon is observed in the oscillators of the first ring and three-phase synchronization phenomenon is observed in the oscillators of the second ring. Hear, when we increase γ_3, the three oscillators of the first ring exhibit in-phase synchronization and the oscillators of the second ring exhibit three-phase synchronization.

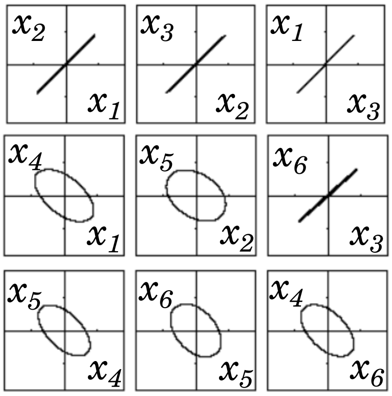

Figure 5: Phase difference ($\gamma_1 = 0.001, \gamma_2 = 0.001, \gamma_3 = 0.02$)

The time waveform of the voltage of each NC after sufficient time has elapsed is shown in Fig.6. This result shows that the three oscillators of the second ring exhibit three-phase synchronization.

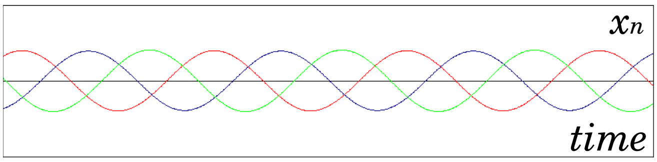

Figure 6: Time waveform of the voltage of each NC

In the case of $\gamma_1 = 0.02, \gamma_2 = 0.005$ and $\gamma_3 = 0.02$, synchronization phenomenon is observed in circuit 4-6 regardless of the initial value. Here, when we increase two of γ_1, γ_2 and γ_3, the three oscillators of the first ring and two of the three oscillators of the second ring exhibit synchronization.

We investigate various synchronization phenomena by changing the coupling strengths. Here, when we require two oscillators of the second ring to synchronize, we increase two of γ_1, γ_2 and γ_3. When we require three oscillators of the second ring to three-phase synchronization, we increase two of γ_1, γ_2 and γ_3. Therefore, we can control the synchronization phenomena via the coupling strengths.

Next, we define the synchronization condition by the following equation:

$$|x_a - x_b| < 0.05 \tag{7}$$

we denote the synchronization rate. We define the synchronization rate (P) by the following equation:

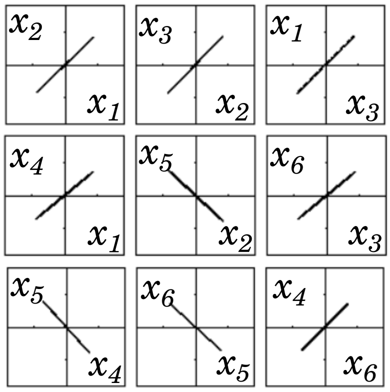

Figure 7: Phase difference ($\gamma_1 = 0.02, \gamma_2 = 0.005, \gamma_3 = 0.02$)

$$P = \frac{N}{\text{Number of trials}} \times 100[\%] \tag{8}$$

where the number of trials is 1,000,000. N denotes the number of oscillator being synchronized.

We change the value of γ_2 from 0 to 0.03 at intervals of 0.001. We fix the other coupling strengths and the initial value. Figure 8 shows the synchronization rate of the first ring. Figure 9 shows the synchronization rate of the oscillators of the first ring with these of the second ring. Figure 10 shows the synchronization rate of the second ring.

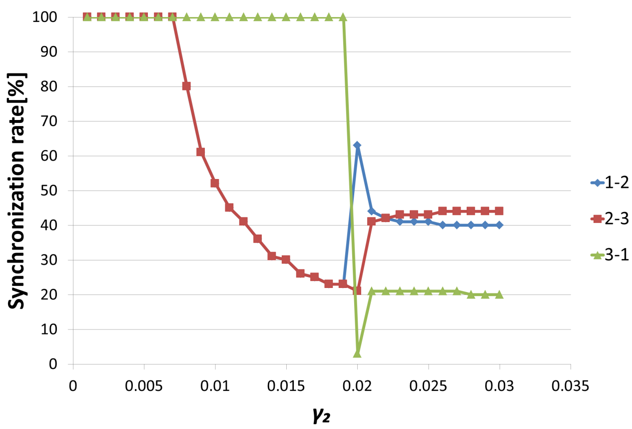

Figure 8: Synchronization rate (first ring)

In Fig.8, when γ_2 is from 0 to 0.007, circuit 1 - circuit 2, circuit 2 - circuit 3 and circuit 3 - circuit 1 exhibit in-phase synchronization. When γ_2 is 0.008, the synchronization rates of circuit 1 - circuit 2 and circuit 2 - circuit 3 begin to decrease. When γ_2 is 0.02, the synchronization rate of circuit 1 - circuit 2 decreases sharply. When γ_2 is from 0.021 to 0.03, the synchronization rates of circuit 1 - circuit 2, circuit 2 - circuit 3 and circuit 3 - circuit 1 reach steady-state value.

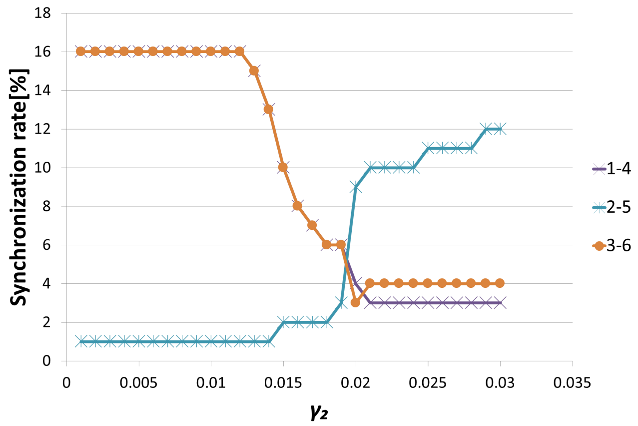

Figure 9: Synchronization rate (second ring)

In Fig.9, when γ_2 is from 0.012, the synchronization rates of circuit 1 - circuit 4 and circuit 3 - circuit 6 begin to decrease. When γ_2 is from 0.015, synchronization rate of circuit 2 - circuit 5 begins to increase. When γ_2 is from 0.021 to 0.03, synchronization rates of circuit 1 - circuit 4 and circuit 3 - circuit 6 reach steady-state values.

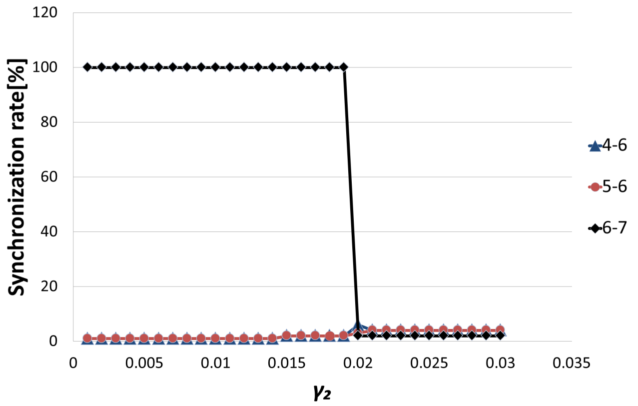

Figure 10: Synchronization rate (between first and second rings)

In Fig.10, when γ_2 is even slightly smaller than γ_1 and γ_3, circuit 4 - circuit 5 exhibits in-phase synchronization. However, when γ_1, γ_2 and γ_3 are equal, circuit 4 - circuit 5 do not exhibit in-phase synchronization. When γ_2 is larger than γ_1 and γ_3, none of the oscillators do exhibit in-phase synchronization.

4. Conclusion

We have proposed a system model using two rings of three van der Pol oscillators coupled by resistors or inductors. We can control the synchronization phenomena by changing the coupling strengths. When the three coupling strengths (γ_1, γ_2, γ_3) are equal, synchronization phenomenon was observed after changing the initial value. However, when we increased one of γ_1, γ_2 and γ_3, the oscillators of the first ring exhibit in-phase synchronization and the oscillators of the second ring exhibited three-phase synchronization. When we increased two of γ_1, γ_2 and γ_3, the three oscillators of the first ring and two oscillators of the second ring exhibited synchronization phenomena. In the future, we will investigate synchronization phenomena using other parameters and analyze the proposed circuit model.

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