

Paper

M-ary modulation scheme based on separation of deterministic chaotic dynamics for noncoherent chaos-based communications

Shintaro Arai^{1a)}, Yoshifumi Nishio^{2b)}, and Takaya Yamazato^{3c)}

¹ Dept. of Communication Network Engineering, Kagawa National College of Technology

551 Kohda, Takuma-cho, Mitoyo, Kagawa 769-1192, Japan

² Dept. of Electrical and Electronic Engineering, Tokushima University

2-1 Minami-Josanjima, Tokushima 770-8506, Japan

³ Institute of Liberal Arts and Science, Nagoya University

Furo-cho, Chikusa-ku, Nagoya 464-8063, Japan

a) arai@cn.kagawa-nct.ac.jp

b) nishio@ee.tokushima-u.ac.jp

c) yamazato@m.ieice.org

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Abstract: The present paper proposes a novel *M*-ary modulation scheme based on separation and reconstruction of deterministic chaotic dynamics for noncoherent chaos-based communications. The *M*-ary modulation scheme can transmit *b*-bit data at a time by using $M = 2^b$ distinct sequences. In order to generate *M* distinct sequences and recover *b*-bit data, the proposed system separates the chaotic dynamics having the chaotic sequence by using *M* interleavers, and reconstructs the original sequence based on the chaotic dynamics from the separated one by using *M* deinterleavers. In advance, each data symbol among *M* symbols is allocated to each interleaver-deinterleaver pair. The transmitter selects an interleaver corresponding to a data symbol and separates the order of samples of the chaotic sequence using the selected interleaver. The receiver feeds the received sequence into all *M* deinterleavers and outputs *M* reconstructed sequences. The proposed system can reconstruct the original chaotic sequence only when the correct deinterleaver, which becomes paired with the interleaver, is selected. Therefore, the receiver can recover *b*-bit data by analyzing the chaotic dynamics of each reconstructed sequence. We carry out computer simulations and evaluate performances of the proposed *M*-ary modulation scheme.

Key Words: chaos, noncoherent chaos-based communications, *M*-ary modulation, deterministic chaotic dynamics

1. Introduction

Communications using chaotic systems are one of interesting topics in the field of engineering chaos [1–24]. Chaotic signals, which are generated in a deterministic rule, have following several properties: non-periodic, random-like, bounded, and wide-band signals, sensitivity to initial conditions, difficulty to predict their future behavior from past observations. During the past few decades, many researchers have focused these properties and have addressed several topics about chaos-based communications: spread-spectrum communication systems and their related modulation schemes [1–9], multiple-access schemes [10–13], chaos-based coding schemes [14–19], their applications [20–22], etc.

In general, chaos-based communication systems are classified into two main categories: *coherent* and *noncoherent* systems. Coherent systems need identical chaotic sequence between the transmitter and receiver for demodulation. In other words, the receiver must reproduce replicas of the basis signals (unmodulated carriers) to recover data. On the other hand, noncoherent systems need not reproduce the basis signals for demodulation at the receiver. Thus, demodulation is performed based on the received signal. Additionally, the noncoherent systems are categorized according to decoding operation: *correlator-type* and *non-correlator-type*. In correlator-type systems, data is demodulated by correlation operation without reproduction of replicas of the basis signals, in whereas in the non-correlator-type systems, data is recovered without correlation operation. Instead of correlation operation, demodulation is performed based on characteristics of chaos having the received sequence. We consider that the noncoherent system is the unique communication system using chaos. Differential chaos shift keying (DCSK) receiver [1] and the optimal receiver [2] are well-known as typical noncoherent correlator-type and non-correlator-type systems, respectively. Moreover, it is also important to develop a suboptimal receiver, which has a performance equivalent to or similar to the optimal receiver, using more efficient algorithms [4, 23]. In this study, we focus on the noncoherent non-correlator-type system as the chaos-based communication. Note that the noncoherent system has different meanings between chaos-based communications and standard ones. The noncoherent system in standard communications means that the receiver recovers data without carrier phase information. Namely, the concept of noncoherent system differs from each other.

Development of the noncoherent chaos-based communication system is essential to differentiate from standard ones. However, the communication property of the noncoherent system is lower as compared with the standard one since the noncoherent system recovers data without the basis signals, as described above. Therefore, we consider that it is important to advance noncoherent chaos-based communication systems for improving the communication property.

In our previous study, we focused on the deterministic chaotic dynamics, which is one of characteristics of chaos, and proposed the error-correcting scheme based on the chaotic dynamics [24]. Our error correction focuses on two successive chaotic sequences generated from the same chaotic map; the second sequence is generated with an initial value that is determined from the last value of the first sequence. Here, we connect the initial value of the second sequence and the last value of the first one, and create a long sequence. In this case, the created sequence is regarded as the successive chaotic sequence having the same chaotic dynamics. This feature gives the receiver additional information to correctly recover data and thus improves the bit error performance. Based on this feature, we designed the error-correcting scheme for noncoherent chaos-based communications. Computer simulations confirmed that the advantage gained in the bit error rate (BER) performance of our error correction is about 1–1.5 dB compared to a conventional method (w/o coding). From the previous results, we concluded that the chaotic dynamics had a great influence on the demodulation of chaos communications.

The key point of our error correction is to apply successive chaotic sequences based on the chaotic dynamics between identical symbols. Here, let us focus on the order of the chaotic sequence of each symbol. As a matter of fact, samples of the chaotic sequence are generated based on the identical chaotic dynamics. Next, let us consider that the order of the samples is separated using a specific rule. In this case, the separated sequence loses the characteristic of the chaotic dynamics. In order to recover the original sequence based on the chaotic dynamics, we need to reconstruct the order of the

samples using the inverse of the rule. We consider that these operations are applied for improving the communication property of the noncoherent chaos-based communications as additional information.

The purpose of this study is to propose and examine a novel M -ary modulation scheme using the separation and reconstruction of chaotic dynamics for noncoherent chaos-based communications. The M -ary modulation scheme can transmit b -bit data at a time by using $M = 2^b$ distinct sequences, i.e., one sequence for each of the 2^b expresses b -bit sequences [25]. This study generates M distinct sequences by separating the chaotic dynamics having the chaotic sequence. In addition, the receiver of this study recovers b -bit data by reconstructing the original sequence based on the chaotic dynamics from the received one. In order to separate and reconstruct the chaotic dynamics, we use an interleaver π_m and deinterleaver π_m^{-1} , respectively ($m : 0, 1, \dots, M - 1$). In the proposed M -ary modulation, we prepare M interleavers and M deinterleavers for the proposed transmitter and receiver. In advance, each data symbol among M symbols is allocated to each interleaver-deinterleaver pair. The proposed transmitter selects an interleaver corresponding to a data symbol and separates the order of samples of the chaotic sequence using the selected interleaver. The proposed receiver feeds a received sequence into all M deinterleavers and outputs M reconstructed sequences. In the proposed system, we can reconstruct the original chaotic sequence only when the correct deinterleaver, which becomes paired with the interleaver, is selected. Thus, the receiver can recover b -bit data by analyzing the chaotic dynamics of each reconstructed sequence. We carry out computer simulations and evaluate performances of the proposed M -ary modulation scheme.

Note that the present paper does not compare the communication performance between the proposed M -ary modulation scheme and the traditional M -ary one. As one of traditional M -ary communication schemes, there is the M -ary spread-spectrum (M -ary/SS) communication system [26]. The M -ary/SS system is known as the noncoherent communication system. However, the concept of this noncoherent system differs from chaos-based one, as described above. In addition, this system performs correlation operation using shared spreading sequences between the transmitter and receiver. Thus, it is difficult to easily compare the performance of our scheme to the traditional one due to different concepts and constraint conditions. The present paper puts emphasis on comparison with conventional noncoherent chaos-based communication and evaluation of the communication property of our scheme.

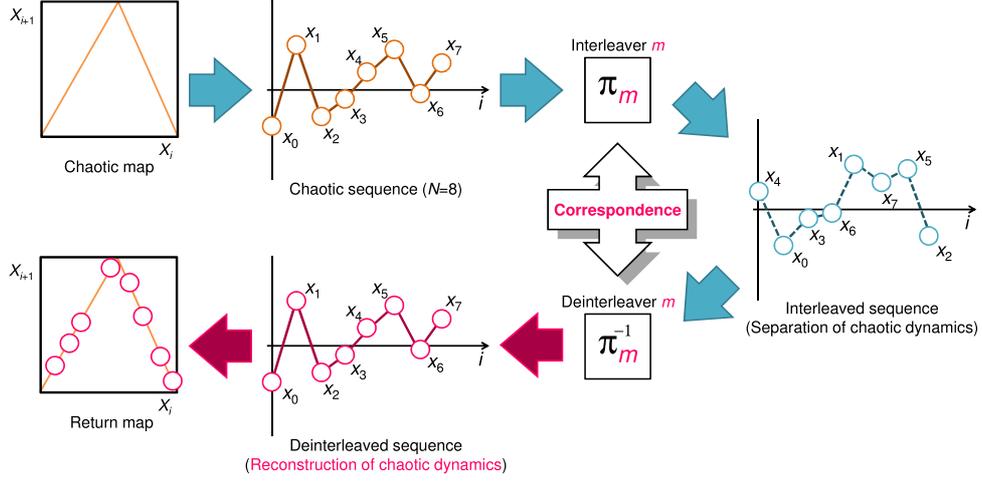
The present paper is organized as follows. In Section 2, we introduce a concept of our proposed M -ary modulation scheme. The proposed method is presented in Section 3, and its performance is evaluated in Section 4. Finally, we conclude the present paper in Section 5.

2. Concept of the proposed M -ary modulation scheme

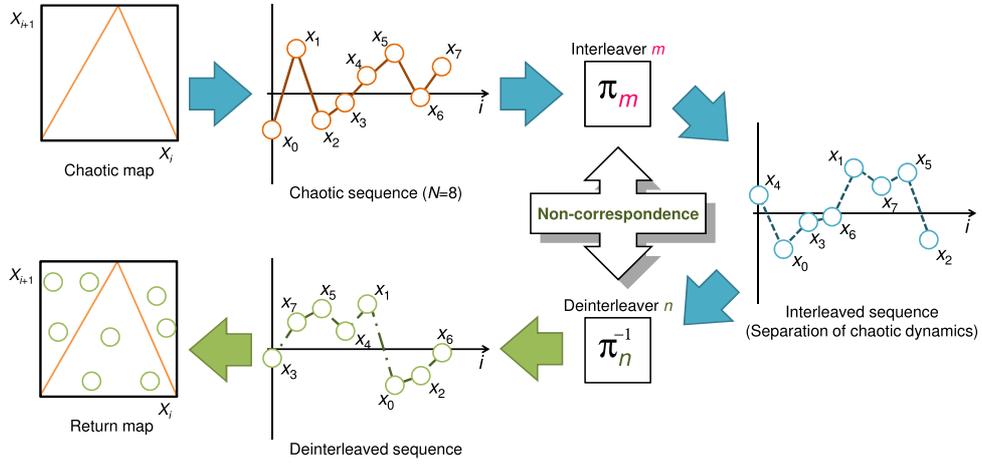
This section introduces a concept of the proposed M -ary modulation scheme using Fig. 1. This figure shows the separation and reconstruction of the chaotic sequence. In this figure, x_i denotes a sample of the chaotic sequence. As described in Section 1, the proposed scheme applies the chaotic dynamics and its separation and reconstruction using the interleaver π_m and deinterleaver π_m^{-1} . Note that a sequence separated by m -th interleaver can be reconstructed to the original sequence by m -th deinterleaver.

Here, let us consider two different situations: *correct* or *incorrect* pair of the interleaver and deinterleaver. In the situation of the correct pair (see Fig. 1(a)), the order of the deinterleaved sequence corresponds to the original chaotic sequence. When we draw a return map using the deinterleaved sequence, all (x_k, x_{k+1}) points of the sequence overlap to lines of the original chaotic map. On the other hand, in the situation of the incorrect pair, almost points of the sequence do not overlap to the original chaotic map since the incorrect deinterleaver cannot reconstruct the original chaotic sequence, as shown in Fig. 1(b). Thus, we can distinguish whether the sequence based on the chaotic dynamics or not by analyzing the chaotic property of the reconstructed sequence.

We use this characteristic (i.e., the loss of the chaotic dynamics) as M -ary modulation for noncoherent chaos-based communications. This characteristic gives the receiver additional information to correctly recover data.



(a) Correct pair of the interleaver and deinterleaver.



(b) Incorrect pair of the interleaver and deinterleaver.

Fig. 1. Concept of the proposed M -ary modulation scheme.

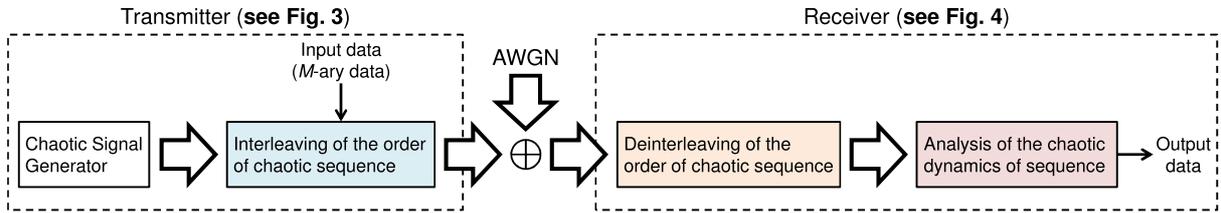


Fig. 2. Block diagram of the proposed M -ary modulation.

3. System model of the proposed M -ary modulation scheme

Figure 2 shows a block diagram of the proposed M -ary modulation scheme. In this scheme, the transmitter and receiver have M interleavers and M deinterleavers, respectively, for separating and reconstructing the chaotic dynamics.

The communication performance of the proposed method depends on the design of interleaver-deinterleaver pairs. Thus, it is important that how to design separation patterns of interleavers. In order to increase the decoding property, we design separation patterns of interleavers with following constraint conditions. We randomly allocate numbers for a separation pattern and set a unique separation pattern with each interleaver-deinterleaver pair. Note that each separation pattern never

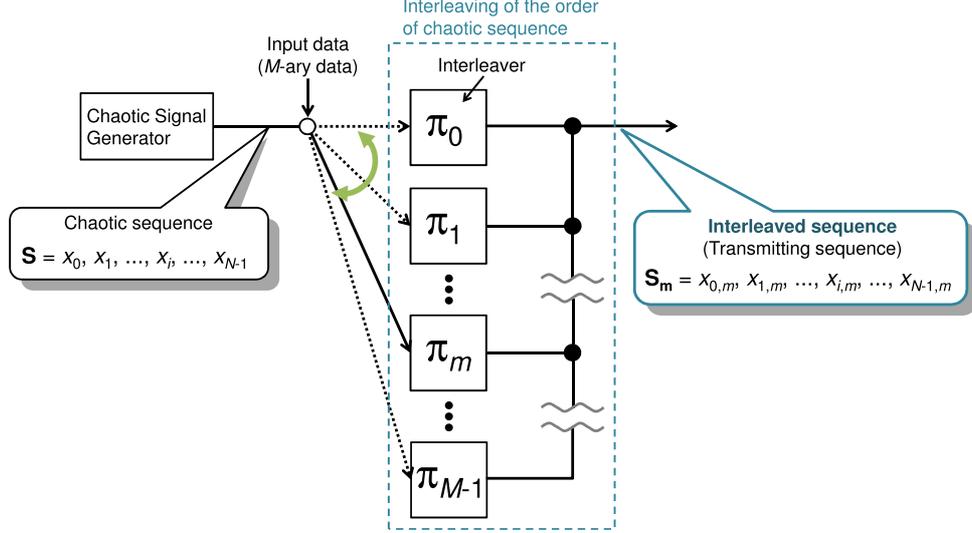


Fig. 3. M -ary modulation using separation of the chaotic dynamics.

matches other patterns except oneself. In addition, it is preferable that even a part of each pattern does not overlap other ones. As an example, we consider following separated sequences when $N = 8$.

$$\begin{aligned}
 \text{Original sequence: } & x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7 \\
 \text{Separated sequence A: } & x_2, x_1, x_4, x_3, x_7, x_6, x_0, x_5 \\
 \text{Separated sequence B: } & \underline{x_2, x_1, x_4, x_3}, x_6, x_5, x_7, x_0
 \end{aligned}$$

As one can see, four samples (x_2, x_1, x_4, x_3) fully overlap between two separated sequences. This case influences the decoding process because a part of the sequence is reconstructed as the original one by the incorrect deinterleaver. Thus, we have to avoid overlaps among separation patterns of interleavers wherever possible. In advance, each data symbol among M symbols is allocated to each interleaver and deinterleaver pair.

3.1 Transmitter (M -ary modulation)

The transmitter consists of a chaotic signal generator and M interleavers, as shown in Fig. 3. A chaotic sequence is first generated by the chaotic signal generator. In this study, we use a skew tent map, which is one of simple chaotic maps, for the generator, and it is described by Eq. (1).

$$x_{i+1} = \begin{cases} \frac{2x_i + 1 - a}{1 + a} & (-1 \leq x_i \leq a), \\ \frac{-2x_i + 1 + a}{1 - a} & (a < x_i \leq 1), \end{cases} \quad (1)$$

where $i = 0, 1, \dots, N - 1$ (N is a chaotic sequence length) and a denotes a position of the top of the skew tent map. Next, an arbitrary interleaver π_m is selected according to an input data m . Finally, the chaotic sequence is fed into the selected π_m , and the order of samples of the sequence is separated according to the separation pattern of π_m for separating of the chaotic dynamics of the sequence. The first-generated-chaotic sequence \mathbf{S} and the interleaved sequence \mathbf{S}_m are given as follows.

$$\mathbf{S} = x_0, x_1, \dots, x_i, \dots, x_{N-1}, \quad (2)$$

$$\mathbf{S}_m = x_{0,m}, x_{1,m}, \dots, x_{i,m}, \dots, x_{N-1,m}, \quad (3)$$

where x_0 denotes an initial value that is selected at random, $x_{i,m}$ denotes a signal sample interleaved by π_m . Thus, \mathbf{S}_m is outputted as the transmitting sequence.

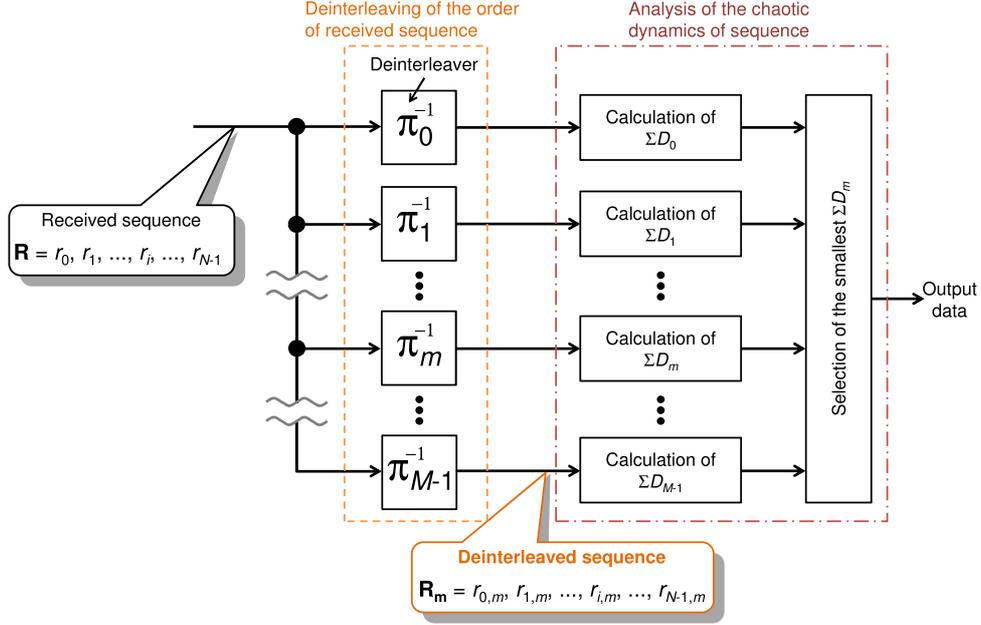


Fig. 4. M -ary demodulation using reconstruction of the chaotic dynamics.

3.2 Channel and noise

The channel distorts the signal and corrupts it by noise. In this study, noise of the channel is assumed to be the additive white Gaussian noise (AWGN). Thus, the received sequence is given by $\mathbf{R} = (r_0, r_1, \dots, r_{N-1}) = \mathbf{S}_m + \text{AWGN}$.

3.3 Receiver (M -ary demodulation)

The receiver consists of M deinterleavers and an analysis unit of the chaotic dynamics of the sequence (second unit), as shown in Fig. 4. Since we consider the noncoherent system, the receiver memorizes the chaotic map used at the transmitter. However, the receiver never knows the initial value of the chaotic sequence generated at the transmitter. In other words, the receiver cannot reproduce replicas of the basis signals.

The received sequence \mathbf{R} is first fed into all M deinterleavers, and the order of the sequence is separated by each deinterleaver π_m^{-1} . The deinterleaved sequence \mathbf{R}_m is given as follows.

$$\mathbf{R}_m = (r_{0,m}, r_{1,m}, \dots, r_{i,m}, \dots, r_{N-1,m}), \quad (4)$$

where $r_{i,m}$ denotes a sample of the sequence deinterleaved by π_m^{-1} .

Each deinterleaved sequence \mathbf{R}_m is fed into the second unit, and the chaotic dynamics of each sequence is analyzed. In this study, we apply the suboptimal detection algorithm [23, 24] for analyzing the chaotic dynamics. This algorithm calculates the shortest distance between N_d -successive signal samples of the received sequence and N_d -dimensional space made from N_d successive chaotic signal samples generated by the skew tent map ($N_d : 2, 3, \dots$). Moreover, this algorithm outputs the sum of the distance, which achieves the minimum shortest distance within the set of l , from $i = 0$ to $N - N_d$. Here, we define following two parameters: $\mathbf{R}'_{i,m}$ as the N_d -successive signal samples beginning with i of the sequence deinterleaved by π_m^{-1} , and l as the number of the straight lines in the N_d -dimensional space. These parameters are given as follows.

$$\mathbf{R}'_{i,m} = (r_{i,m}, r_{i+1,m}, \dots, r_{i+N_d-1,m}), \quad (5)$$

$$l = 2^{N_d-1}. \quad (6)$$

For calculating the shortest distance, we find the closest point \mathbf{P}_l between $\mathbf{R}'_{i,m}$ and the l th line using the scalar product of the vector. When both edges of the l th line are defined as \mathbf{P}'_l and \mathbf{P}''_l shown in Fig. 5, the closest point \mathbf{P}_l is calculated by the following equation.

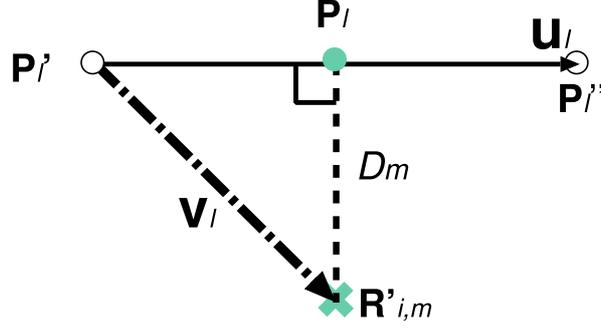


Fig. 5. Calculation of the shortest distance.

$$\mathbf{P}_l = \left(p_0^{(l)}, p_1^{(l)}, \dots, p_{N_d-1}^{(l)} \right) = (\mathbf{u}_l \cdot \mathbf{v}_l) \mathbf{u}_l + \mathbf{P}'_l, \quad (7)$$

where

$$\text{unit vector } \mathbf{u}_l = \frac{\mathbf{P}''_l - \mathbf{P}'_l}{\|\mathbf{P}''_l - \mathbf{P}'_l\|}, \quad (8)$$

$$\mathbf{v}_l = \mathbf{R}'_{i,m} - \mathbf{P}'_l. \quad (9)$$

Then, each outputted distance $\sum D_m$, which is calculated using the sequence deinterleaved by π_m^{-1} , is expressed as

$$\sum D_m = \sum_{i=0}^{N-N_d} \min_l \|\mathbf{P}_l - \mathbf{R}'_{i,m}\|, \quad (10)$$

Next, the receiver selects the smallest $\sum D_m$ from all outputted $\sum D_m$ values and outputs the decoded data. As described above, a separating pattern of each interleaver-deinterleaver pair does not correspond with other pairs. Namely, the single (correct) deinterleaver within the second unit can reconstruct the original chaotic sequence generated at the transmitter. When the successive signal samples based on the chaotic dynamics is used for the calculation of $\sum D_m$, the calculation value becomes the smallest value from other calculations. Finally, the receiver outputs a decoded symbol corresponding to π_m^{-1} , which reconstructs \mathbf{R}_m for the smallest $\sum D_m$.

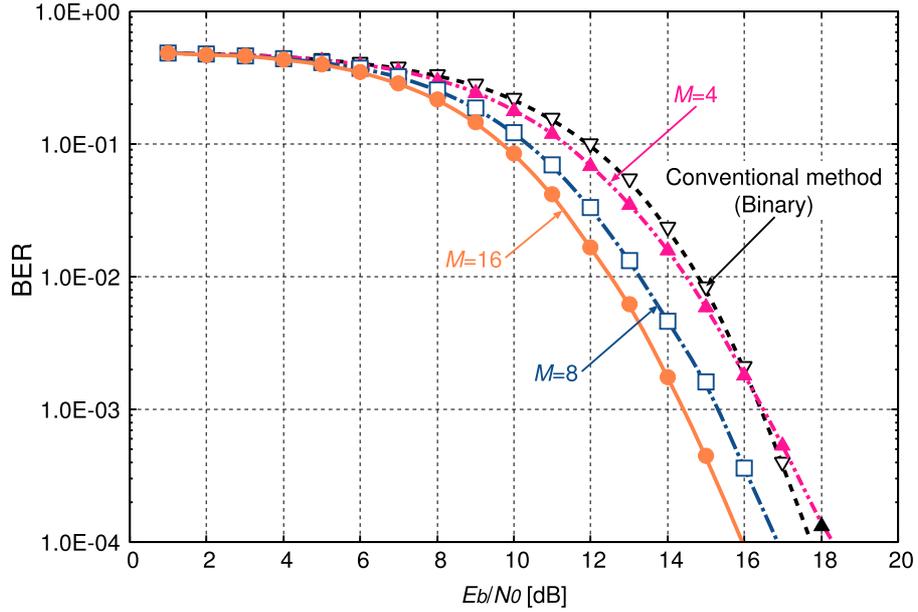
This suboptimal detection algorithm have been proposed based on the optimal receiver [2], which is based on an optimal classifier which optimizes the bit error rate by selecting the symbol that minimizes the *a posteriori* probability. Our suboptimal algorithm approximates the calculation of the probability by calculating shortest distances. As simulation results, we have already confirmed that the performance of our suboptimal algorithm is equivalent to the optimal one [23]. Therefore, it is considered that the calculation of the sum of the shortest distance is suitable for analyzing the chaotic dynamics in our M -ary demodulation scheme although we do not try to use other decision criteria.

4. Evaluation of the proposed M -ary modulation

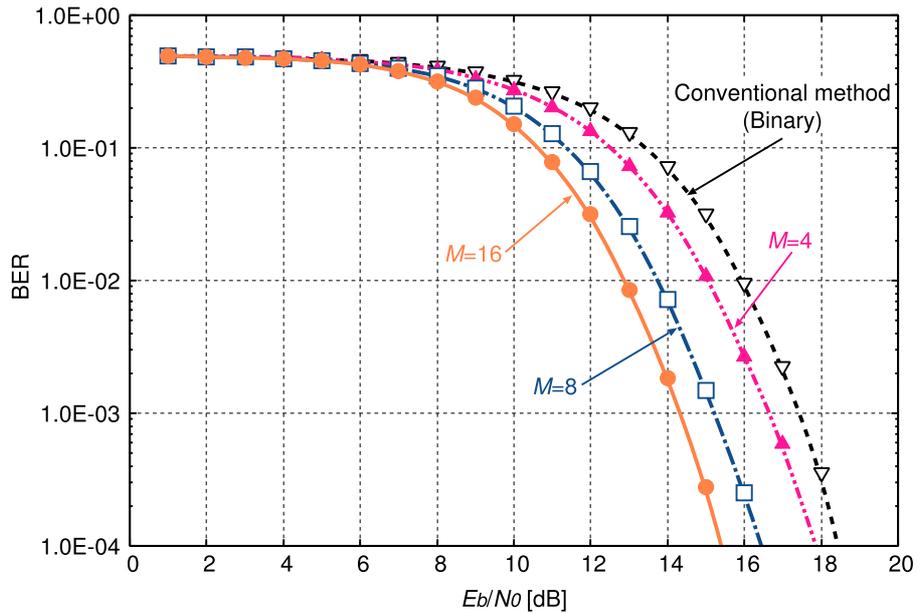
This section explains the computer simulation of the proposed M -ary modulation and evaluates its performances. Experimental conditions are as follows. The parameter of the skew tent map is fixed as $a = 0.05$. The initial value of the chaotic sequence is set at random every transmitting data.

We assume following parameters: $b = 2, 3$ and 4 , $M = 4, 8$ and 16 , and $N = 4-256$. For calculation of the shortest distance, we use 4-dimensional space ($N_d = 4$). Based on these conditions, we iterate the simulation 10,000 times and calculate BER performance.

Figures 6(a) and (b) show the BERs versus E_b/N_0 (SNR per bit) for $N = 32$ and $N = 64$, respectively. We plot the performance of the proposed scheme with $M = 4, 8$ and 16 and the performance of the suboptimal receiver as the binary noncoherent chaos-based communication (i.e., conventional method). From both figures, we can observe that the BER performance of the proposed scheme improves with increasing the number of M . Let us focus on $\text{BER} = 1 \times 10^{-4}$ of both figures. To achieve this BER performance, the required E_b/N_0 of the binary noncoherent system is about 18 dB.



(a) $N=32$.

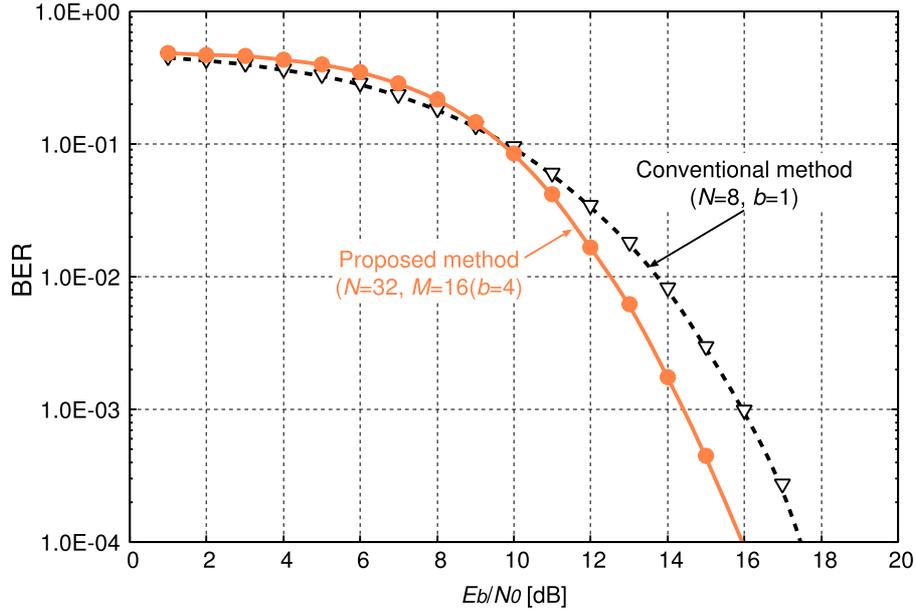


(b) $N=64$.

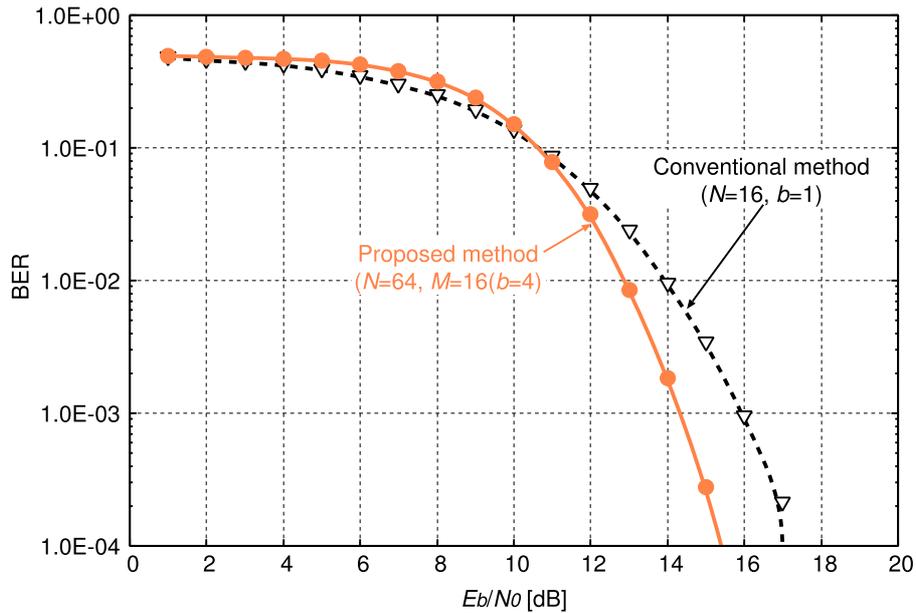
Fig. 6. BER vs. E_b/N_0 ($M = 4, 8$ and 16).

On the other hand, in the case of the proposed scheme, the required E_b/N_0 is 16 dB or less when $M = 16$ ($b = 4$). Namely, to achieve $\text{BER} = 1 \times 10^{-4}$, the proposed scheme with $M = 16$ realizes a saving of energy about 2 dB as compared with the binary system.

Figures 7(a) and (b) show the BER performances of the proposed and conventional one with the same E_b/N_0 for $N = 32$ and $N = 64$, respectively. To achieve the same E_b/N_0 , we set each different N for each method. When we assume $N = 32$ and 64 of the proposed scheme with $M = 16$ ($b = 4$), N of the conventional one becomes 8 ($= 32/b$) and 16 ($= 64/b$), respectively, for performing simulations with the same E_b/N_0 . As shown in Figs. 7(a) and (b), BERs of the proposed scheme show gains over the conventional one. The maximum 1.4dB gain in error performance is obtained by using the propose scheme. As the reason for this, we consider that the separation and reconstruction of the



(a) $N=32$.



(b) $N=64$.

Fig. 7. BER vs. E_b/N_0 (Comparison with the same E_b/N_0).

chaotic dynamics work effectively. As described above, the correct deinterleaver can only reconstruct the original chaotic sequence generated at the transmitter. In this case, other incorrect deinterleavers further separate the order of samples of the received sequence, which has already separated the chaotic dynamics at the transmitter. Namely, the sequence separated by the incorrect deinterleaver mostly loses the characteristics of chaos. Thus, we consider that the error performance of the proposed scheme is better than the conventional one with the same E_b/N_0 since the interleaver and deinterleaver give rise to great difference in the characteristic between the correct sequence and other incorrect sequences.

Figure 8 plots the BERs versus the chaotic sequence length N for different E_b/N_0 values. For fair comparison, we show the performance of the conventional system with the same E_b/N_0 . For example, we compare the performance of the proposed scheme with $N = 16$ to the conventional one with $N = 4$

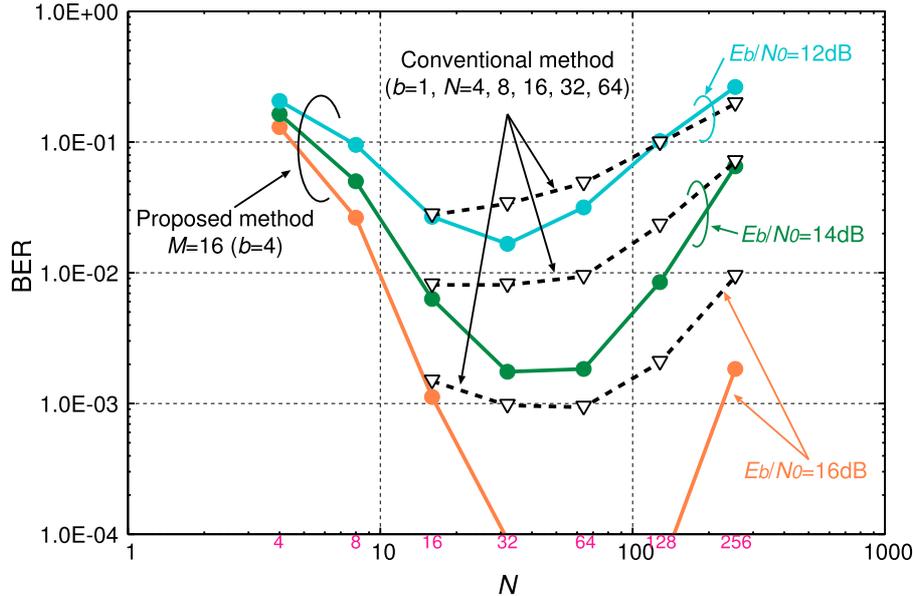


Fig. 8. BER vs. N ($M = 16$).

when $M = 16$. From Fig. 8, it can be confirmed that the performance of the proposed one clearly increases as compared with the conventional one when $N = 32, 64$ and 128 . Especially, the BER of $N = 64$ with $E_b/N_0 = 16$ dB improves from 1×10^{-3} to 1×10^{-4} or more using the proposed scheme. From this result, we find that the effectiveness of using the separation and reconstruction of the chaotic dynamics for M -ary modulation. However, we also find the degradation of the error performance of the proposed scheme when $N = 4, 8$ and 256 . First let us focus on BERs in the region of small N ($N = 4$ and 8). As the reason for degradation of the performance in this region, we consider the decrease of the number of separation patterns using the interleaver and deinterleaver. For example, when $N = 4$, the number of the patterns is $N! = 4! = 24$. When we choose 16 patterns from 24 patterns, some samples certainly overlap among chosen patterns since the number of the patterns is very low. In other words, the interleaver cannot fully separate the chaotic dynamics of the sequence in the case of low number of the patterns. Thus, the analysis error increases due to the influence of low number of the patterns. Next, we focus on the BER when $N = 256$ (i.e., the region of large N). As the reason for the performance degradation, we consider an accumulation of calculation error of the shortest distance. In general, the transmitting sequence is distorted in the channel and is arrived at the receiver. Due to the influence of the channel, the calculation error of the shortest distance occurs even if the chaotic dynamics of the received sequence is reconstructed by the correct deinterleaver. As described in Eq. (10), the outputted shortest distance is the sum of D_m from $i = 0$ to $N - N_d$. Namely, the accumulation of calculation error increases with increasing N . Thus, the accumulation of calculation error affects an accuracy of demodulation. This degradation phenomenon is often found in other noncoherent chaos-based communication systems [3, 5–7]. Several studies about noncoherent systems have reported that the BER increases because the noise component dominates over the gain in the required signal component when N becomes large at constant E_b/N_0 . This conclusion is similar to our discussion. We consider that the degradation with increasing N is specific phenomenon of noncoherent chaos-based communication systems.

Therefore, in order to achieve the better error performance, we need to find and set the following suitable parameters: number of separation patterns of the interleaver and deinterleaver, and the chaotic sequence length N .

5. Conclusions

The present paper has proposed the novel M -ary modulation scheme using the characteristics of chaos for noncoherent chaos-based communications. We have focused on the successive samples of the chaotic sequence based on the chaotic dynamics and have addressed the separation and reconstruction

of the chaotic dynamics of the sequence, which can be applied as additional information to correctly recover b -bit data, by using the interleaver and deinterleaver for M -ary modulation.

As results of the computer simulation, we have observed that the proposed scheme with $M = 16$ realizes a saving of energy about 2 dB as compared with the conventional system for achieving $\text{BER} = 1 \times 10^{-4}$. Moreover, we have confirmed that the advantage gained in BER performance of the proposed scheme is about 1.4 dB compared to the conventional one with the same E_b/N_0 . Finally, it has been concluded that it is necessary to find and set the suitable parameters of the proposed system in order to achieve the better error performance. We currently consider that the design method of interleaver-deinterleaver pairs with the constraint conditions, which is described in Section 3, is a suboptimum method in this study. As an important future work, we would like to discuss whether our design method is optimum.

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