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Dependence of Clustering Patterns on Density of Chaotic Circuits in Networks

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Abstract

In this study, we investigate clustering patterns generated in coupled chaotic circuits in networks. In these networks, the coupling strength reflects the distance information, and each chaotic circuit is connected to all other chaotic circuits. We consider the relationship between coupling strength and phase difference by changing the scaling parameter of coupling strength. Furthermore, we determine the various phase synchronization patterns when we change the number of chaotic circuits.

1. Introduction

Recently, it has become necessary to deal with a large amount of information in our lives. Our society can be considered as an advanced information network society. Because of this, a large amount of information is gradually processed day by day. Therefore, the ideas of clustering algorithms have been proposed and applied to information processing. Clustering algorithms have widespread applications in different fields, such as business data mining, image processing and analysis of biological data. There are a variety of different clustering algorithms along with many applications. Many algorithms were proposed to utilize the synchronization phenomenon, for instance, in coupled map lattices (CMLs), for clustering [1]-[3]. Previously, many of these studies used a discrete-time model for clustering, and hardly any analysis using a continuous-time model has been carried out. Therefore, we have focused on research on clustering phenomena using electronic circuits using a continuous-time model.

On the other hand, the synchronization phenomenon is a typical phenomenon when we analyze coupled chaotic circuits. This phenomenon can be widely observed and has been studied in the fields of natural and technical sciences. In order to understand the synchronization phenomenon in detail, we analyze electronic circuits. Coupled chaotic circuits are composed of electronic circuits and a suitable model to analyze the synchronization phenomenon. Moreover, we can observe various phenomena in addition to the synchronization phenomenon. However, not all phenomena have been sufficiently investigated. Therefore, we consider that our study provides a new approach to investigating the synchronization and clustering phenomena in coupled chaotic circuits.

In a previous study, we investigated the relationship between clustering and the density of coupled chaotic circuits in two-dimensional space [4]-[6]. In our previous investigation, the coupling strength reflected the distance information and we changed the number of circuits in a cluster. We showed that clustering phenomena affected other clusters when the density in the chaotic circuits was high. We also observed that networks of coupled chaotic circuits could be split into different synchronized groups.

In this study, we investigate clustering patterns generated in coupled chaotic circuits in networks. In these networks, chaotic circuits are connected to all other chaotic circuits. For this investigation, we change the parameters of coupling strength and the density of chaotic circuits. From the results, three clustering patterns can be observed in the networks. We consider the relationship between the scaling parameter and the phase difference by changing the scaling parameter of coupling strength. Additionally, we determine the various phase synchronization patterns when we change the density of chaotic circuits.

2. Circuit Model

Figure 1 shows the model of the chaotic circuit investigated in [7]-[9].

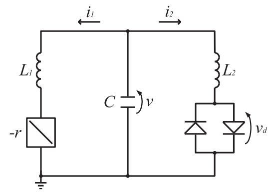


Figure 1: Chaotic circuit

The following equations are the circuit equations when each chaotic circuit is coupled globally with all other circuits.

$$
\begin{aligned}
\frac{dx_i}{d\tau} &= \alpha x_i + z_i \\
\frac{dy_i}{d\tau} &= z_i + f(y) \\
\frac{dz_i}{d\tau} &= -x_i - \beta y_i - \sum_{j=1}^{N} \gamma_{ij}(z_i - z_j) \\
&(i, j = 1, 2, \cdots, N)
\end{aligned}
\tag{1}
$$

For the computer simulation, we set the parameters as $\alpha = 0.460$, $\beta = 3.0$ and $\delta = 470$. The function $f(y)$ can be described as a three-segment piecewise-linear function. The value of γ_{ij} reflects the distance between the circuits in an inverse manner, described by the following equation:

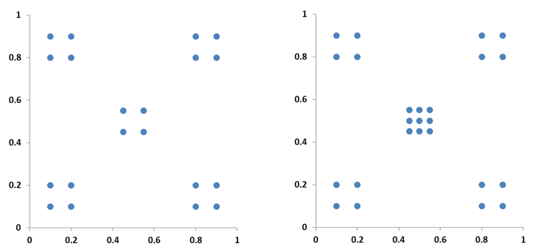
$$
\gamma_{ij} = \frac{g}{d_{ij}^2} \tag{2}
$$

Here, d_{ij} denotes the Euclidean distance between the ith circuit and the jth circuit. The parameter g is a scaling parameter that determines the coupling strength.

3. Clustering Phenomena

3.1 Clustering phenomena

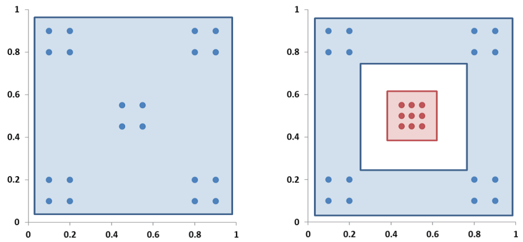
In this section, we investigate clustering phenomena when we configure a network of coupled chaotic circuits in two-dimensional space. In our previous study [6], we investigated the relationship between clustering and the density of coupled chaotic circuits by changing the density of chaotic circuits. The arrangements of chaotic circuits are shown in Fig. 2. Figure 2(a) is composed of groups with the same number of chaotic circuits; however, Fig. 2(b) is composed of a group with a high density of chaotic circuits inside and some groups with a low density of chaotic circuits outside. In these networks, we replace chaotic circuits with a simple model such as a small circle.



(a) Same number of groups (b) Different numbers of groups

Figure 2: Arrangements of chaotic circuits

The simulation results of clustering in these networks are shown in Fig. 3. From the simulation results, all chaotic circuits are synchronized in one cluster as shown in Fig. 3(a); however, we can observe two-clusters from chaos synchronization between the high-density group and the low-density groups in Fig. 3(b).



(a) One cluster (b) Two clusters

Figure 3: Clustering results

From these results, clustering phenomena are related to the density of coupled chaotic circuits.

3.2 Investigation of clustering phenomena

Next, we investigate the clustering result shown in Fig. 3(b) in detail. This network can be divided into two-clusters with high density and low density as a result of chaos synchronization. We consider the state of this network when we change the parameter g determined by Eq. (2). Furthermore, we calculate the phase difference between chaotic circuits by computer simulation.

For this simulation, the number of iterations is set to $\tau_p = 10,000$ to calculate the result more precisely. Figure 4 shows the phase difference between the two chaotic circuits when we change the value of parameter g.

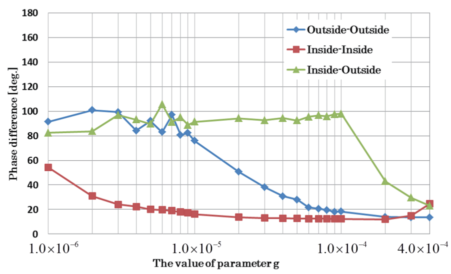


Figure 4: Relationship between g and phase difference

We define the state of synchronization patterns from the average phase difference when we perform $\tau_p = 10,000$ iterations. A synchronized state can be defined if the average phase difference is below 40°. Similarly, we define an asynchronous state as having an average phase difference between 70° and 110°. In the region between $g = 2.0 \times 10^{-6}$ and

Table 1: State of networks

Value of g ($\times 10^{-4}$)	In - In	In - Out	Out - Out
$0.02 \leq g \leq 0.3$	Syn.	Not-syn.	Not-syn.
$0.4 \leq g \leq 2.0$	Syn.	Not-syn.	Syn.
$3.0 \leq g \leq 4.0$	Syn.	Syn.	Syn.

$g = 3.0 \times 10^{-5}$, the inside group of chaotic circuits is synchronized in one cluster; however, the other chaotic circuits composing the outside groups are not synchronized. In the region between $g = 4.0 \times 10^{-5}$ and $g = 2.0 \times 10^{-4}$, both of the inside and outside groups of chaotic circuits are synchronized in one cluster. Therefore, clustering phenomena can be observed in this region. Finally, all chaotic circuits are synchronized in one cluster in the region between $g = 3.0 \times 10^{-4}$ and $g = 4.0 \times 10^{-4}$. Thus, three-clustering patterns can be observed in this network from the average phase difference.

4. Relationship between Phase Difference and Density

In this section, we calculate the phase difference between inside chaotic circuits, and between inside chaotic circuits and outside chaotic circuits when we change the density of chaotic circuits in the inside group shown in Fig. 3. Moreover, we calculate the maximum and minimum phase difference and its range.. Here, we change the number of chaotic circuits in the inside group from 1 to 9.

First, we calculate the phase difference inside chaotic circuits shown in Tab. 2. Table 2 shows the phase difference, maximum value and minimum value. Figure 5 shows the obtained relationship between the phase difference and the density of chaotic circuits. In this result, the average phase difference is below 40°. Thus, the inside chaotic circuits are synchronized regardless of the density.

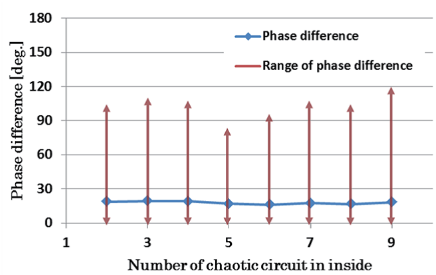


Figure 5: Phase difference and range of phase difference (in-in)

Next, we calculate the phase difference between the inside chaotic circuits and outside chaotic circuits as shown in Table 3. Table 3 shows the phase difference, maximum value and minimum value. Figure 6 shows the obtained relationship between the phase difference and the density of chaotic circuits. From this result, the inside chaotic circuits are synchronized

Table 2: The phase difference (in-in)

Density of circuits	Ave.	Max.	Min.
2	18.894°	102.898°	0.001°
3	19.502°	109.258°	0.001°
4	19.029°	107.431°	0.002°
5	17.067°	86.363°	0.001°
6	16.272°	94.816°	0.002°
7	17.554°	106.801°	0.002°
8	16.678°	101.588°	0.001°
9	18.375°	118.286°	0.003°

with the outside circuits when the density of the inside circuits is between 1 and 5; however, the state for other densities is not clear. Therefore, we use the frequency distribution to reveal the synchronized state.

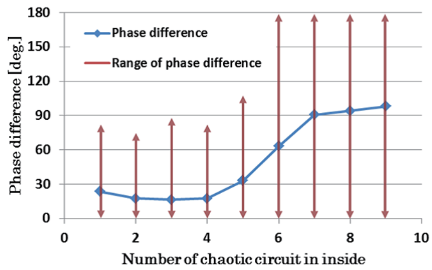
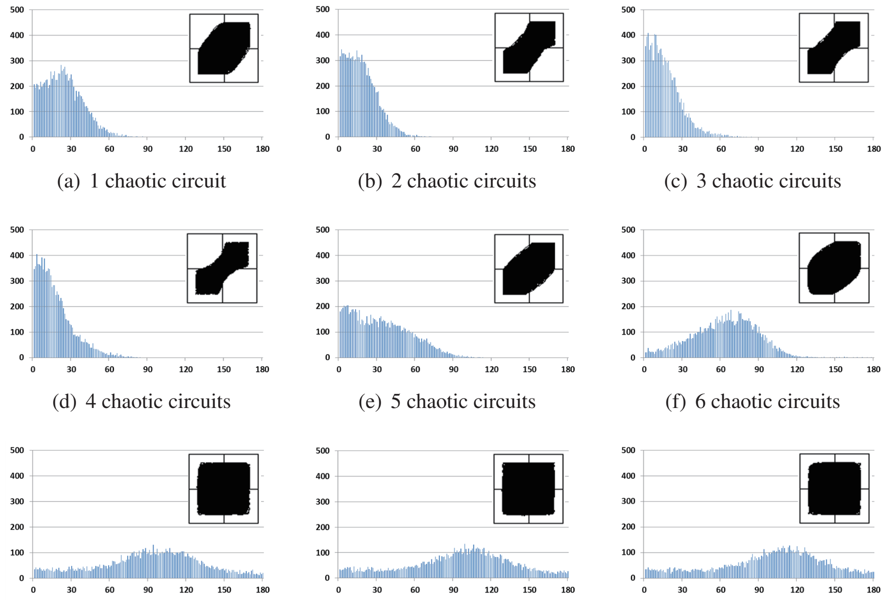


Figure 6: Phase difference and range of phase difference (in-out)

Table 3: The phase difference (in-out)

Density of circuits	Ave.	Max.	Min.
1	23.640°	85.773°	0.003°
2	17.425°	77.697°	0.009°
3	16.442°	87.398°	0.001°
4	17.500°	83.591°	0.008°
5	32.938°	112.724°	0.006°
6	63.203°	178.035°	0.135°
7	90.618°	179.972°	0.032°
8	94.031°	179.998°	0.043°
9	97.988°	179.996°	0.007°

Figure 7 shows the synchronization patterns obtained from Lissajous figures. Each pattern in these figures corresponds to the values in Fig. 6 and Table 3. We calculate the number of phase domains as a function of each phase difference. The number of iterations is set to $\tau_p = 10,000$. From these figures, in the small region of the average phase difference are synchronized with small value for phase domains. Also, for a phase difference of approximately 90°, are not synchronized from the Lissajous figures and the phase domain. However, there are six chaotic circuits, the synchronization state is unclear. Additionally, the maximum value of the phase difference is 178.035° in the phase domain, and the phase domain



(a) 1 chaotic circuit (b) 2 chaotic circuits (c) 3 chaotic circuits

(d) 4 chaotic circuits (e) 5 chaotic circuits (f) 6 chaotic circuits

(g) 7 chaotic circuits (h) 8 chaotic circuits (i) 9 chaotic circuits

Figure 7: State of synchronicity (horizontal axis: phase difference, vertical axis: frequency)

is between 0.135° and 178.035°; namely, when there are six chaotic circuits, they are not synchronized. Therefore, we define the state of synchronicity or asynchronicity using the average phase difference and phase domain.

As mentioned before, we define the synchronized state as having a phase difference is below 40°. Also, we define the asynchronous state as having a phase difference between 70° and 110° and a phase domain between 0° and 180°. Thus, we define synchronization patterns using the average phase difference and phase domain. Furthermore, some clustering patterns can be observed in the coupled chaotic circuits in networks as a result of chaos synchronization.

5. Conclusions

In this study, we have investigated clustering patterns generated in coupled chaotic circuits in networks. In these networks, the coupling strength reflects the distance information, and each chaotic circuit is connected to all other chaotic circuits. For this investigation, we have changed the scaling parameter of coupling strength and the number of chaotic circuits composing a cluster. We have observed some clustering patterns resulting from chaos synchronization. From computer simulation results, we have confirmed that the state of clustering patterns depends on the scaling parameter g and the density of chaotic circuits in networks. Furthermore, we have clarified that it is efficient to use the average phase difference and phase domain to define synchronization patterns.

In our future work, we intend to study the clustering phenomena in large-scale networks. Additionally, we hope to apply this clustering method for data mining, image processing and practical applications to enrich our lives.

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