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Synchronization Phenomena of Two Simple RC Chaotic Circuits Coupled by a Capacitor

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Abstract

In this study, two coupled chaotic oscillators composed of RC circuits are investigated. We carry out computer simulations and circuit experiments and observe both in-phase synchronization and anti-phase synchronization. Moreover, when we change the coupling strength between oscillators, we observe a change in synchronization state.

1. Introduction

Recently, many researchers have shown interest in chaotic systems [1],[2]. In particular, chaos synchronization has attracted many researchers' attention and its mechanism has been gradually clarified [3],[4]. In the field of electrical and electronic engineering, researchers have proposed various applications using chaos, for example, chaos communication systems [5], chaos cryptosystems [6], and so forth. In order to realize such chaotic engineering systems, it is important to investigate simple coupled chaos-generating circuits.

In our previous study, a simple chaotic oscillator using two RC circuits was proposed [7]. When we change the parameter, we can observe not only periodic attractors but also chaotic attractors from this simple oscillator. Furthermore, we investigated the chaotic behavior by increasing when the number of coupled RC circuits [8]. In addition, we focused on the cross-correlation characteristics between neighboring oscillators. Because this chaotic oscillator has an extremely simple structure, it is important to investigate its basic nonlinear phenomena.

In this study, we investigate synchronization phenomena observed from two chaotic oscillators composed of RC circuits coupled by a capacitor. We carry out computer simulations and circuit experiments for the two cases in which the external forces have the same phase states and inverse phase states.

2. Circuit Model

Figure 1 shows the circuit model. In this figure, two chaotic oscillators, as proposed in [6], are coupled via one capacitor C_0. Two independent rectangular voltage sources are connected to the two comparators of the oscillators. Hence, the whole circuit consists of two rectangular voltage sources, four comparators, four resistors, four capacitors, and one coupling capacitor. Figure 2(a) shows the rectangular voltage signals $V_S(t)$. $E\alpha$ is the amplitude of the rectangular voltage and T is the period of the waveform. E is the output voltage of the comparators, namely, the DC supply voltage of the operational amplifiers. The circuit equations are described as follows:

$$RC\frac{dv_1}{dt} = \begin{cases} -v_1 + E & (v_2 > V_{S1}) \\ -v_1 - E & (v_2 < V_{S1}) \end{cases} \tag{1}$$

$$RC\frac{dv_2}{dt} = \begin{cases} -\frac{C+C_0}{C+2C_0}v_2 - \frac{C_0}{C+2C_0}v_4 - E & (v_1 > V_{S1}) \\ -\frac{C+C_0}{C+2C_0}v_2 - \frac{C_0}{C+2C_0}v_4 + E & (v_1 < V_{S1}) \end{cases} \tag{2}$$

$$RC\frac{dv_3}{dt} = \begin{cases} -v_3 + E & (v_4 > V_{S2}) \\ -v_3 - E & (v_4 < V_{S2}) \end{cases} \tag{3}$$

$$RC\frac{dv_4}{dt} = \begin{cases} -\frac{C_0}{C+2C_0}v_2 - \frac{C+C_0}{C+2C_0}v_4 - E & (v_3 > V_{S2}) \\ -\frac{C_0}{C+2C_0}v_2 - \frac{C+C_0}{C+2C_0}v_4 + E & (v_3 < V_{S2}) \end{cases} \tag{4}$$

By using the following variables and the parameters

$$v_n = Ex_n,\ \gamma = \frac{C_0}{C+2C_0},\ t = RC\tau,\ T = RC\beta \quad (n = 1,\ 2,\ 3,\ 4) \tag{5}$$

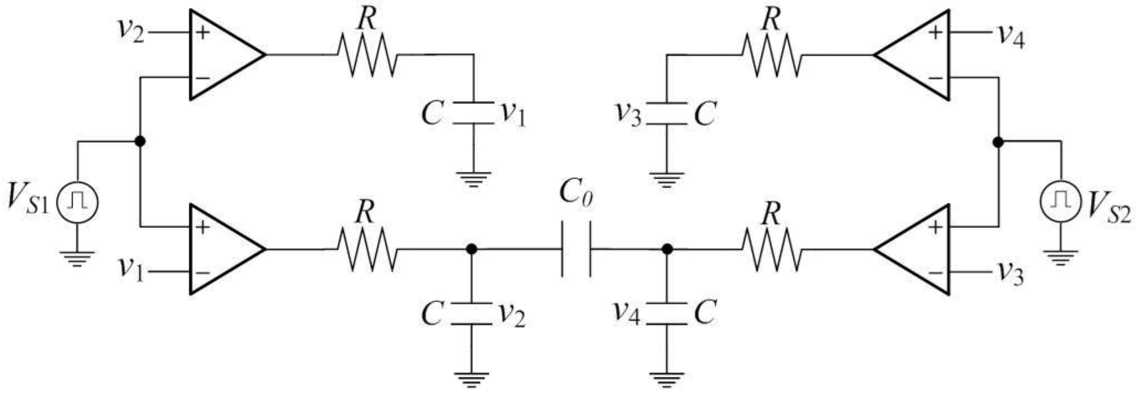

Figure 1: Circuit model

we obtain the normalized circuit equations. Because the circuit equations are linear in each region, the rigorous solution of the circuit equations can be derived as follows:

$$x_1 = \begin{cases} (x_{10} - 1)\, e^{-\tau} + 1 & (x_2 > V_\alpha) \\ (x_{10} + 1)\, e^{-\tau} - 1 & (x_2 < V_\alpha) \end{cases} \tag{6}$$

$$x_2 = \begin{cases} \left(\dfrac{x_{20} + x_{40}}{2} + 1\right) e^{-\tau} + \left(\dfrac{x_{20} - x_{40}}{2}\right) e^{-\gamma\tau} - 1 \\ \qquad\qquad\qquad\qquad (x_1 > V_\alpha) \\ \left(\dfrac{x_{20} + x_{40}}{2} - 1\right) e^{-\tau} + \left(\dfrac{x_{20} - x_{40}}{2}\right) e^{-\gamma\tau} + 1 \\ \qquad\qquad\qquad\qquad (x_1 < V_\alpha) \end{cases} \tag{7}$$

$$x_3 = \begin{cases} (x_{30} - 1)\, e^{-\tau} + 1 & (x_4 > V_\alpha) \\ (x_{30} + 1)\, e^{-\tau} - 1 & (x_4 < V_\alpha) \end{cases} \tag{8}$$

$$x_4 = \begin{cases} \left(\dfrac{x_{20} + x_{40}}{2} + 1\right) e^{-\tau} - \left(\dfrac{x_{20} - x_{40}}{2}\right) e^{-\gamma\tau} - 1 \\ \qquad\qquad\qquad\qquad (x_3 > V_\alpha) \\ \left(\dfrac{x_{20} + x_{40}}{2} - 1\right) e^{-\tau} - \left(\dfrac{x_{20} - x_{40}}{2}\right) e^{-\gamma\tau} + 1 \\ \qquad\qquad\qquad\qquad (x_3 < V_\alpha) \end{cases} \tag{9}$$

where V_α corresponds to V_S and is shown in Fig. 2(b). x_{10}, x_{20}, x_{30}, and x_{40} are the initial values in the corresponding linear regions.

3. Computer Simulations

First, we present and discuss the computer-simulated results. In the following results, the parameters corresponding to the rectangular voltage signals are fixed as $\alpha = 0.060$ and $\beta = 4.0$ and the parameter corresponding to the coupling strength γ is varied in order to investigate the synchronization phenomena of chaotic oscillations.

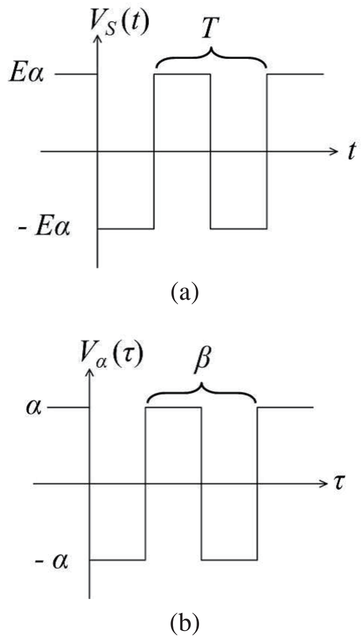

Figure 2: Rectangular voltage signals

3.1 Case of $V_{S1} = V_{S2}$

Figure 3 shows computer-simulated results for the case in which the two rectangular voltage signals are the same, namely $V_{S1} = V_{S2}$. Figures 3(a) and 3(b) show the attractor ($x_1 - x_2$) observed from the left oscillator in Fig. 1 and the attractor ($x_3 - x_4$) observed from the right oscillator in Fig. 1, respectively. Figure 3(c) shows the phase states between the two attractors ($x_1 - x_3$).

For strong-coupling parameters, we can observe complete in-phase synchronization as shown in Fig. 3(1), although the two oscillators exhibit chaotic oscillations. As decreasing the coupling parameter γ, the in-phase synchronization state

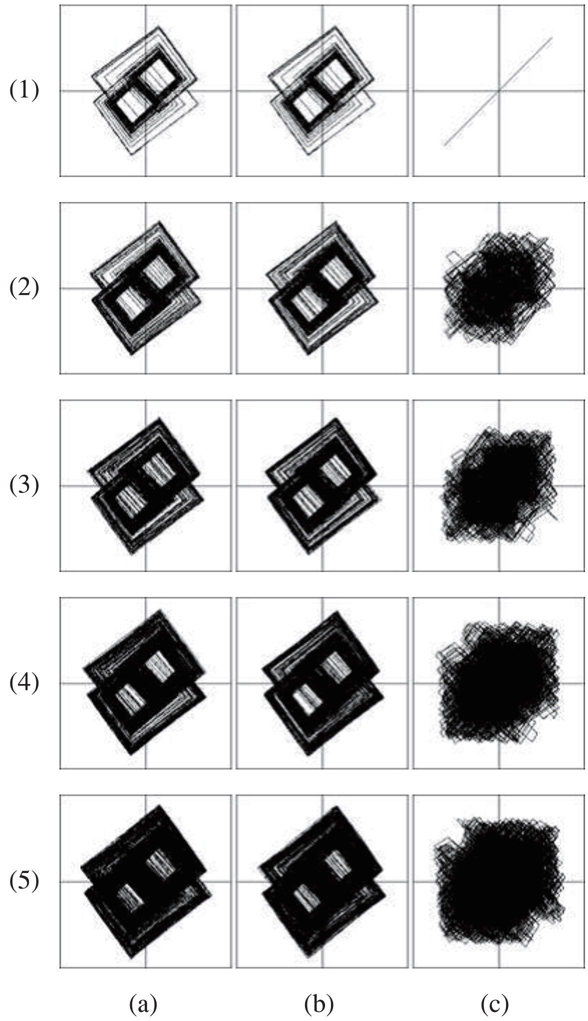

Figure 3: Attractors and phase difference for $V_{S1} = V_{S2}$: (1) $\gamma = 0.80$, (2) $\gamma = 0.75$, (3) $\gamma = 0.50$, (4) $\gamma = 0.45$, and (5) $\gamma = 0.01$

gradually becomes weak and changes to an asynchronous state.

In order to evaluate the synchronization states quantitatively, we introduce the degree of synchronization by using the distance between the two solutions as follows:

$$D_{syn} = \max\{|x_1 - x_3|\} - \min\{|x_1 - x_3|\} \qquad (10)$$

Figure 4 shows the relationship between the coupling strength and the degree of synchronization. We can see that the degree of synchronization immediately changes after breaking the synchronization state.

3.2 Case of $V_{S1} = -V_{S2}$

Next, we consider the case in which the two rectangular voltage signals have inverse phase states, namely $V_{S1} = -V_{S2}$. It is interesting to investigate how the two coupled oscillators behave for the inverse phase external signals, because such a complicated situation was not considered in past studies. Figure 5 shows the computer-simulated results. Figures 5(a) and 5(b) show the attractors $(x_1 - x_2)$ and $(x_3 - x_4)$, respectively. Figure 5(c) shows the phase states between the two attractors $(x_1 - x_3)$.

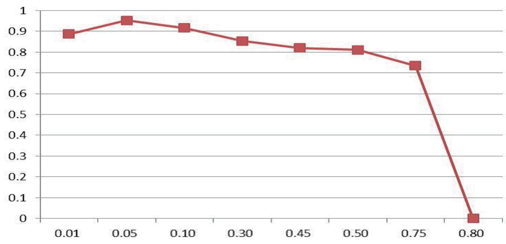

Figure 4: Relationship between coupling strength and degree of synchronization

For strong-coupling parameters, we can observe complete anti-phase synchronization as shown in Fig. 5(1). As decreasing the coupling parameter γ, the anti-phase synchronization state becomes weak, similar to the case of $V_{S1} = V_{S2}$.

4. Circuit Experiments

Finally, we show the circuit experimental results. Figures 6(a) and 6(b) show the phase states and time waveforms for the case of $V_{S1} = V_{S2}$ and Figs. 7(a) and 7(b) show the phase states and time waveforms for the case of $V_{S1} = -V_{S2}$.

From all of the time waveforms, we can confirm that chaotic oscillations appear in the circuit. We also observe in-phase synchronization and anti-phase synchronization similar to those in the computer-simulated results.

5. Conclusions

In this study, we have investigated two coupled chaotic oscillators composed of RC circuits. We carried out computer simulations and circuit experiments for the two cases of $V_{S1} = V_{S2}$ and $V_{S1} = -V_{S2}$. For the case of $V_{S1} = V_{S2}$, we confirmed the generation of in-phase synchronization states, while for the case of $V_{S1} = -V_{S2}$, we confirmed the generation of anti-phase synchronization states.

Detailed research on the synchronization states such as stability analysis and the coexistence region of the in-phase and anti-phase states will be carried out as future research.

Acknowledgments

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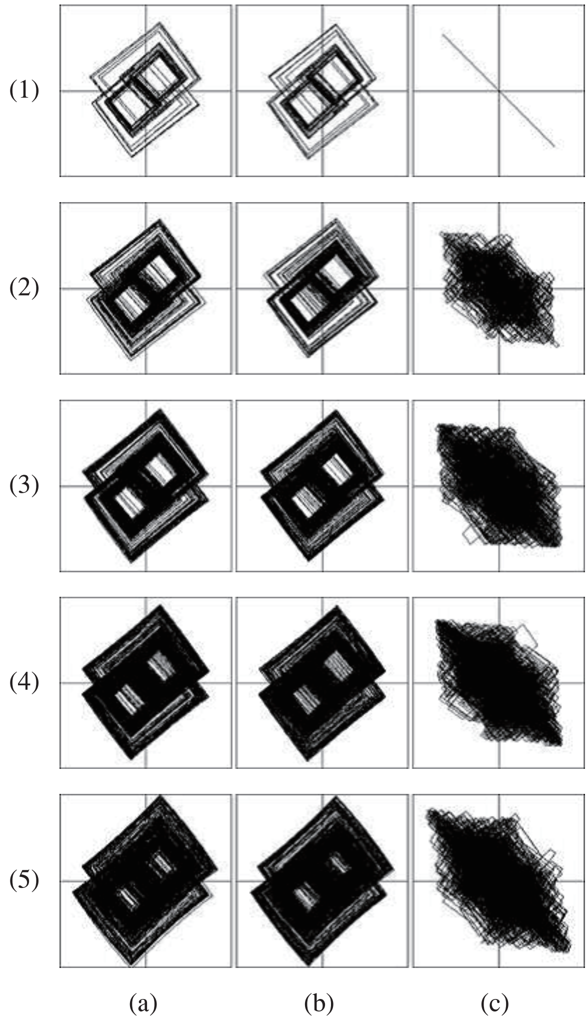
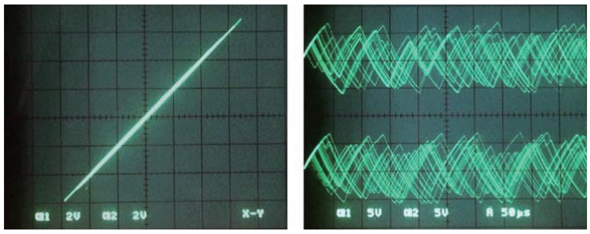
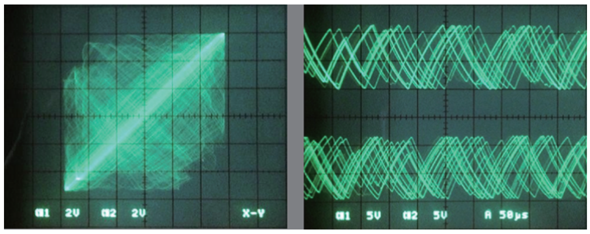


Figure 5: Attractors and phase difference for $V_{S1} = -V_{S2}$: (1) $\gamma = 0.80$, (2) $\gamma = 0.75$, (3) $\gamma = 0.50$, (4) $\gamma = 0.45$, and (5) $\gamma = 0.01$

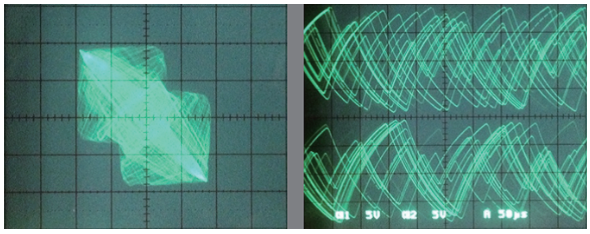


(a) $C_0 = 67.9$[nF]

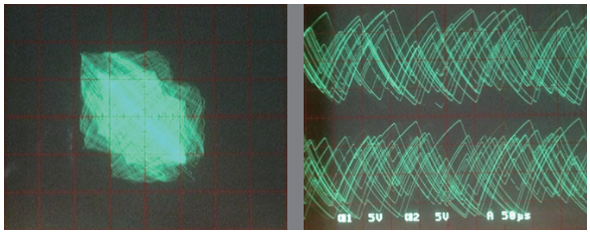


(b) $C_0 = 1.5$[nF]

Figure 6: Circuit experimental results for $V_{S1} = V_{S2}$



(c) $C_0 = 48.6$[nF]



(d) $C_0 = 10.0$[nF]

Figure 7: Circuit experimental results for $V_{S1} = -V_{S2}$

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