# Spice-Oriented Algorithm for Peak Search and Stability Assessment for Frequency Response

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**Keywords:** Spice, nonlinear circuit, harmonic balance method, peak search, stability assessment

In this study, we propose a Spice-oriented algorithm for peak search and stability assessment for the frequency response. We obtain the frequency response by using the sine-cosine circuit, Fourier transformation circuit and solution tracing-curve circuit, which have been developed in our previous studies. We combine the idea using a differentiator and a nonlinear limiter with them in order to search the exact peaks and propose a stability assessment algorithm based on the Floquet theory. In our algorithm, we can simulate almost all tasks with Spice.

In this article, we introduce our algorithm to the circuit that is shown in Fig. 1.

Figure 2 shows the simulation result of the frequency response obtained by the proposed algorithm. our algorithm is very effective to search exact peak values of the frequency response curve. The obtained peaks are summarized in Table 1.

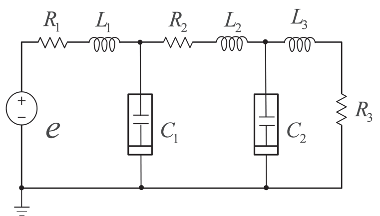


Fig. 1. Example circuit with strong nonlinearity.

Table 1. Frequencies and voltages at peaks.

| | peak1 | peak2 | peak3 |
|---|---|---|---|
| $\omega[rad/s]$ | 5.067 | 6.151 | 6.420 |
| $i_1[A]$ | 2.4545 | 2.5000 | 3.4450 |

We assess the stability of the solutions at the points that are marked in Fig. 2. Table 2 shows the calculated eigenvalues of $\Phi(T)$ for the three cases which obtained by using our algorithm.

We can see that the point $(\omega, i_1) = (4.570, 1.902)$ is stable, because all of eigenvalues satisfy $|\lambda| < 1$. However, for the other two points, $(\omega, i_1) = (4.927, 0.628)$ and $(5.560, 2.000)$, the solutions are unstable, because some eigenvalues do not satisfy $|\lambda| < 1$.

The simulation results for example circuits showed that the proposed algorithm worked well.

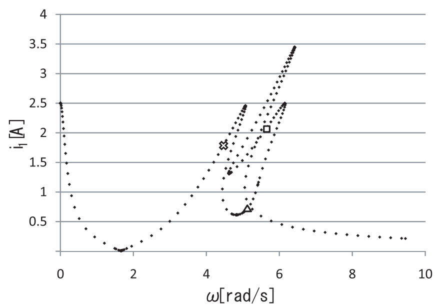


Fig. 2. Frequency response obtained by the proposed algorithm.

Table 2. Eigenvalues of $\Phi(T)$.

| | $\omega = 4.570$ | $\omega = 4.927$ | $\omega = 5.560$ |
|---|---|---|---|
| $\lambda_1$ | $-0.622 + j0.554$ | $0.726 + j0.825$ | 2.530 |
| $\lambda_2$ | $-0.622 - j0.554$ | $0.726 - j0.825$ | $0.545 + j0.638$ |
| $\lambda_3$ | $0.495 + j0.625$ | $0.373 + j0.499$ | $0.545 - j0.638$ |
| $\lambda_4$ | $0.495 - j0.625$ | $0.373 - j0.499$ | 0.8662 |
| $\lambda_5$ | 0.8394 | 0.8501 | 0.2862 |

Paper

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In this study, we propose a Spice-oriented algorithm for peak search and stability assessment for the frequency response. We obtain the frequency response by using the sine-cosine circuit, Fourier transformation circuit and solution tracing-curve circuit, which have been developed in our previous studies. We combine the idea using a differentiator and a nonlinear limiter with them in order to search the exact peaks and propose a stability assessment algorithm based on the Floquet theory. In our algorithm, we can simulate almost all tasks with Spice.

**Keywords:** Spice, nonlinear circuit, harmonic balance method, peak search, stability assessment

## 1. Introduction

Spice is a very convenient tool for circuit simulation and is used by many researches. However, when we analyze nonlinear circuits, we have to make consideration for harmonic components. Because the frequency response is distorted by the influence of the nonlinear elements. In that case, the Harmonic Balance (HB) method [(1)–(3)] is useful and we have developed Spice-oriented HB method in our past studies [(4)–(6)].

When we simulate nonlinear circuits, it is very important to find out the locations and the frequencies giving large peak values of the voltages. If the circuit includes some nonlinear elements, resonance curve becomes complicated. In that case, the stability assessment is also very important. In this article, we propose a Spice-oriented algorithm for peak search and stability assessment for frequency response of nonlinear circuits.

Section 2 reviews the Spice-oriented HB method algorithm for nonlinear circuits. Section 3 shows the peak search algorithm for frequency response. Section 4 shows the algorithm for stability assessment based on the Floquet theory [(3)(7)]. Illustrated examples of the proposed algorithm for the peak search and the stability assessment are shown in Sec. 5. Section 6 gives discussions and Sec. 7 concludes the article.

## 2. Spice-Oriented HB Method

In this section, we review the algorithm of Spice-oriented HB method, which has been developed by the authors [(4)–(6)].

**2.1 Sine-Cosine Circuit** The sine-cosine circuit has been introduced in order to solve the determining equations of the HB method by using Spice. In this subsection, we explain how we can derive the sine-cosine circuit for simple passive element cases.

First, we set a voltage and a current with Fourier series;

$$\begin{cases} v = V_0 + \sum_{k=1}^{n}(V_{s_k}\sin k\omega t + V_{c_k}\cos k\omega t) \\ i = I_0 + \sum_{k=1}^{n}(I_{s_k}\sin k\omega t + I_{c_k}\cos k\omega t) \end{cases} \quad \cdots (1)$$

A current through a capacitor is given by

$$i = C\,\frac{dv}{dt}. \quad \cdots (2)$$

From Eqs. (1) and (2), we can express the current as

$$i = \sum_{k=1}^{n}(-k\omega CV_{c_k}\sin k\omega t + k\omega CV_{s_k}\cos k\omega t). \quad \cdots (3)$$

From Eq. (3), we can express the relation between the coefficients of sine and cosine components as follows;

$$\begin{cases} I_{s_k} = -k\omega CV_{c_k} \\ I_{c_k} = k\omega CV_{s_k} \end{cases} \quad \cdots (4)$$

In the case of an inductor, we can express the voltage across an inductor as

$$v = \sum_{k=1}^{n}(-k\omega LI_{c_k}\sin k\omega t + k\omega LI_{s_k}\cos k\omega t) \cdots (5)$$

where the current through an inductor is given by

$$v = L\,\frac{di}{dt}. \quad \cdots (6)$$

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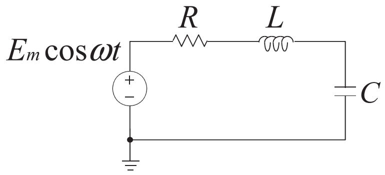


Fig. 1. Example RLC circuit.

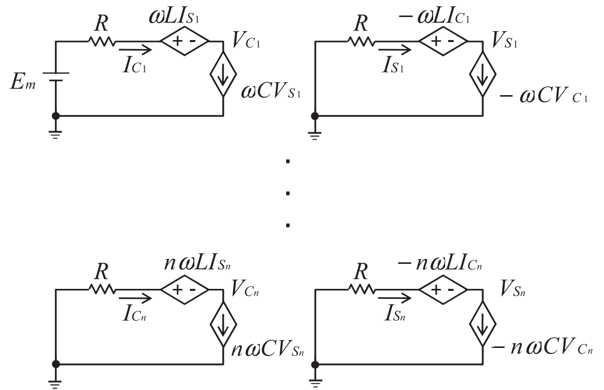

Fig. 2. Sine-cosine circuit for Fig. 1

Equations for coefficient of $\sin k\omega t$ and $\cos k\omega t$ are given by

$$\begin{cases} V_{s_k} = -k\omega L I_{c_k} \\ V_{c_k} = k\omega L I_{s_k} \end{cases} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (7)$$

As an example, we consider a simple linear RLC circuit as Fig. 1. In this circuit, coefficients of $\sin k\omega t$ and $\cos k\omega t$ for the capacitor and the inductor are given by Eqs. (4) and (7), respectively. We make the sine-cosine circuits satisfying Eqs. (4) and (7) to obtain the values of the coefficients as shown in Fig. 2.

In Fig. 2, the capacitor is replaced by coupled voltage-controlled current sources and the inductor is replaced by coupled current-controlled voltage sources in the sine-cosine circuit.

**2.2 Fourier Transformation Circuit** When we analyze nonlinear circuits with Spice-oriented HB method, we introduce the Fourier transformation circuit in order to obtain the nonlinear characteristics of the elements including many harmonic components.

Let us consider the nonlinear capacitor whose characteristics are given by

$$v = f(q). \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (8)$$

Suppose the input and the output waveforms are written as follows;

$$\begin{cases} q = Q_0 + \displaystyle\sum_{k=1}^{n} (Q_{sk} \sin k\omega t + Q_{ck} \cos k\omega t) \\ v = V_0 + \displaystyle\sum_{k=1}^{n} (V_{sk} \sin k\omega t + V_{ck} \cos k\omega t) \end{cases} \quad \cdots (9)$$

The coefficients can be calculated by the Fourier series expansion as follows;

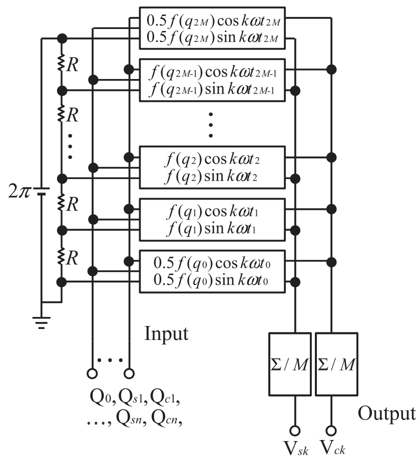


Fig. 3. Fourier transformation circuit.

$$\begin{cases} V_0 = \dfrac{1}{2\pi} \displaystyle\int_0^{2\pi} f(q) d(\omega t) \\ V_{ck} = \dfrac{1}{\pi} \displaystyle\int_0^{2\pi} (f(q) \cos k\omega t) d(\omega t) \\ V_{sk} = \dfrac{1}{\pi} \displaystyle\int_0^{2\pi} (f(q) \sin k\omega t) d(\omega t) \\ (k = 1, 2, \ldots, n) \end{cases} \quad \cdots\cdots\cdots (10)$$

In Spice, the coefficients cannot be given in the analytical forms. Therefore, we apply the trapezoidal formula.

$$\int_a^b g(\omega t) d(\omega t) = \frac{h}{2}(g(\omega t_0) + g(\omega t_{2M})) + h(g(\omega t_1) + g(\omega t_2) + \cdots + g(\omega t_{2M-1})),$$
$$\omega t_m = 0,\ \pi/M,\ ...,\ (2M-1)\pi/M,\ 2\pi, \cdots\cdots (11)$$

where $g(\omega t)$ is the function of $(\omega t)$, $h = (b-a)/2M$, and $M$ is the number of the division for an interval $(0, \pi)$.

From Eqs. (8), (9), (10) and (11), we can obtain the equations for the coefficient of $\sin k\omega t$ and $\cos k\omega t$ as

$$V_{ck} = \frac{1}{\pi} \int_0^{2\pi} (f(q) \cos k\omega t) d(\omega t) = \frac{1}{2M}(f(q_0) + f(q_{2M})) + \frac{1}{M}(f(q_1) \cos k\omega t_1 + f(q_2) \cos k\omega t_2 + \ldots + f(q_{2M-1}) \cos k\omega t_{2M-1}), \quad \cdots\cdots\cdots (12)$$

$$V_{sk} = \frac{1}{\pi} \int_0^{2\pi} (f(q) \sin k\omega t) d(\omega t) = \frac{1}{M}(f(q_1) \sin k\omega t_1 + f(q_2) \sin k\omega t_2 + \cdots + f(q_{2M-1}) \sin k\omega t_{2M-1}) \cdots\cdots (13)$$

where $f(q_m) = f(q_{\omega t_m})$.

Fig. 3 shows the Fourier transformation circuit. The circuit is realized to satisfy Eqs. (12) and (13).

In the Fourier transformation circuit, we obtain the Fourier expansion as voltage values by describing the nonlinear characteristics with ABM (Analog Behavior Model). Namely, if we can describe the characteristics with ABM, the nonlinear model can be handled in

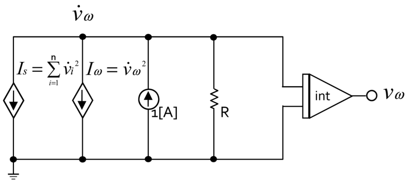


Fig. 4. Solution tracing circuit.

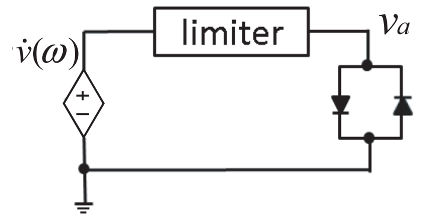


Fig. 5. Nonlinear limiter.

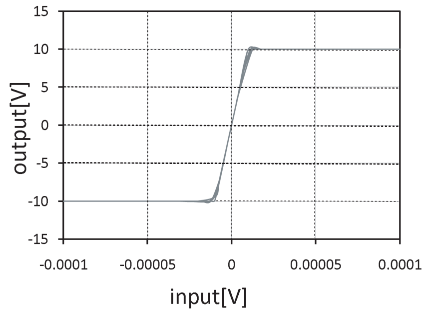


Fig. 6. Characteristics of nonlinear limiter.

the proposed method even for complicated characteristics. However, it may be better to develop a software to derive the Fourier transformation circuit automatically, if we often handle such complicated models. Further, if we perform a multi-tone analysis, we have to introduce two dimensional Fourier transformation circuit. It is theoretically possible with a similar concept to the one-dimensional case, although the circuit becomes very large in order to treat a huge number of frequency components.

**2.3 Solution Tracing Circuit** Even we combine the sine-cosine circuit and the Fourier transformation circuit, we cannot obtain unstable region of the frequency curve, because we set the frequency as time in Spice to solve the sine-cosine circuits. In this section, we explain the Solution Tracing Circuit (STC) (8) (9) realizing the arc-length method (10) (11).

First, we express the ark length in $(n+1)$ dimensional euclidean space as Eq. (14)

$$ds = \sqrt{(dx_1)^2 + (dx_2)^2 + \cdots + (dx_{n+1})^2} \cdots \quad (14)$$

In order to trace the solutions curve by Spice, we replace the differentiation with respect to the arc-length $s$ by the time $t$. Also we replace $x_i$ by voltages $v_i$ in Spice. From this, we can obtain Eq. (15).

$$\sum_{i=1}^{n} \left(\frac{dv_i}{dt}\right)^2 + \left(\frac{dv_\omega}{dt}\right)^2 = 1 \cdots\cdots\cdots\cdots\cdots\cdots \quad (15)$$

where $v_i$ $(i = 1, 2, \ldots, n)$ are coefficients for the values of the voltages and the currents in Eqs. (4) and (7) and $v_\omega$ corresponds to $\omega$. They are realized by using differentiators (simply realized by capacitors with 1 $(F)$ in Spice). The circuit in Fig. 4 realizes to satisfy the arc-length method Eq. (15). In this circuit, the voltages corresponding to the coefficients are inputted after differentiated and squared via the voltage-controlled current source (VCCS). If we set the voltage of the node $a$ as $\dot{v}_\omega$, $I_\omega = \dot{v}_\omega^2$ can be obtained by multiplier and voltage-controlled current source VCCS (MVCCS). Then, the node voltage $\dot{v}_\omega$ is integrated to obtain $v_\omega$. Note that $R$ in Fig. 4 is a very large resistor used only to avoid the $L - J$ cut-set.

## 3. Spice-Oriented Peak Search Algorithm

In this section, we explain the proposed peak search algorithm for frequency response.

In order to search the exact peaks of the frequency characteristics curve obtained by using the above-mentioned algorithm, we use a differentiator and a nonlinear limiter. The frequencies $\omega$ at the peak voltages satisfy

$$\frac{dv(\omega)}{d\omega} = 0, \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \quad (16)$$

on the frequency characteristics curve. Hence, $v(\omega)$ needs to be firstly differentiated. In order to find the exact peak points, the output of the differentiator is limited and expanded with a nonlinear limiter, which consists of a limiter and diodes as shown in Fig. 5.

We suppose the characteristics of the limiter as follows;

$$v_a = \begin{cases} -V_{max} & : \text{ for } \; v_{in} < -V_L \\ K\, v_{in} & : \text{ for } \; -V_L \le v_{in} \le V_L \\ V_{max} & : \text{ for } \; V_L < v_{in} \end{cases} \cdots \quad (17)$$

In our example, we set $V_L = 0.1\,[mV]$, $V_{max} = 10\,[V]$, and $K = 1.0 \times 10^5$. Also, the current through the nonlinear resistor of the diodes is assumed to be given by

$$i_{out} = \begin{cases} I_s \exp(\lambda v_a) & : \text{ for } \; v_a > 0 \\ -I_s \exp(-\lambda v_a) & : \text{ for } \; v_a < 0 \end{cases} \cdots\cdots \quad (18)$$

where $I_s = 10^{-12}$ and $\lambda = 40$. Then, the output characteristics of the nonlinear limiter is obtained as shown in Fig. 6.

The expansion factor of $K$ is set to be large enough. By virtue of the effect of the nonlinear limiter, the transient analysis around the zero points of the input is executed with a very small step size in Spice. As a result, we can obtain the exact peak values of the curve automatically.

## 4. Assessment of Stability

In this section, we explain the proposed peak stability assessment algorithm for frequency response.

We suppose that the circuit equation is given as

$$f(\dot{x},\ x,\ y,\ \omega t) = 0, \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \quad (19)$$

and make the variational equation for the regular periodic solution of $(\hat{x}, \hat{y})$. First, we assume the small change quantity as $(\Delta x, \Delta y)$ as

$$\begin{cases} x = \hat{x} + \Delta x \\ y = \hat{y} + \Delta y \end{cases} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (20)$$

and substitute Eq. (20) to Eq. (19). We obtain the equation as

$$f(\dot{\hat{x}},\ \hat{x},\ \hat{y},\ \omega t) + \left[ \frac{\partial f}{\partial \dot{x}} \frac{\partial f}{\partial x} \frac{\partial f}{\partial y} \right] \Bigg|_{\substack{x=\hat{x} \\ y=\hat{y}}} \begin{bmatrix} \dot{\Delta x} \\ \Delta x \\ \Delta y \end{bmatrix} = 0. \quad \cdots\cdots\cdots\cdots (21)$$

In Eq. (21), the first term is regular periodic solution and the second term is variational equation. We change the second term as

$$\dot{\Delta x} = A(t)\Delta x. \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (22)$$

In Eq. (22), $A(t)$ is the periodic function. We apply the Floquet theory for this periodic function. We write the elementary matrix of the periodic solution as $\Phi(t)$. From this reason, the solution after one period from initial value of $\Delta x(0)$ is given as follows;

$$\Delta x(T) = \Phi(T)\Delta x(0). \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (23)$$

Hence, when the eigenvalues $\lambda_k$ $(k = 1, 2, \ldots, n)$ of $\Phi(T)$ satisfy $|\lambda_k| < 1$ for all $k$, the regular periodic solution $\hat{x}$ can be assessed as stable.

In this study, we derive the variational circuit which corresponds to the variational equation and perform the transient analysis of Spice just for one period in order to obtain the components of the elementary matrix. We should repeat the transient analysis by giving different initial conditions to obtain all the components numerically. However, the number of the repeat is at most the same as the number of the state variables of the circuit. Further, it should be mentioned that we do not have to change the structure of the variational circuit even when the voltages of the regular periodic solution are changed.

## 5. Illustrative Examples

**5.1 Example 1** As the first example, we consider an LRC ladder circuit including nonlinear capacitors as Fig. 7. We consider the case that all inductors are linear and have the same inductance as $L_1 = L_2 = L_3 = L_4 = 0.1\,[H]$, and that all nonlinear capacitors have the same nonlinear characteristics as $G(q) = q + 0.8q^3$. The other parameters are set as $R = 0.05\,[\Omega]$ and $E = 0.35\,[V]$. We also set the parameter of the Fourier transformation circuit as $M = 10$.

Fig. 8 shows the sine-cosine circuit derived from the circuit in Fig. 7. In this example, for simplify, we consider the basic harmonic component only in the sine-cosine circuit and the Fourier transformation circuit. If we make the circuit for harmonic component by the same way, we can perform multi-tone analysis.

The simulated results obtained by the conventional method and the proposed algorithm are shown in Figs. 9 and 10, respectively. The horizontal axis is $\omega$ and the vertical axis is the voltage through the nonlinear capacitor $C_1$. We can see that the step size around the peaks becomes smaller in the proposed algorithm. We should mention that the frequency characteristics curve can be obtained by a single run of the transient analysis of Spice. When we name the peaks of the curve as peak1, peak2, peak3 and peak4 from the left, the frequencies and the voltages, which should be the exact values, are summarized in Table 1.

**5.2 Example 2** We consider a circuit including a transistor as shown in Fig. 11. We use the Gummel-

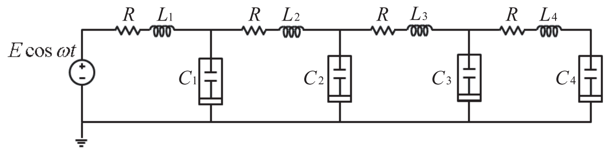

Fig. 7. LRC ladder circuit including nonlinear capacitors.

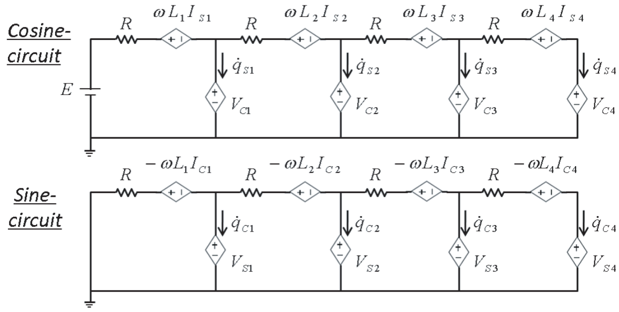

Fig. 8. Sine-cosine circuit for Fig. 7.

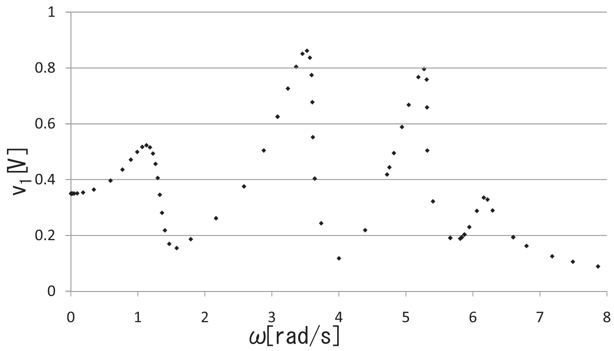

Fig. 9. Frequency response obtained by the conventional method in Sec. 2 (circuit in Fig. 7).

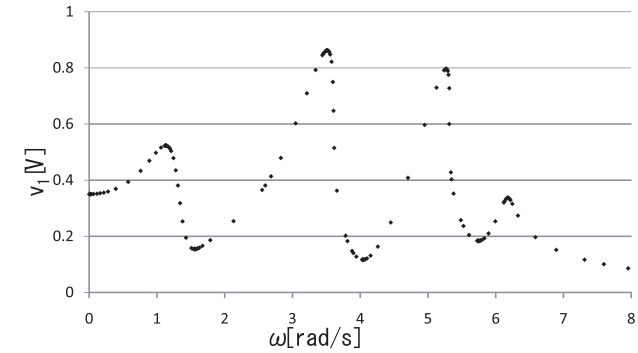

Fig. 10. Frequency response obtained by the proposed algorithm (circuit in Fig. 7). Calculation time: 4.30 [sec].

Table 1. Frequencies and voltages at peaks (circuit in Fig. 7).

| | peak1 | peak2 | peak3 | peak4 |
|---|---|---|---|---|
| $\omega[rad/s]$ | 1.1269 | 3.516 | 5.2664 | 6.1803 |
| $v_1[mV]$ | 523.1 | 862.4 | 796.3 | 337.3 |

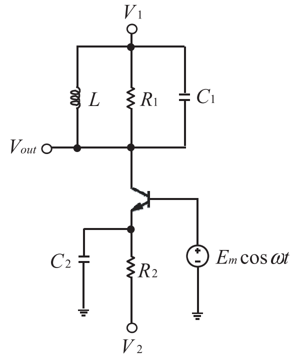


Fig. 11. Amplifier circuit including transistor.

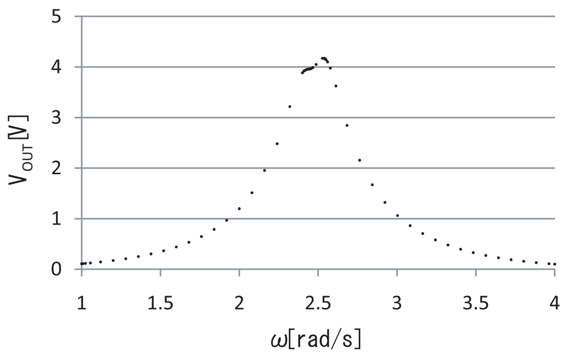


Fig. 12. Frequency response obtained by the proposed algorithm (circuit in Fig. 11). Calculation time: 3.69 [sec].

Poon model for the transistor, whose characteristics are given as follows;

$$i_C = \frac{I_S}{Q_B}\left(\exp\left(\frac{v_{BE}}{V_t}\right)-1\right) - I_S\left(\frac{1}{Q_B}+\frac{1}{B_R}\right)\left(\exp\left(\frac{v_{BE}}{V_t}\right)-1\right) \quad \cdots\cdots (24)$$

$$i_B = \frac{I_S}{B_F}\left(\exp\left(\frac{v_{BE}}{V_t}\right)-1\right) - \frac{I_S}{B_R}\left(\exp\left(\frac{v_{BE}}{V_t}\right)-1\right) \quad \cdots\cdots (25)$$

where

$$\frac{1}{Q_B} \simeq 1 - \frac{v_{BC}}{v_{AF}} - \frac{v_{BE}}{v_{AF}} \quad \cdots\cdots (26)$$

and $I_S = 2.64\,[pA]$, $\tau = 100\,[ps]$, $V_t = 0.026\,[V]$, $\Phi_B = 0.75$, $B_F = 1940$, $C_{j0} = 24.1\,[pF]$, $V_{AF} = 150\,[V]$, $F_C = 0.44$, $B_R = 1$.

The circuit parameters in Fig. 12 are set as $E_m = 0.05$, $V_1 = 10\,[V]$, $V_2 = -10\,[V]$, $R_1 = 1.5\,[k\Omega]$, $R_2 = 2\,[k\Omega]$, $L = 20\,[\mu H]$, $C_1 = 50\,[pH]$ and $C_2 = 1\,[\mu H]$.

The simulated results obtained by the proposed algorithm is shown in Fig. 11. We can see that the step size around the peaks becomes smaller as the previous example. The peak value $V_{out} = 4.164\,[V]$ for $\omega = 2.54\,[rad/sec]$ is obtained.

**5.3 Example 3** Fig. 13 shows the third example circuit including strong nonlinear capacitors. This circuit is the same one which has been investigated in [10]. In order to confirm the correctness of our algorithm, we use the same circuit parameters as [10]; $E_m = 0.35\,[V]$, $\alpha_1 = 2.4$, $\beta_1 = 12.0$, $\alpha_2 = 1.0$, $\beta_2 = 5.0$, $R_1 = 0.1\,[\Omega]$, $R_2 = R_3 = 0.02\,[\Omega]$, $L_1 = 0.2\,[H]$, $L_2 = 0.8\,[H]$, $L_3 = 0.10101\,[H]$,

$$\begin{cases} e = E_m \sin \omega t \\ C_1\ ;\ v_1 = \alpha_1 q_1 + \beta_1 {q_1}^3 \\ C_2\ ;\ v_2 = \alpha_2 q_2 + \beta_2 {q_2}^3 \end{cases} \quad \cdots\cdots (27)$$

Fig. 14 shows the simulation result of the frequency response obtained by the proposed algorithm. Although we consider the basic harmonic component only again, the curve is quite similar to that reported in [10]. Similar to the case of the previous example, our algorithm is very effective to search exact peak values of the frequency response curve. The obtained peaks are summarized in Table 2.

Next, we assess the stability of the solutions of this strong nonlinear circuit.

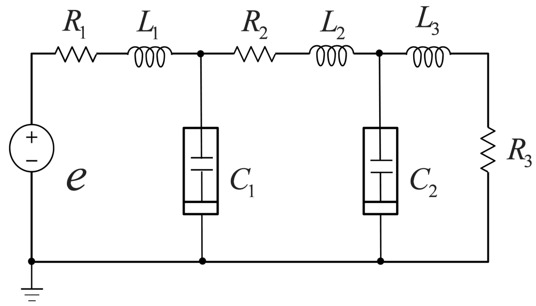


Fig. 13. Example circuit with strong nonlinearity.

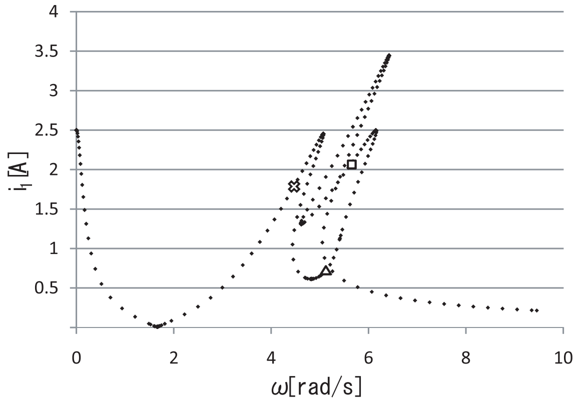


Fig. 14. Frequency response obtained by the proposed algorithm (circuit in Fig. 13). Calculation time: 2.58 [sec].

Table 2. Frequencies and currents at peaks (circuit in Fig. 13).

| | peak1 | peak2 | peak3 |
|---|---|---|---|
| $\omega[rad/s]$ | 5.067 | 6.151 | 6.420 |
| $i_1[A]$ | 2.4545 | 2.5000 | 3.4450 |

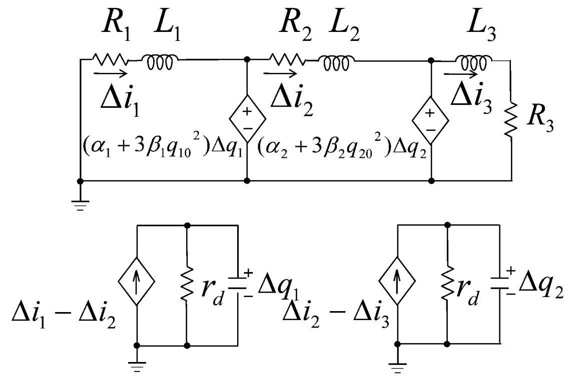


Fig. 15. Variational circuit of Fig. 13.

The circuit equation can be written as

$$\begin{cases} E_m \sin\omega t = R_1 i_1 + L_1 \dfrac{di_1}{dt} + \alpha q_1 + \beta {q_1}^3 \\ \alpha q_1 + \beta {q_1}^3 = R_2 i_2 + L_2 \dfrac{di_2}{dt} + \alpha q_2 + \beta {q_2}^3 \\ \alpha q_2 + \beta {q_2}^3 = R_3 i_3 + L_3 \dfrac{di_3}{dt} \\ \dfrac{dq_1}{dt} = i_1 - i_2 \\ \dfrac{dq_2}{dt} = i_2 - i_3. \end{cases} \quad \cdots\cdots (28)$$

If we write the variables as periodic solutions with small variations:

$$\begin{cases} i_k = i_{k0} + \Delta i_k \\ q_k = q_{k0} + \Delta q_k \end{cases} \quad \cdots\cdots (29)$$

we obtain the following variational equations as

$$\begin{cases} 0 = R_1 \Delta i_1 + L_1 \dfrac{d\Delta i_1}{dt} + \alpha \Delta q_1 + \beta 3 {q_{10}}^2 \Delta q_1 \\ \alpha \Delta q_1 + \beta 3 {q_{10}}^2 \Delta q_1 \\ \quad = R_2 \Delta i_2 + L_2 \dfrac{d\Delta i_2}{dt} + \alpha \Delta q_2 + \beta 3 {q_{20}}^2 \Delta q_2 \\ \alpha \Delta q_2 + \beta 3 {q_{20}}^2 \Delta q_2 = R_3 \Delta i_3 + L_3 \dfrac{d\Delta i_3}{dt} \\ \dfrac{d\Delta q_1}{dt} = \Delta i_1 - \Delta i_2 \\ \dfrac{d\Delta q_2}{dt} = \Delta i_2 - \Delta i_3 \end{cases} \quad \cdots\cdots (30)$$

where we neglect higher-order small terms. From these equations, we can make the variational circuit of Fig. 13 as shown in Fig. 15. In Fig. 15, $q_{10}$ and $q_{20}$ are the steady periodic solution and are calculated as

$$q_0 = Q_c \cos\omega t + Q_s \sin\omega t \quad \cdots\cdots (31)$$

where $Q_c$ and $Q_s$ are given by the sine-cosine circuit obtained from the circuit in Fig. 13.

We perform the transient analysis of this circuit just for one period and obtain the components of the elementary matrix $\Phi(T)$ as the values of the state variables of the variational circuit. Note that we have to carry out the transient analysis five times with different initial conditions to obtain all the components.

After obtaining the full matrix $\Phi(T)$, we calculate the eigenvalues by using MATLAB and assess the stability of the periodic solutions.

We assess the stability of the solutions at $(\omega, i_1) = (4.570, 1.902)$, $(4.927, 0.628)$ and $(5.560, 2.000)$ as examples. These points are marked in Fig. 14.

First, we show the elementary matrix for the case of $\omega = 4.570\,[rad/s]$ (Calculation time: 0.72 [sec]).

$$\Phi(T) = \begin{bmatrix} 0.475 & 0.222 & -1.124 & 0.130 & 0.066 \\ 0.732 & 0.515 & 1.219 & -0.131 & 0.075 \\ -0.366 & 0.096 & 0.813 & -0.021 & -0.135 \\ -2.755 & 0.630 & 1.504 & -0.496 & 0.345 \\ -1.246 & -0.551 & 6.490 & 0.3054 & -0.723 \end{bmatrix}$$

Second, for the case of $\omega = 4.927\,[rad/s]$ (Calculation time: 0.95 [sec]).

$$\Phi(T) = \begin{bmatrix} 0.480 & 0.104 & 0.041 & -0.178 & -0.016 \\ 0.407 & 0.827 & -0.640 & 0.043 & 0.301 \\ 0.012 & -0.081 & 1.360 & 0.121 & -0.273 \\ 1.833 & -0.026 & -3.077 & 0.392 & 0.317 \\ 0.485 & -0.407 & 2.228 & -0.045 & -0.010 \end{bmatrix}$$

Lastly, for the case of $\omega = 5.560\,[rad/s]$ (Calculation time: 0.53 [sec]).

$$\Phi(T) = \begin{bmatrix} 1.450 & -0.083 & -0.611 & 0.154 & -0.058 \\ -0.135 & 1.001 & -0.630 & -0.204 & 0.289 \\ -0.486 & -0.037 & 2.084 & 0.035 & -0.219 \\ -2.186 & 0.990 & -2.772 & -0.072 & 0.4769 \\ 1.537 & -0.312 & -0.707 & 0.124 & 0.309 \end{bmatrix}$$

Table 3. Eigenvalues of $\Phi(T)$.

| | $\omega = 4.570$ | $\omega = 4.927$ | $\omega = 5.560$ |
|---|---|---|---|
| $\lambda_1$ | $-0.622 + j0.554$ | $0.726 + j0.825$ | 2.530 |
| $\lambda_2$ | $-0.622 - j0.554$ | $0.726 - j0.825$ | $0.545 + j0.638$ |
| $\lambda_3$ | $0.495 + j0.625$ | $0.373 + j0.499$ | $0.545 - j0.638$ |
| $\lambda_4$ | $0.495 - j0.625$ | $0.373 - j0.499$ | 0.8662 |
| $\lambda_5$ | 0.8394 | 0.8501 | 0.2862 |

Table 3 shows the calculated eigenvalues of $\Phi(T)$ for the three cases.

We can see that the point $(\omega, i_1) = (4.570, 1.902)$ is stable, because all of eigenvalues satisfy $|\lambda| < 1$. However, for the other two points, $(\omega, i_1) = (4.927, 0.628)$ and $(5.560, 2.000)$, the solutions are unstable, because some eigenvalues do not satisfy $|\lambda| < 1$. These results agree with the results obtained in [10]. Namely, we can say that our algorithm gives the correct results with much simpler Spice-oriented algorithm.

## 6. Discussions

In 5.3, we assess the stability with all of variables. However, the states of the circuit elements should be in interdependency. From this reason, in this section, we try to assess the stability of the solutions with only limited variables in this section.

For the circuit in Fig. 13, we choose only three variables out of five state variables and perform the algorithm; namely run transient analysis of Spice for the variational circuits only three times with three sets of

Table 4. Eigenvalues when we choose $(i_2, i_3, q_1)$.

| | $\omega = 4.570$ | $\omega = 4.927$ | $\omega = 5.560$ |
|---|---|---|---|
| $\lambda_1$ | 0.975 | 1.0926 | 0.212 |
| $\lambda_2$ | 0.260 | $0.743 + j0.358$ | 0.756 |
| $\lambda_3$ | -0.402 | $0.743 - j0.358$ | 2.0449 |

Table 5. Eigenvalues when we choose $(i_3, q_1, q_2)$.

| | $\omega = 4.570$ | $\omega = 4.927$ | $\omega = 5.560$ |
|---|---|---|---|
| $\lambda_1$ | $0.187 + j0.652$ | $0.738 + j0.759$ | 2.141 |
| $\lambda_2$ | $0.187 - j0.652$ | $0.738 - j0.759$ | 0.286 |
| $\lambda_3$ | -0.780 | 0.265 | -0.106 |

initial values. The simplified elementary matrix becomes $3 \times 3$ and we assess the stabilities of the solutions by only three eigenvalues.

Firstly, for the solution at $(\omega, i_1) = (4.570, 1.902)$, which is stable, we tried to assess the solution using different ten patterns of variables. Eight patterns out of ten gave the correct result (stable), while two gave the wrong result (unstable). Secondly, for $(\omega, i_1) = (4.927, 0.628)$, which is unstable, only four patterns gave the correct result. Lastly, for $(\omega, i_1) = (5.560, 2.000)$, which is also unstable, nine patterns gave the correct result.

Although we do not have obtained any criteria, correct results are always obtained when we choose $(i_2, i_3, q_1)$ or $(i_3, q_1, q_2)$. Table 4 and Table 5 show the obtained eigenvalues when we choose $(i_2, i_3, q_1)$ and $(i_3, q_1, q_2)$, respectively.

If we analyze a circuit with weak nonlinearity and with relatively uniform parameters, we may assess the stability with limited variables and we may give some guidelines to choose the necessary variables. However, if a circuit has strong nonlinearity or nonuniform parameters, it must be very difficult to do that. For the next step of our research, we need to work on the necessary number and important nodes of the variables to be chosen under a certain conditions on the circuit.

## 7. Conclusions

We have proposed a Spice-oriented peak search and stability assessment algorithm for nonlinear circuits. By combining the sine-cosine circuit, the Fourier transformation circuit, the solution tracing circuit with the differentiator and the nonlinear limiter, we can search the exact peaks efficiently for nonlinear circuits. By introducing Floquet theory, we can perform the stability assessment of the solutions. The simulation results for example circuits showed that the proposed algorithm worked well. We also considered the case that we chose only limited numbers of the valuables for the stability assessment.

The proposed method cannot be applied to oscillator analysis directly, because we have to input the oscillation frequency to the circuit. However, if we combine the proposed method with the algorithm proposed in our previous research (4), which can obtain oscillation frequencies using Newton homotopy circuit, the proposed method would be extended to the oscillator analysis. As a future research for our proposed method, we would like to assess stability for printed circuit boards using our proposed method that includes limited conditions.

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