Journal of Signal Processing, Vol. 8, No. 6, pp. 439–447, November 2004

PAPER

Search of Many Good Solutions of QAP by Connected Hopfield NNs with Intermittency Chaos Noise

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Abstract **The method of adding chaos noise to the Hopfield neural network (NN) in application to combinatorial optimization problems has been proposed, and many researchers have suggested that adding intermittency chaos noise near the three-periodic window of the logistic map will yield the best performance. However, the Hopfield NN with chaos noise sometimes remains in a group of several solutions. In this study, some Hopfield NNs with intermittency chaos noise are connected like hierarchical networks in order to find good solutions of quadratic assignment problems (QAPs). It is confirmed that connected Hopfield NNs with intermittency chaos noise provide numerous nearly optimal solutions.**

Keywords: intermittency chaos, Hopfield neural network, parallel connection, combinatorial optimization problems, quadratic assignment problems

1. Introduction

Combinatorial optimization problems have can describe various actual problems mathematically. However, there is a large disparity between describing problems mathematically and solving them. Solving a very difficult problem takes a long time using current computer systems. (e.g, longer than the age of the universe) which defeats the purpose. The method of using the Hopfield neural network (NN) [1] has been proposed for combinatorial optimization problems as an approximate means based on search. In this method, if we choose appropriately connection weights between neurons according to the given problem, we can obtain a good solution on the basis of the energy minimization principle. However, the solutions are often trapped in a local minimum and do not reach the global minimum. In order to avoid this critical problem, several people has proposed the addition of some kinds of noise to the Hopfield NN. Many researchers have suggested that adding intermittency chaos noise near the three-periodic window of the logistic map will yield the best performance [2]. In order to clarify why the addition of intermittency chaos noise is better than that of fully developed chaos noise, we have investigated the performance using the burst noise generated by the Gilbert model for traveling salesman problems (TSPs) [3] and quadratic assignment problems (QAPs), which is said to be one of most difficult problem to solve among combinatorial optimization problems [4]. In the results, we confirmed that the Hopfield NN with intermittency chaos noise can provide a large variety of solutions. It is very important to find a variety of solutions for very difficult problems with a large number of elements, such as QAPs. However, it is extremely difficult to obtain a good parameter set for the network. Furthermore, the Hopfield NN with intermittency chaos noise has some problems: the solution of the network remains at the same state during a certain period; the solution of the network arrives at states found a number of previously times. If we can avoid these problems, the network would provide many more nearly optimal solutions.

In this study, we connect some Hopfield NNs with intermittency chaos noise in the manner of hierarchical networks to provide many nearly optimal solutions of QAPs. If we connect some networks in an appropriate way, the search space of solutions would be expanded while maintaining the energy minimization principle of the Hopfield NN. We prepare multiple Hopfield NNs with the same weight pattern between neurons for a given QAP. Namely, each network operates independently to find solutions of the QAP. In order to give the networks the task of searching different parts of the solution space, we propose a method of setting outputs of some neurons to be zero according to the firing patterns in the previous networks. From the results of computer simulations, we confirm that connected Hopfield NNs with intermittency chaos noise can search a broad range of energy functions and provide many nearly optimal solutions.

2. Solving QAP with Hopfield NN

Various methods are proposed for solving a QAP, which is one of the NP-hard combinatorial optimization problems. QAP is expressed as follows: given two matrices, distance matrix $\mathbf{C}$ and flow matrix $\mathbf{D}$, find the permutation $\mathbf{p}$ which corresponds to the minimum value of the objective function $f(\mathbf{p})$ in

$$f(\mathbf{p}) = \sum_{i=1}^{N}\sum_{j=1}^{N} C_{ij} D_{p(i)p(j)} \tag{1}$$

where C_{ij} and D_{ij} are the (i,j)-th elements of $\mathbf{C}$ and $\mathbf{D}$, respectively, $p(i)$ is the i-th element of vector $\mathbf{p}$, and N is the size of the problem. There are many real applications which are formulated by Eq. (1). One example of QAPs is to find an arrangement of factories to minimize the cost. The cost is given by the distance between cities and the flow of products between factories (Fig. 1). Other examples are the placement of logical modules on an IC chip and the distribution of medical services in a large hospital.

Because a QAP is very difficult, it is almost impossible to determine the optimal solutions in large problems. The largest problem whose optimal solution can be obtained may be only 36 according for a recent study [5]. Furthermore, computation time is very long for obtaining exact optimal solutions. Therefore, it is common to develop heuristic methods which search for nearly optimal solutions in a reasonable time.

To solve an N-element QAP using the Hopfield NN, $N\times N$ neurons are required, and the following energy function is defined:

$$E = \sum_{i,m=1}^{N}\sum_{j,n=1}^{N} w_{im;jn} x_{jn} + \sum_{i,m=1}^{N} \theta_{im} x_{im} \tag{2}$$

The neurons are coupled to each other by the synaptic connection weight. The weight between the (i,m)-th neuron and (j,n)-th neuron and the threshold of the (i,m)-th neuron are described by

$$\begin{aligned} w_{im;jn} &= -2\Big\{ A(1-\delta_{mn})\delta_{ij} \\ &\quad + B\delta_{mn}(1-\delta_{ij}) + \frac{C_{ij}D_{mn}}{q} \Big\} \\ \theta_{im} &= A + B \end{aligned} \tag{3}$$

where A and B are positive constants, q is a normalization parameter to correspond given problems, and δ_{ij} is Kronecker's delta. The states of $N\times N$ neurons are asynchronously updated according to the following difference equation:

$$\begin{aligned} x_{im}(t+1) = g\Bigg(&\sum_{j,n=1}^{N} w_{im;jn} x_{jn}(t) \\ &- \theta_{im} + \beta z_{im}(t) \Bigg) \end{aligned} \tag{4}$$

where g is a sigmoidal function defined as

$$g(x) = \frac{1}{1+\exp\left(-\frac{x}{\varepsilon}\right)} \tag{5}$$

z_{im} is additional intermittency chaos noise, and β limits the amplitude of the intermittency chaos noise. Figure 2 shows a conceptual neuron model for this NN.

We also use the method suggested by Sato *et al.* (1.1 in [6]) to decide the firing of neurons.

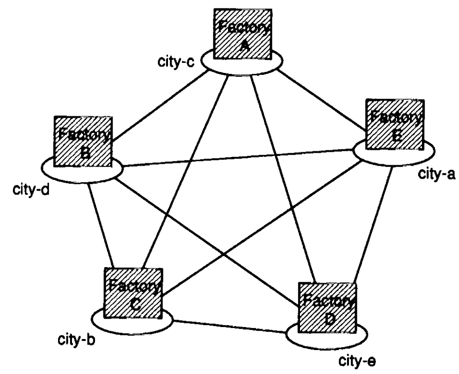


Fig. 1 An example of QAP

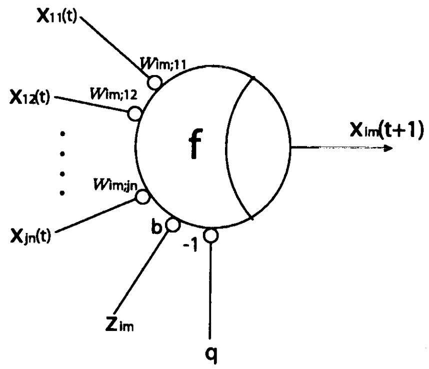


Fig. 2 A neuron model

3. Intermittency Chaos Noises

In this section, we describe the intermittency chaos noise injected into the Hopfield NN. The logistic map is used to generate intermittency chaos noise:

$$\hat{z}_{im}(t+1) = \alpha \hat{z}_{im}(t)(1 - \hat{z}_{im}(t)) \tag{6}$$

Varying parameter α, Eq. (6) behaves chaotically via a period-doubling cascade. When we inject intermittency chaos noise into the Hopfield NN, we normalize $\hat{z}_{im}$ by

$$z_{im}(t+1) = \frac{\hat{z}_{im}(t) - \bar{z}}{\sigma_z} \tag{7}$$

where $\bar{z}$ is the average of $\hat{z}(t)$, and σ_z is the standard deviation of $\hat{z}(t)$. Figure 3 shows an example of the time series of the intermittency chaos noise near the three-periodic window for α=3.8274 As we can see from the figure, the chaotic time series can be divided into two phases: laminar parts of periodic behavior with a period of three and burst parts of spread points over the invariant interval. We have confirmed that intermittency chaos near the three-periodic window gives better performance than fully developed chaos for TSP [3] and QAP [4].

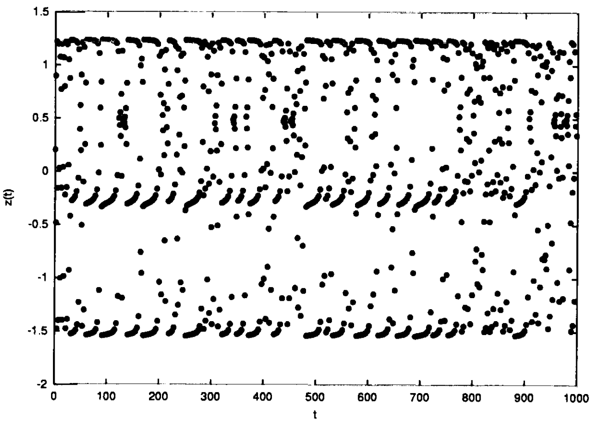


Fig. 3 Intermittency chaos noise near the three-periodic window for α=3.8274

4. Connected Hopfield NNs

In this study, we connect some Hopfield NNs with intermittency chaos noise in the manner of hierarchical networks in order to find many nearly optimal solutions. We consider that reflecting the firing pattern of one network to the firing pattern of the next network by connecting the neurons between two networks is important. In this study, we propose a method of setting the output of some neurons to zero according to the firing patterns in the previous networks, in order to give the networks the tasks of searching different parts of the solution space. By this method, connected Hopfield NNs with intermittency chaos noise can search a broad range of the energy function.

The connected Hopfield NNs with intermittency chaos noise are shown in Fig. 4. The weights of neurons are configured the same and each network operates independently. In this figure, • represents a firing neuron and ⊗ a connection neuron. We now explain the updating of the states of the neurons. First, all of the neurons in the $(K-1)$-th network are asynchronously updated. Next, one neuron is selected from among the firing neurons of the $(K-1)$-th network at random, and we connect the selected neuron to the neurons at the same position in the K-th, $(K+1)$-th, $\cdots$ networks. The output of the connected neurons is set to zero, namely, firing of the neurons is stopped while updating. Next, all of the neurons in the K-th network are asynchronously updated.

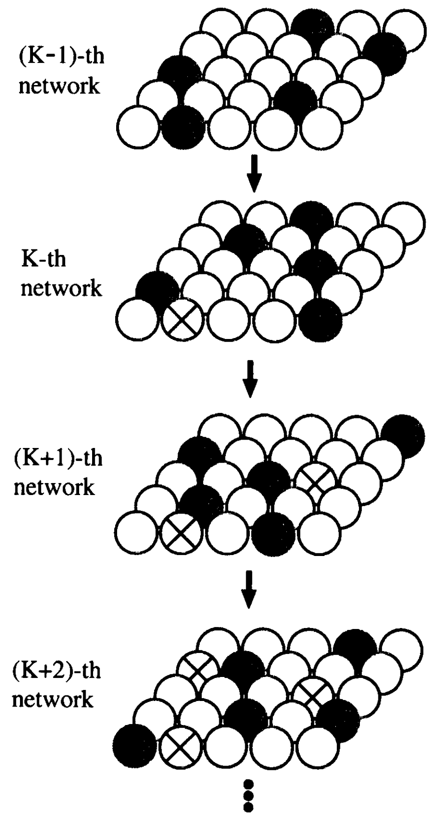


Fig. 4 Connected Hopfield NNs

5. Simulated Results

The problem used here was chosen from the site QAPLIB which provides a collection of benchmark problems [5]. We carried out computer simulations for "Tai12a" using 2~8 connected Hopfield NNs with intermittency chaos noise. The global minimum of this problem is known to be 224416. The parameters of the Hopfield NNs are fixed at $A = 0.86$, $B = 0.86$, $q = 12000$ and $\varepsilon = 0.02$, and the amplitude of the injected intermittency chaos noise is fixed at $\beta = 0.5$. These parameters are selected in order to find many nearly optimal solutions when we operate only 1network for the Tai12a problem. The total number of updates of the network is 12000. For example, for 4 connected networks, the number of updates is 3000 per network, and the total number of updates becomes 12000.

Next, we explain how to accept solutions. The connected Hopfield NNs with intermittency chaos noise search various solutions. However, the state of the Hopfield NNs sometimes remains around a group of several solutions. We consider that such behavior is not useful for finding optimal or nearly optimal solutions. Therefore, we adopt the only-different-solutions method. Namely, we look only at solutions that have been previously not found.

5.1 Changing the number of networks

The simulated results of the frequency distribution of the accepted solutions are shown in Fig. 5 for different numbers of connected Hopfield NNs with intermittency chaos noise. For comparison, the result obtained from only one Hopfield NN with intermittency chaos noise is also shown in Fig. 5(a). The horizontal axis is the cost of the QAP calculated using Eq. (1) and the vertical axis is the frequency of the solutions, which means the number of accepted solutions with the corresponding cost found during 12000 iterations.

We can see that many nearly optimal solutions are found for the case of connected Hopfield NNs with intermittency chaos noise. When the number of connected networks is large, many good solutions are found. However, when the number of the networks exceeds four, the performance of the networks gradually becomes worse because the number of connected neurons increases and the networks become sluggish.

From these results, we confirmed that the connected Hopfield NNs with intermittency chaos noise can provide many good solutions in an expanded are of the solution space, when an appropriate number of connected networks is chosen according to the given problem. This is because each network is given the task of searching different parts of the solution space. We believe that it is very important to find many nearly optimal solutions for very difficult problems, such as QAPs with a large number of elements, by connecting several Hopfield NNs with intermittency chaos noise.

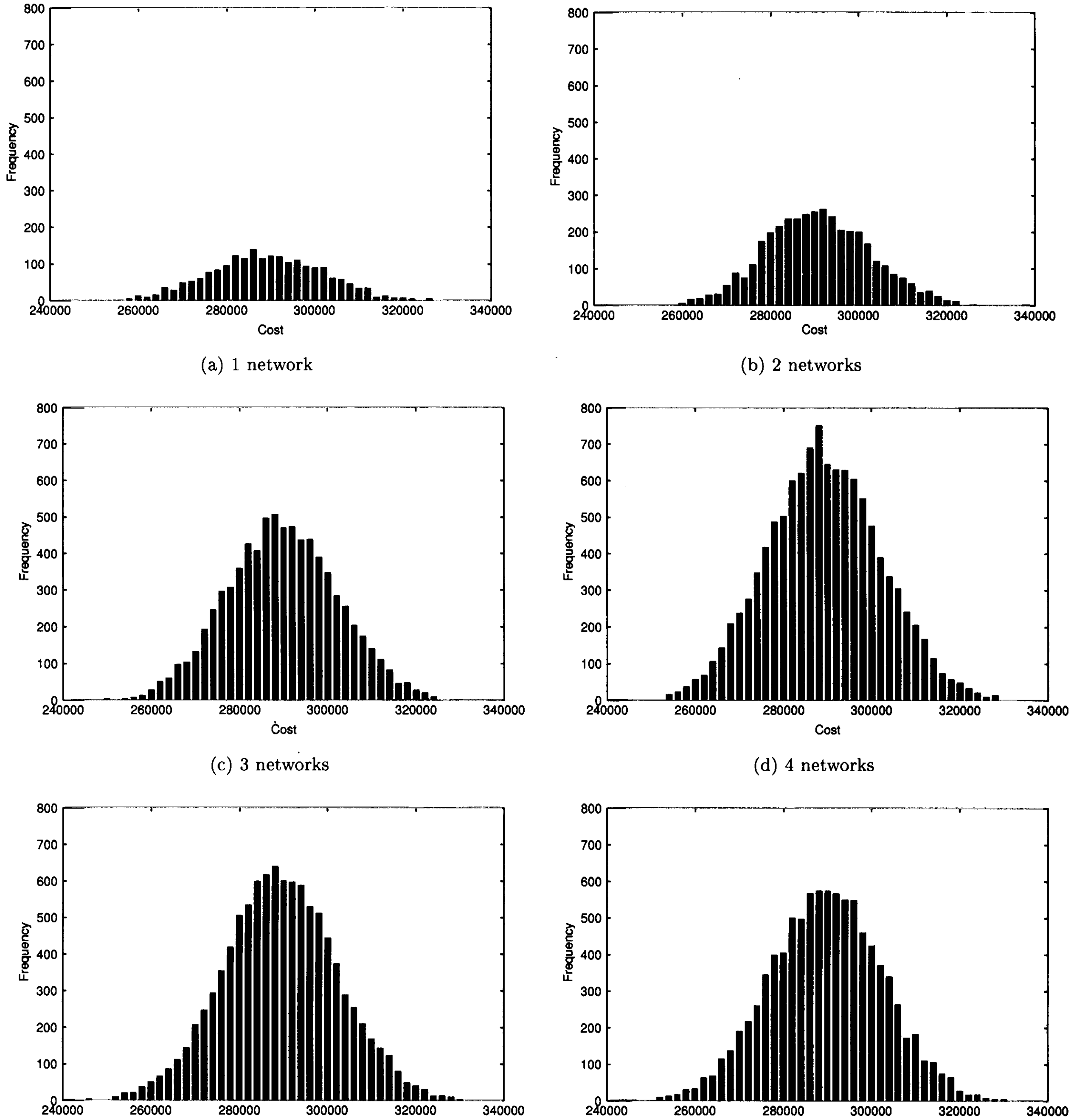
We also consider that it is not difficult to obtain a good parameter set for the case of connected Hopfield NNs, because connecting multiple networks has the same effect as modifying the parameter set of each network. Namely, the solution space is expanded by increasing the number of connected networks. This is similar to the effect of changing the parameter that governs cost q and the amplitude of noise β. This is one of the most practical points of the proposed network, because it is usually extremely difficult to obtain a good parameter set of the Hopfield NNs for solving difficult combinatorial optimization problems.

5.2 Changing the number of connected neurons

In this section, we investigate the frequency distribution of the accepted solutions when the number of connected neurons is increased. When the number of connected neurons is 6, 6 neurons are newly connected in each network after the 2nd network. Namely, when the number of connected networks becomes large, the networks become sluggish because the output of many neurons is set to zero. The simulated frequency distribution of the accepted solutions provided by 2 networks is shown in Fig. 6. In this figure, we confirmed that an appropriate number of connected networks exists. The number of accepted solutions increases as the number of connected neurons increase. However, when the number of the connected neurons exceeds 9, the performance of the networks gradually becomes worse. In the case of 2 networks, the networks gain good performance when the number of connected neurons is appropriately set. Furthermore, we investigate the ratio of the number of accepted solutions found by each network to the total number of accepted solutions to clarify how many solutions each network finds in each set of connected network. The results in the case of 2 networks are shown in Fig. 7. We consider that an appropriate number of connected networks exists.

Next, the simulated results of the frequency distribution of the accepted solutions found by 8 connected networks are shown in Fig. 8, when the number of connected neurons was increased. In this figure, we confirmed that the number of accepted solutions decreases as the number of connected neurons increases. Namely, when the number of the connected networks is 8, the networks operate well to find various solutions when the number of connected neurons is small. The results of the the ratio of the number of accepted solutions found by each network to the total number of accepted solutions in the case of 8 connected networks are shown in Fig. 9. We can see that when the number of connected neurons is small, the networks find many solutions and each network can find various kinds of solution equally well. However, when the number of connected neurons becomes large, the performance of networks becomes worse and the networks rarely find solutions. For reference, the results of the frequency distribution of accepted solutions in the case of 3 and 6 connected networks are shown in Fig. 10, when the number of connected neurons was increased.

On the basis of the above results, we consider that although the performance of the networks is dependent on the problem itself, it is not very effective to increase the number of connected neurons in an effort



(a) 1 network

(b) 2 networks

(c) 3 networks

(d) 4 networks

(e) 6 networks

(f) 8 networks

Fig. 5 Frequency distributions of accepted solutions for various numbers of connected networks (total number of updates is 12000): (a) 1 network (number of updates is 12000): (b) 2 networks (number of updates is 6000 per network), (c) 3 networks (number of updates is 4000 per network), (d) 4 networks (number of updates is 3000 per network), (e) 6 networks (number of updates is 2000 per network), (f) 8 networks (number of updates is 1500 per network)

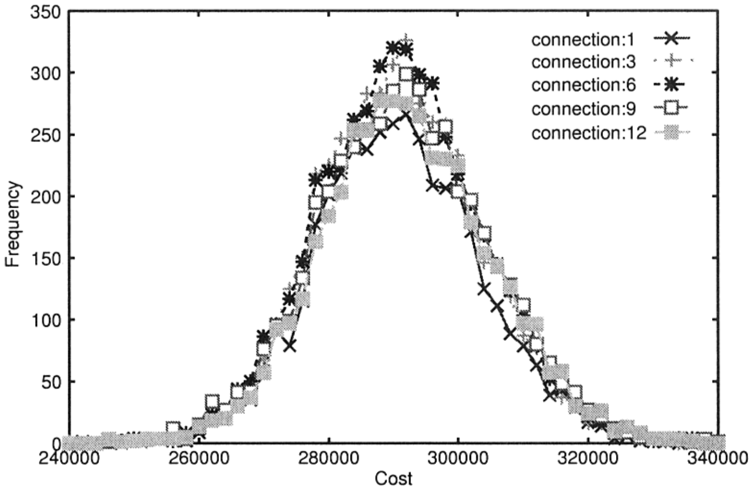
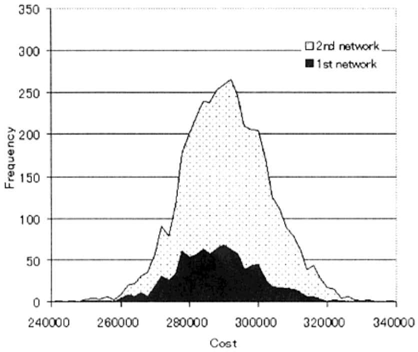
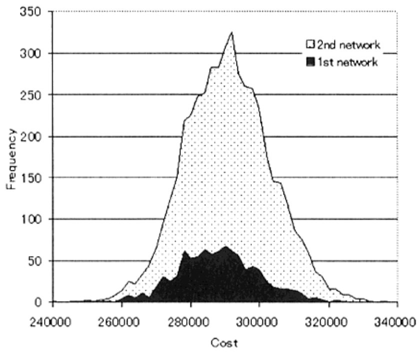


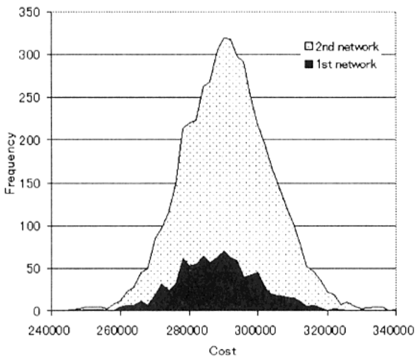
Fig. 6 Frequency distribution of accepted solutions for various numbers of connected neurons (2 networks)



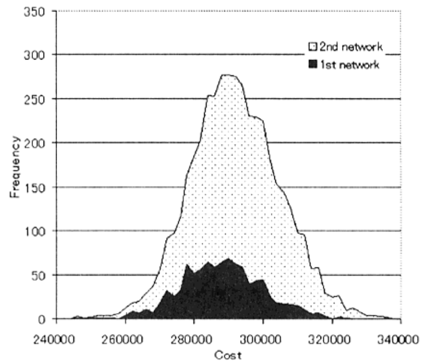
(a) connection: 1



(b) connection: 3



(c) connection: 6



(d) connection: 12

Fig. 7 Ratio of the number of accepted solutions found by each network to the total number of accepted solutions (2 networks): (a) connection: 1, (b) connection: 3, (c) connection: 6, (d) connection: 12

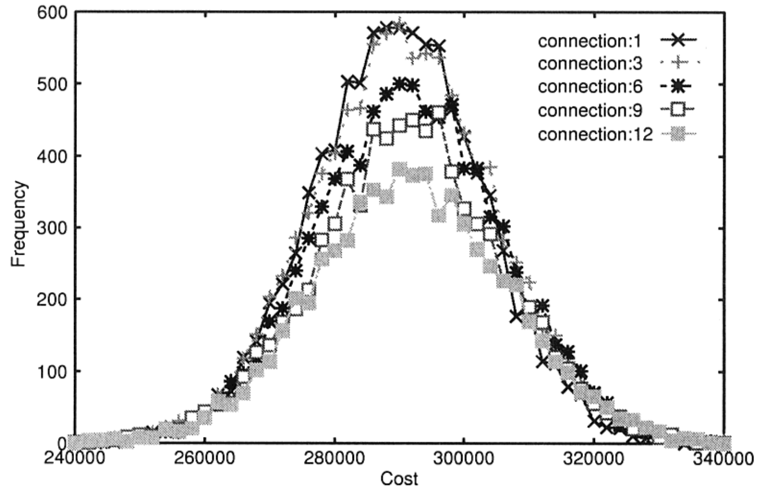
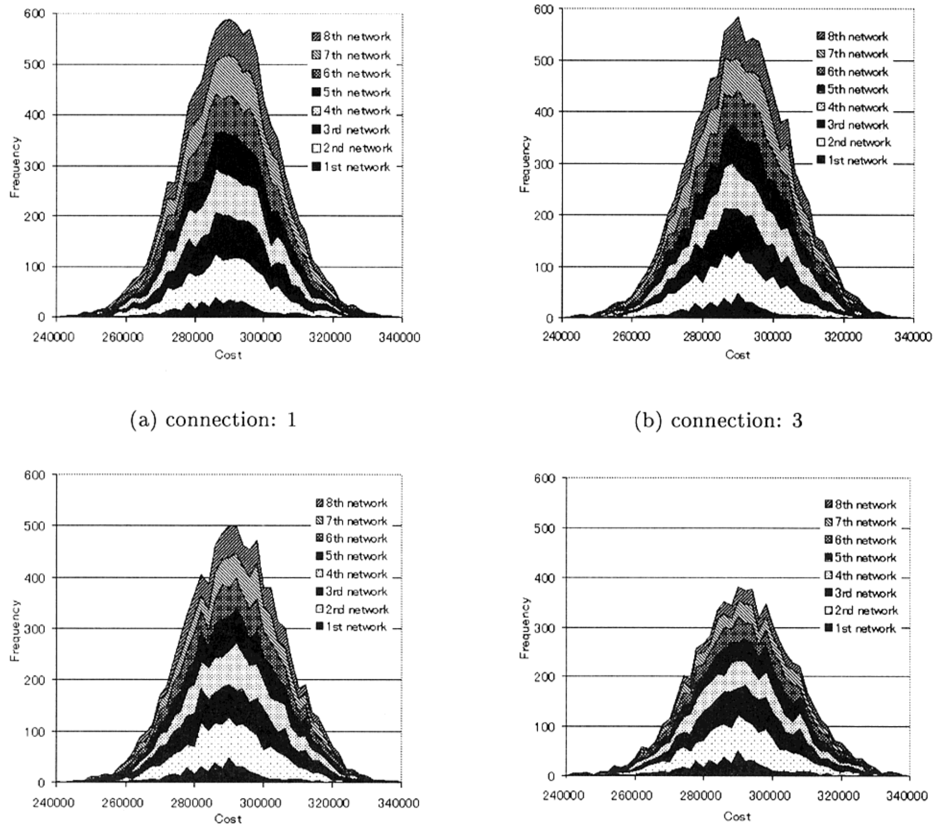


Fig. 8 Frequency distribution of solutions for various numbers of connected neurons (8 networks)



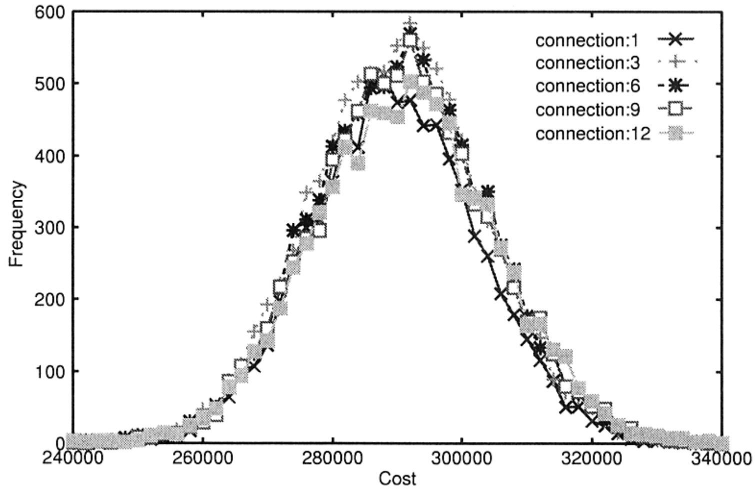
(a) connection: 1

(b) connection: 3

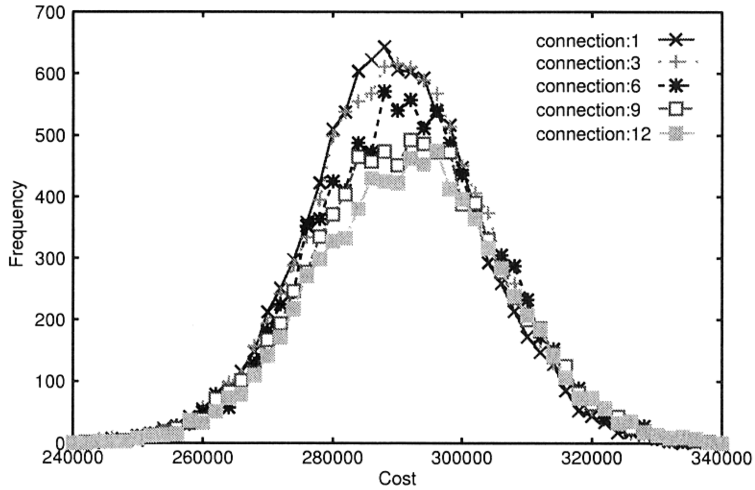
(c) connection: 6

(d) connection: 12

Fig. 9 Ratio of the number of accepted solutions found by each network to the total number of accepted solutions (8 networks): (a) connection: 1, (b) connection: 3, (c) connection: 6, (d) connection: 12



(a) 3 networks



(b) 6 networks

Fig. 10 Frequency distribution of accepted solutions for various numbers of connected neurons: (a) 3 networks, (b) 6 networks to improve the performance of the networks. However, it is a certainty that all of the networks can search a broad range of energy functions and find numerous nearly optimal solutions by connecting neurons between each network.

6. Conclusion

In this study, we connected some Hopfield NNs with intermittency chaos noise in the manner of hierarchical networks in order to find many nearly optimal solutions of QAPs. We prepared multiple Hopfield NNs with the same weight pattern which is decided for any given QAP between neurons. Namely, each network operates to find solutions of the QAP independently. In order to give the networks the task of searching different parts of the solution space, we proposed a method of setting output of some neurons to zero according to the firing patterns in the previous networks. Through computer simulations, we confirmed that the connected Hopfield NNs with intermittency chaos noise could search a broad range of energy functions and obtained many nearly optimal solutions.

Acknowledgement

This work was partly supported by the Kayamori Foundation of Informational Science Advancement's Research Grant.

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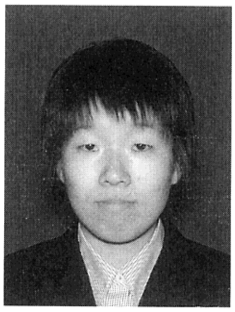

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(Received May 31, 2004; revised August 12, 2004)