PAPER

Analysis of Phase-Inversion Waves in Coupled Oscillators Synchronizing at In-and-Anti-Phase

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SUMMARY Recently, we have discovered wave propagation phenomena which are continuously existing waves of changing phase states between two adjacent oscillators from in-phase to anti-phase or from anti-phase to in-phase in van der Pol oscillators coupled by inductors as a ladder. We named the phenomena as "phase-inversion waves." In this study, phase-inversion waves which exist in the state of in-and-anti-phase synchronization have been found. We observe the phenomena by circuit experiments and computer calculations, and investigate them.

key words: *coupled oscillators, phase difference, phase-inversion waves, in-and-anti-phase synchronization*

1. Introduction

Many studies on synchronization phenomena of coupled oscillators have been carried out up to now in various fields, physics [1]–[3], biology [4], electrical engineering [5]–[14], and so on. Endo et al. have reported details of theoretical analysis and circuit experiments about some coupled oscillators as a ladder, a ring and a two-dimensional array [6]–[8].

Recently, the authors have discovered a wave propagation phenomena that phase states between adjacent oscillators change from in-phase to anti-phase or from anti-phase to in-phase in oscillators coupled by inductors as a ladder [11]. We named the phenomena as "phase-inversion waves." It is very important to analyze the phase-inversion waves and to clarify the mechanism of the generation, because it is similar to propagation phenomena of electrical information in an axial fiber of nervous system. In [11], we explained the mechanism of the generation of the phase-inversion waves by using the relationship between phase states and instantaneous oscillation frequencies. Further, we confirmed that the phenomena could be explained by a simplified mathematical model [12].

In this study, we investigate the phase-inversion waves observed from the same van der Pol oscillators ladder but in different synchronization mode; in-and-anti phase synchronization, which is similar to the phenomenon reported in [10]. In this synchronization mode, in-phase and anti-phase synchronizations exist alternately. An interesting point of this synchronization is that the edge oscillators and their adjacent oscillators must be synchronized at anti-phase. This causes various limitations of the generated phenomena with respect to the phase-inversion waves. For example, in the past studies, only even numbers of the phase-inversion waves can exist and the number of the oscillators do not affect the observed phenomena. On the other hand, in this study, even numbers of the phase-inversion waves exist in even numbers of the oscillators and odd numbers of the phase-inversion waves exist in odd numbers of the oscillators. The reason of this difference is explained in Sect. 4. Further, the mechanisms of propagation, reflection at an edge of the array and reflection by collision of two phase-inversion waves are different from those reported in [11]. They are explained by using a relationship between phase states and instantaneous oscillation frequencies in Sect. 5. The computer simulated results are confirmed by circuit experiments.

2. Circuit Model

The circuit model used in this study is shown in Fig. 1. N van der Pol oscillators are coupled by inductors L_0. We carried out computer calculations for the cases of $N = 9$ and 20. Further, we carried out circuit experiments for the case of $N = 9$. In the computer calcu-

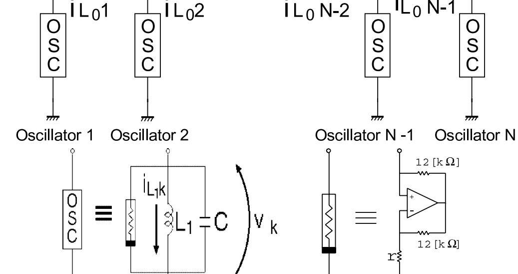

Fig. 1 Coupled van der Pol oscillators as a ladder.

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lations, we assume the $v-i$ characteristics of the nonlinear negative resistors in the circuit by the following function.

$$i_r(v_k) = -g_1 v_k + g_3 v_k^3 \quad (g_1, g_3 > 0). \tag{1}$$

The circuit equations governing the circuit in Fig. 1 are written as

[First Oscillator]

$$\dot{x}_1 = y_1 \tag{2}$$
$$\dot{y}_1 = -x_1 + \alpha(x_2 - x_1) + \varepsilon\left(y_1 - \frac{1}{3}y_1^3\right)$$

[Middle Oscillators]

$$\dot{x}_k = y_k \tag{3}$$
$$\dot{y}_k = -x_k + \alpha(x_{k+1} - 2x_k + x_{k-1}) + \varepsilon\left(y_k - \frac{1}{3}y_k^3\right)$$
$$(k = 2 \sim N–1)$$

[Last Oscillator]

$$\dot{x}_N = y_N \tag{4}$$
$$\dot{y}_N = -x_N + \alpha(x_{N-1} - x_N) + \varepsilon\left(y_N - \frac{1}{3}y_N^3\right),$$

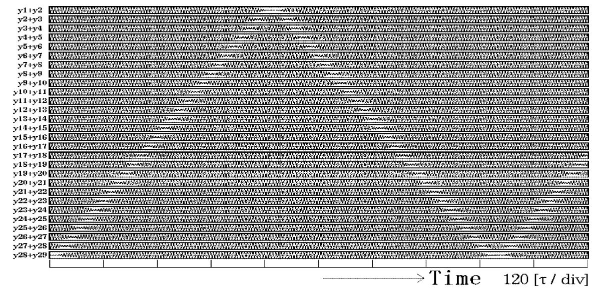
where

$$t = \sqrt{L_1 C}\tau,\ i_{L_1 k} = \sqrt{\frac{Cg_1}{3L_1 g_3}}x_k, v_k = \sqrt{\frac{g_1}{3g_3}}y_k,$$
$$\alpha = \frac{L_1}{L_0},\ \varepsilon = g_1\sqrt{\frac{L_1}{C}},\ \frac{d}{d\tau} = \text{“ . ”}. \tag{5}$$

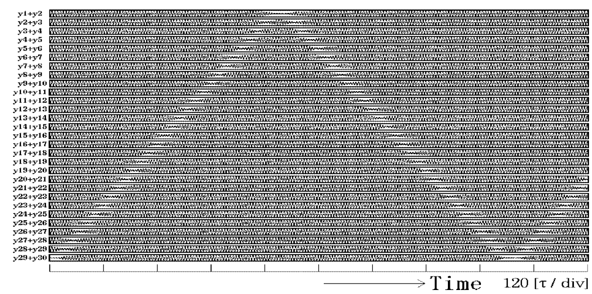
It should be noted that α corresponds to the coupling of the oscillators and ε corresponds to the nonlinearity of the oscillators. Throughout the paper, we fix $\alpha = 0.10$ and $\varepsilon = 0.30$ and calculate (2)–(4) by using the fourth-order Runge-Kutta method with the stepsize $\Delta\tau = 0.01$.

3. Example of Phase-Inversion Waves

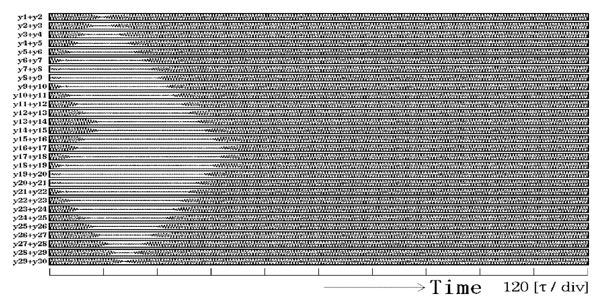
Figure 2 shows some typical examples of phase-inversion waves reported previously. They are computer calculated results from the circuit with the sizes of $N = 29$ and 30. In the figures, the vertical axes are the sum of the voltages of adjacent oscillators and the horizontal axis is time. White regions in the diagram correspond to the states that the sum of the voltages is close to zero, namely the adjacent two oscillators are synchronized at anti-phase. While, black regions correspond to the states that the sum of the voltages has large amplitude. We can see that the adjacent two oscillators are synchronized at in-phase in the black regions from Fig. 3, which shows the same phenomenon as Fig. 2(b), but the vertical axes are the difference between the voltages of adjacent oscillators. Namely, we



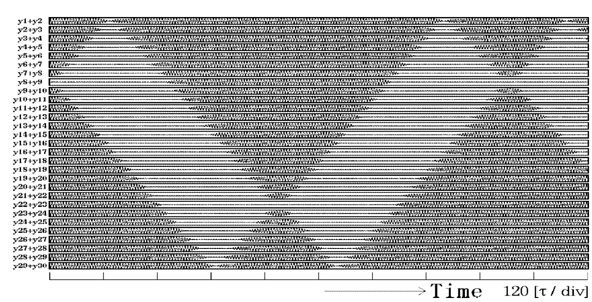
(a) Double waves ($N = 29$).



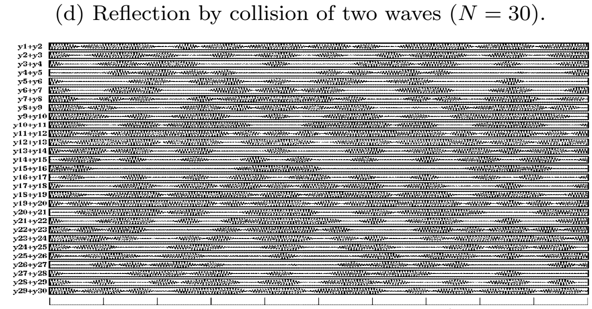
(b) Double waves ($N = 30$).



(c) Extinction by collision of waves ($N = 30$).

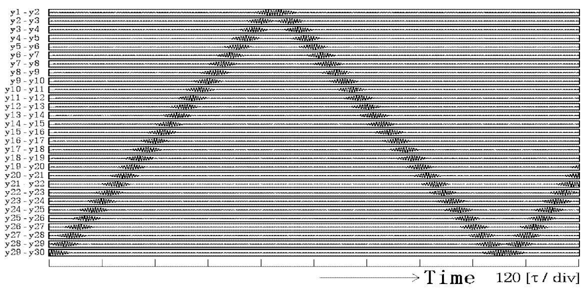


(d) Reflection by collision of two waves ($N = 30$).



(e) Decuple waves ($N = 30$).

Fig. 2 Examples of phase-inversion waves. $y_k + y_{k+1}$ vs. time ($k = 1 \sim N-1$). $\alpha = 0.10$, $\varepsilon = 0.30$ and $\Delta\tau = 0.01$.



Double waves ($N = 30$).

Fig. 3 Examples of phase-inversion waves. $y_k - y_{k+1}$ vs. time ($k = 1 \sim N-1$). $\alpha = 0.10$, $\varepsilon = 0.30$ and $\Delta\tau = 0.01$.

can say that the phase-inversion waves in Fig. 2 propagate in the state of in-phase synchronization.

Remark A:
Strictly speaking, when the phase-inversion waves propagate, the whole circuits are not synchronized. However, because the concept of synchronization is useful to explain the phenomena, the phase-inversion waves are described such as they propagate in a synchronization state.

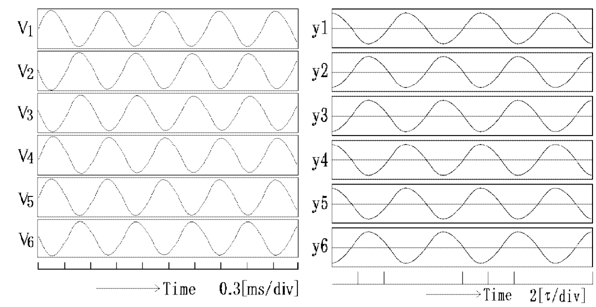
In Figs. 2 and 3, we can see the wave propagation, reflection and extinction. Even numbers of the phase-inversion waves exist in both odd and even numbers of coupled oscillators.

It has been known that two van der Pol oscillators coupled by an inductor have two stable phase states, namely in-phase synchronization and anti-phase synchronization. Further, the oscillation frequency for the in-phase synchronization is smaller than the oscillation frequency for the anti-phase synchronization. By using these features, we could explain the mechanism of the phenomena in Fig. 2 by using the relationship between phase states and instantaneous oscillation frequencies (See [11] for the details).

4. Phase-Inversion Waves in the State of In-and-Anti-Phase Synchronization

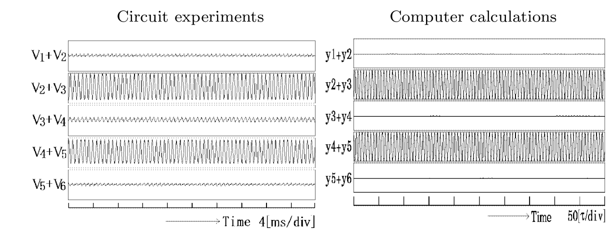
4.1 In-and-Anti-Phase Synchronization

In the circuit model, we can observe another type of synchronization as shown in Fig. 4. The vertical axes are the amplitudes of the voltages and the horizontal axis is time. We name this synchronization state as "in-and-anti-phase synchronization" because in-phase and anti-phase exist alternately. Also, in this synchronization state, the edge oscillators and their adjacent oscillators can not be synchronized at in-phase. Hence, this synchronization state can be observed only when N is an even number. The oscillation frequency for the in-and-anti-phase synchronization f_{mid} is almost the average of the oscillation frequency for the in-phase synchronization f_{low} and the oscillation frequency for the anti-phase synchronization f_{high}.

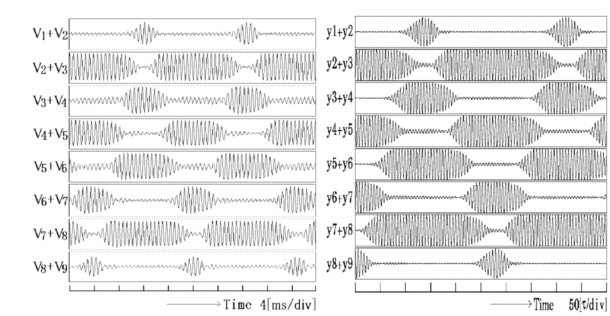


(a) (b)

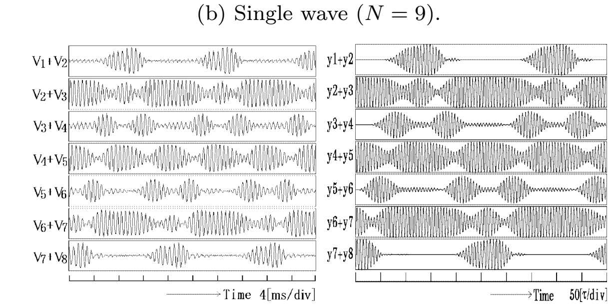
Fig. 4 Example of in-and-anti-phase synchronization. (a) Circuit experimental result for L_0=1170 mH, L_1=200 mH, C=68 nF and r=1 kΩ. (b) Computer calculated result for $\alpha = 0.10$, $\varepsilon = 0.30$ and $\Delta\tau = 0.01$.



(a) No wave ($N = 6$).



(b) Single wave ($N = 9$).

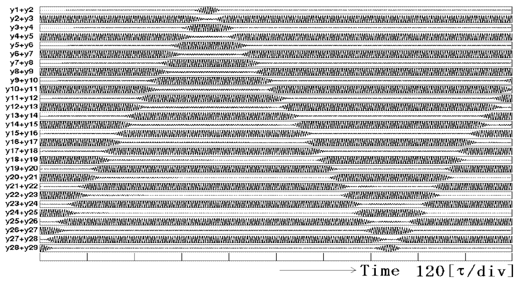


(c) Double waves ($N = 8$).

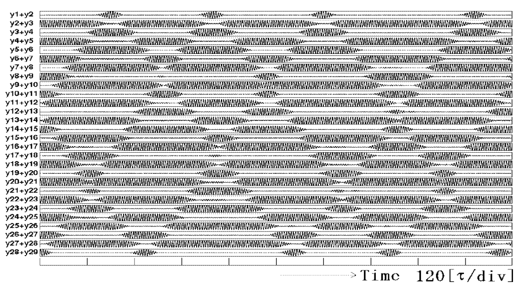
Fig. 5 Experimental results. Circuit experiments: L_0 = 1170 mH, L_1 = 200 mH, C = 68 nF and r=1 kΩ. Computer calculations: $\alpha = 0.10$, $\varepsilon = 0.30$ and $\Delta\tau = 0.01$.

4.2 Phase-Inversion Waves

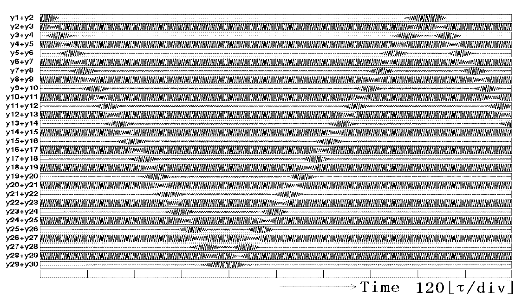
Figures 5 and 6 show examples of phase-inversion waves in the state of in-and-anti-phase synchronization. The vertical axes of Figs. 5 and 6 are the sum of the voltages of adjacent oscillators and the horizontal axis is time.



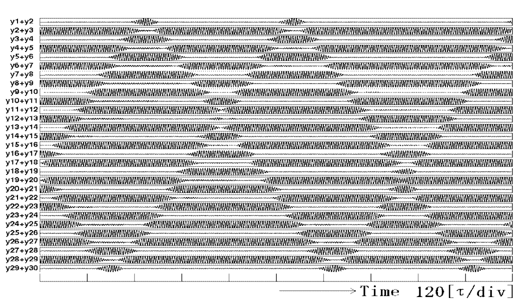
(a) Single wave ($N = 29$).



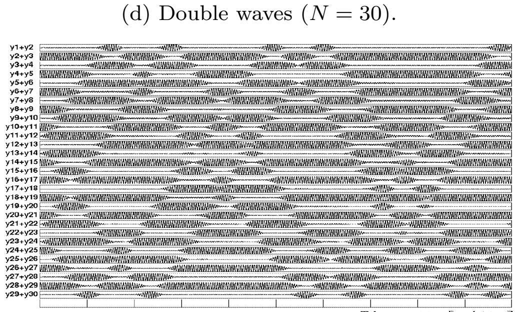
(b) Triple waves ($N = 29$).



(c) Double waves ($N = 30$).



(d) Double waves ($N = 30$).



(e) Quadruple waves ($N = 30$).

Fig. 6 Examples of phase-inversion waves. $\alpha = 0.10$, $\varepsilon = 0.30$ and $\Delta\tau = 0.01$.

Figure 5 shows circuit experimental results and the corresponding computer calculated results obtained for the cases of $N \leq 9$. In Fig. 5(a), phase-inversion waves do not exist. A phase-inversion-wave exists continuously in Fig. 5(b). We can see that the change of the phase states from in-phase to anti-phase or from anti-phase to in-phase propagates. In Fig. 5(c), a pair of phase-inversion-waves exist.

Figure 6 shows computer calculated results obtained for the cases of $N = 29$ and 30. We can observe phase-inversion waves similar to Fig. 5.

In in-and-anti-phase synchronization, we could not observe wave extinction by collision of two waves like Fig. 2(c).

4.3 Single Phase-Inversion Wave

In this subsection, We clarify the relationship between the number of oscillators and the number of phase-inversion waves.

<in In-Phase Synchronization>

In the state of the in-phase synchronization, the edge oscillators and their adjacent oscillators must be synchronized at in-phase. Therefore, in spite of the number of the oscillators in the array, only even numbers of phase-inversion waves can exist [11],

<in In-and-Anti-Phase Synchronization>

In the state of the in-and-anti-phase synchronizing, the edge oscillators and their adjacent oscillators must be synchronized at anti-phase. Therefore, when an even number of oscillators are coupled, even numbers of phase-inversion waves can exist, while when an odd number of oscillators are coupled, odd numbers of phase-inversion waves can exist.

Remark B:
As explained in Sect. 4.1, the in-and-anti-phase synchronization is not stable in the array of odd numbers of oscillators. However, when odd numbers of phase-inversion waves exist in the array, the in-and-anti-phase synchronization can be observed. Furthermore, for the first time, odd numbers of phase-inversion waves including the single wave are observed.

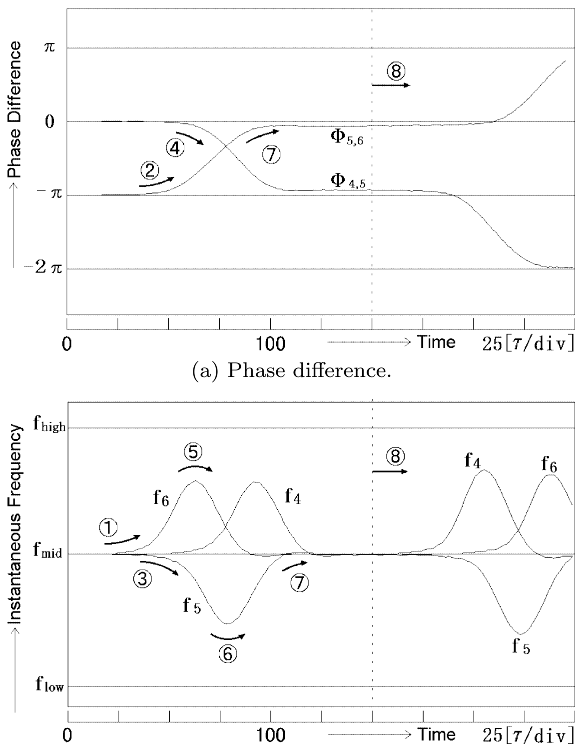
5. Mechanisms of Phase-Inversion Waves

Figures 7, 8 and 9 show phase differences and instantaneous frequencies, where $\Phi_{k,k+1}$ is phase difference between OSC_k and OSC_{k+1} and f_k is instantaneous frequency of OSC_k. We define them as follows:

$$\Phi_{k,k+1} = \frac{\tau_k(n) - \tau_{k+1}(n)}{\tau_k(n) - \tau_k(n-1)} \times \pi \tag{6}$$

$$f_k(n) = \frac{1}{2(\tau_k(n) - \tau_k(n-1))} \tag{7}$$

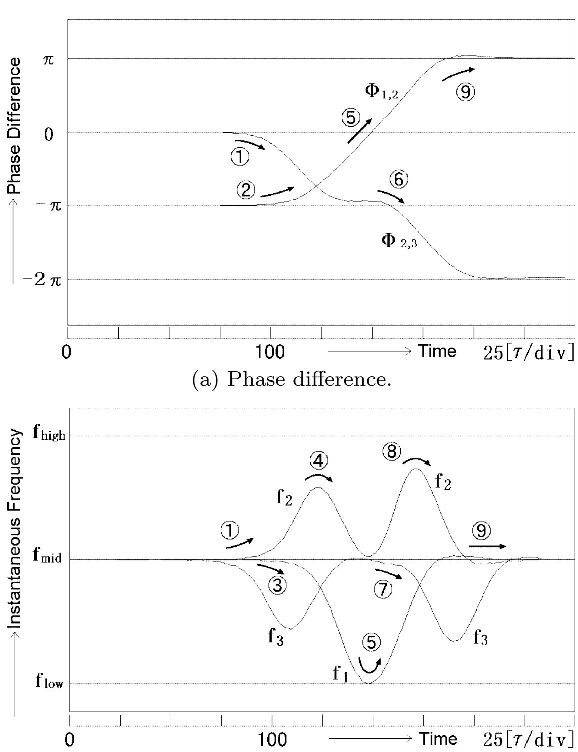
where $\tau_k(n)$ is time when the voltage of OSC_k crosses



(a) Phase difference.

(b) Instantaneous frequency.

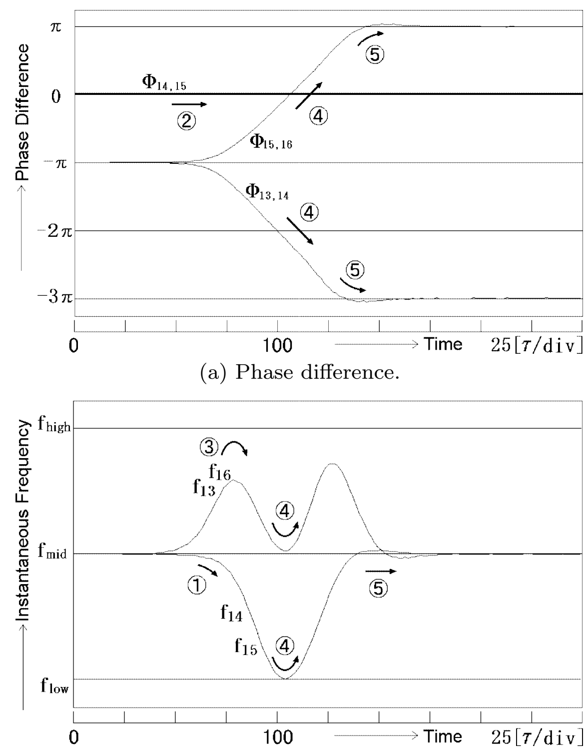
Fig. 7 Mechanism of propagation (computer calculated results).



(a) Phase difference.

(b) Instantaneous frequency.

Fig. 8 Mechanism of reflection at an edge of the array (computer calculated results).



(a) Phase difference.

(b) Instantaneous frequency.

Fig. 9 Mechanism of reflection by collision of two waves (computer calculated results).

0 [V] at n-th time. It has been known that f changes as Φ changes. We explain the mechanisms of the phase-inversion waves by using the relationship between f and Φ.

5.1 Mechanism of Propagation

Figure 7 shows propagating single phase-inversion wave. The mechanism of the wave propagation can be explained in Table 1.

After the wave reflects at an edge of the array, when the propagating wave reaches OSC_4 from the direction of OSC_N, phase states and instantaneous frequencies change in a similar manner (see ⑧ in Fig. 7(a) and (b)).

5.2 Mechanism of Reflection at an Edge

Figure 8 shows reflecting single phase-inversion wave at an edge of the array. The mechanism of the reflection at an edge of the array can be explained in Table 2.

5.3 Mechanism of Reflection by Collision

Figure 9 shows reflection by collision of two phase-inversion waves in the middle of the array. When the two phase-inversion waves reach two adjacent oscillators at the same time, the phase-inversion waves reflect. The mechanism of the wave reflection can be explained

Table 1 Mechanism of propagation.

time [τ]	sequence of propagation
Let us assume that the circuit synchronizes at in-and-anti-phase and that single wave reaches OSC_6 from the direction of OSC_N.	
around 24	f_6 changes from f_{mid} toward f_{high}, because the phase state between OSC_6 and OSC_7 changes from in-phase to anti-phase (see ① in Fig. 7(b)).
around 27	The phase state of OSC_6 advances, because f_6 changes from f_{mid} toward f_{high}. Hence, $\Phi_{5,6}$ starts to change from $-\pi$ to 0 (see Eq. (6) and ② in Fig. 7(a)).
around 32	f_5 starts to change from f_{mid} toward f_{low} (see ③ in Fig. 7(b)).
around 36	$\Phi_{4,5}$ changes from 0 to $-\pi$ (see ④ in Fig. 7(a)).
around 62	f_6 changes to f_{mid} again before f_6 reaches f_{high}, because the phase states between OSC_5 and OSC_6 and between OSC_6 and OSC_7 start to interchange. (see ⑤ in Fig. 7(b)).
around 79	f_5 changes to f_{mid} again before f_5 reaches f_{low} because the phase states between OSC_5 and OSC_6 and between OSC_6 and OSC_7 start to interchange (see ⑥ in Fig. 7(b)).
around 114	f_5 becomes to f_{mid} when the phase states between OSC_4 and OSC_5 and between OSC_5 and OSC_6 finish to interchange (see ⑦ in Fig. 7(a) and (b)).

in Table 3.

5.4 Extinction by Collision

We could observe extinction of the phase-inversion waves by collision in the past study [11] (see Fig. 2(c)). However, such an extinction can not be observed in this study. The reason would be explained as follows. In the past study, when two phase-inversion waves reach an oscillator at the same time, the phase-inversion waves disappeare. Because the instantaneous frequency of the oscillator changes from f_{low} to f_{high} or f_{high} to f_{low}. However, in this study, the instantaneous frequency of the oscillator can not change as the past study, because the changing directions of the instantaneous frequency are different between the two sides of the oscillator, for example, from f_{mid} to f_{high} at the right side and from f_{mid} to f_{low} at the left side. Therefore, we can not observe extinction by collision.

Table 2 Mechanism of reflection at an edge.

time [τ]	sequence of reflection at an edge
Let us assume that the circuit synchronizes at in-and-anti-phase and that single wave reaches OSC_3 from the direction of OSC_N. The phase state between OSC_1 and OSC_2 must be anti-phase and the phase state between OSC_2 and OSC_3 must be in-phase (see Sect. 4.1).	
around 80	f_2 starts to change from f_{mid} toward f_{high}, because the phase state between OSC_2 and OSC_3 changes from in-phase to anti-phase (see ① in Figs. 8(a) and (b)).
around 84	$\Phi_{1,2}$ starts to change from $-\pi$ to 0 (see ② in Fig. 8(b)).
around 89	f_1 starts to degrease toward f_{low} (see ③ in Fig. 8(a)).
around 122	f_2 starts to decrease to f_{mid} again before f_2 reaches f_{high}, because the phase states between OSC_1 and OSC_2 and between OSC_2 and OSC_3 start to interchange (see ④ in Fig. 8(b)).
around 148	When $\Phi_{1,2}$ reaches 0, f_1 reaches f_{low} and f_2 reaches f_{mid}. However, the in-phase synchronization of OSC_1 and OSC_2 is not stable, because f_1 is not equal to f_2. $\Phi_{1,2}$ changes to π, because f_1 is smaller than f_2. Further, f_1 and f_2 start to increase toward f_{high} (see ⑤ in Figs. 8(a) and (b)).
around 154	$\Phi_{2,3}$ starts to change to -2π, because f_2 is larger than f_3 (see ⑥ in Fig. 8(a)).
around 160	f_3 changes toward f_{low}. (see ⑦ in Fig. 8(b)).
around 170	f_2 changes to f_{mid} again before f_2 reaches f_{high}, because the phase states between OSC_1 and OSC_2 and between OSC_2 and OSC_3 start to interchange (see ⑧ in Fig. 8(b)).
around 200	When $\Phi_{1,2}$ reaches π, f_1 and f_2 reaches f_{mid} again (see ⑨ in Figs. 8(a) and (b)).

6. Conclusions

In this study, we observed phase-inversion waves in a ladder of coupled oscillators synchronizing at in-and-anti-phase by both circuit experiments and computer calculations.

In the previous study, wave propagation, reflection

Table 3 Mechanism of reflection by collision.

time [τ]	sequence of reflection by collision
Let us assume that the circuit synchronizes at in-and-anti-phase and that two phase-inversion waves are going to reach OSC_{13} and OSC_{16} at the same time from the direction of OSC_1 and OSC_N, respectively. The phase states between OSC_{13} and OSC_{14} and between OSC_{15} and OSC_{16} must be anti-phase and the phase state between OSC_{14} and OSC_{15} must be in-phase (see Sect. 4.1).	
around 51	f_{14} and f_{15} start to change from f_{mid} toward f_{low}, because $\Phi_{13,14}$ changes from $-\pi$ to -2π. $\Phi_{15,16}$ changes from $-\pi$ to 0 (see ① in Fig. 8(b)).
around 60	$\Phi_{14,15}$ does not change, because f_{14} and f_{15} start to change toward f_{low} at the same time (see ② in Fig. 8(a)).
around 78	f_{13} and f_{16} start to decrease to f_{mid} again before they reach f_{high}, because the phase states between OSC_{12} and OSC_{13} and between OSC_{13} and OSC_{14} start to interchange and the phase states between OSC_{15} and OSC_{16} and between OSC_{16} and OSC_{17} also start to interchange (see ③ in Fig. 8(b)).
around 103	When $\Phi_{13,14}$ and $\Phi_{15,16}$ reach -2π and 0, f_{14} and f_{15} reach f_{low}. However, because f_{13} and f_{16} are f_{mid}, $\Phi_{13,14}$ and $\Phi_{15,16}$ change to -3π and π. Therefore f_{13} and f_{16} change from f_{mid} toward f_{high}, and f_{14} and f_{15} change from f_{low} to f_{mid} (see ④ in Figs. 8(a) and (b)).
around 300	When $\Phi_{13,14}$ and $\Phi_{15,16}$ reach -3π and π, $f_{13} \sim f_{16}$ reach f_{mid}.

at an edge of the array, reflection by collision of two waves and extinction by collision of two waves could be observed. However, the extinction could not be observed in the in-and-anti-phase synchronization. Further, single phase-inversion wave could be observed for the first time.

In the state of the in-and-anti-phase synchronization, instantaneous oscillation frequencies change around only single frequency f_{mid}. We could explain the mechanism of the wave propagation, the reflection by collision of two waves and the reflection at an edge of the array by using the relationship between phase states and instantaneous frequencies.

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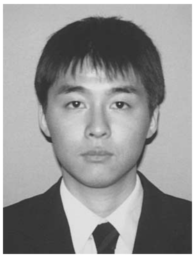

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