PAPER

Phase-Waves in a Ladder of Oscillators

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SUMMARY In this study, wave propagation phenomena of phase differences observed in van der Pol oscillators coupled by inductors as a ladder are investigated. The phenomena are called "phase waves." We classify the observed phenomena and analyze the difference in detail. We observe that the behavior of the phase waves generated by giving a phase difference of positive value is different from the behavior of those generated by giving a phase difference of negative value. We can also observe the generation of two pairs of phase waves. We clarify the mechanisms of these complicated phenomena. Finally, for the case of nine oscillators, we carry out both computer calculations and circuit experiments. Circuit experimental results agree well with computer calculated results qualitatively.

key words: *coupled oscillators, phase difference, phase waves, phase-inversion waves*

1. Introduction

A large number of coupled limit-cycle oscillators are useful as models for a wide variety of systems in natural fields, for example, diverse physiological organs including gastrointestinal tracts and axial fibers of nervous systems, convecting fluids, arrays of Josephson junctions and so on. Hence, it is very important to analyze synchronization and the related phenomena observed in coupled oscillators in order to clarify the mechanisms of the generations or in order to control the generating conditions of various phenomena in such natural systems [1]–[3]. In the field of electrical engineering, numerous studies on synchronization phenomena of coupled van der Pol oscillators have been carried out up to now [4]–[8].

Recently, we have discovered very interesting wave propagation phenomena of phase states between two adjacent oscillators in an array of van der Pol oscillators coupled by inductors [9]–[11]. As far as we know, to date, such phenomena have not been reported in simple real physical systems. In the study, we named the continuously existing wave of changing phase states between two adjacent oscillators from in-phase to anti-phase or from anti-phase to in-phase as "phase-inversion wave" and explained the generation mechanism by using the relation between the instantaneous oscillation frequencies and the phase differences of two adjacent oscillators. In the past study, we researched four basic properties of the phase-inversion waves (propagation, reflection at an edge of the array, reflection by collision of two waves and extinction by collision of two waves) [9]. However, phase propagations observed in transient states have not been analyzed and they have a rich variety of unknown phenomena.

In this study, a propagating wave of phase difference between two adjacent oscillators is observed. It is called "phase wave." The phase waves are observed in transient states and change to the phase-inversion waves or disappear. In the computer calculations, we produce phase waves as follows. Almost the same initial conditions are given for all oscillators to produce complete in-phase synchronization which is one of the stable steady states in the system. After the system settles in the complete in-phase synchronization, the voltage and the current with arbitrary phase difference are input to the first oscillator. The observed phase waves are classified into four patterns and the difference is analyzed in detail. Furthermore, the mechanisms of the complicated phenomena for the phase waves are also explained. Finally, for the case of nine oscillators, we carry out both computer calculations and circuit experiments. Circuit experimental results agree well with computer calculated results qualitatively.

2. Circuit Model

The circuit model used in this study is shown in Fig. 1.

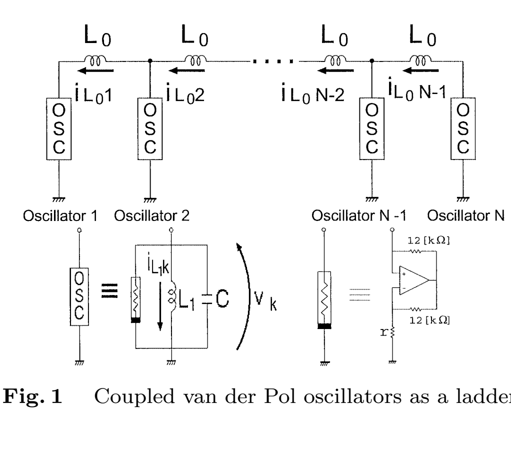

Fig. 1 Coupled van der Pol oscillators as a ladder.

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N van der Pol oscillators are coupled by coupling inductors L_0. We carried out computer calculations for the cases of $N = 9$ and 20 and circuit experiments for the case of $N = 9$. In the computer calculations, we assume the $v - i$ characteristics of the nonlinear negative resistors in each circuit as given by the following function.

$$i_r(v_k) = -g_1 v_k + g_3 v_k^3 \quad (g_1, g_3 > 0) \tag{1}$$

The circuit equations governing the circuit in Fig. 1 are written as

[First Oscillator]

$$\dot{x}_1 = y_1 \tag{2}$$

$$\dot{y}_1 = -x_1 + \alpha(x_2 - x_1) + \varepsilon\left(y_1 - \frac{1}{3}y_1^3\right)$$

[Middle Oscillators]

$$\dot{x}_k = y_k \tag{3}$$

$$\begin{aligned}\dot{y}_k = &-x_k + \alpha(x_{k+1} - 2x_k + x_{k-1}) \\ &+\varepsilon\left(y_k - \frac{1}{3}y_k^3\right) \\ &(k = 2 \sim N\text{–}1)\end{aligned}$$

[Last Oscillator]

$$\dot{x}_N = y_N \tag{4}$$

$$\dot{y}_N = -x_N + \alpha(x_{N-1} - x_N) + \varepsilon\left(y_N - \frac{1}{3}y_N^3\right),$$

where

$$t = \sqrt{L_1 C}\tau,\ i_{L_1 k} = \sqrt{\frac{Cg_1}{3L_1 g_3}}x_k, v_k = \sqrt{\frac{g_1}{3g_3}}y_k,$$

$$\alpha = \frac{L_1}{L_0},\ \varepsilon = g_1\sqrt{\frac{L_1}{C}},\ \frac{d}{d\tau} = \text{“}\cdot\text{”}. \tag{5}$$

It should be noted that α corresponds to the coupling of the oscillators and ε corresponds to the nonlinearity of the oscillators. Throughout the paper, we fix $\alpha = 0.050$ and $\varepsilon = 0.30$ and calculate (2)–(4) by using the fourth-order Runge-Kutta method.

3. Phase Waves

In this section, wave propagation phenomena observed from the circuit with 20 oscillators are investigated.

3.1 Phase Difference

We define phase difference between two adjacent oscillators and instantaneous frequency as

<Phase Difference>

$$\Phi_{k,k+1}(n) = \frac{\tau_k(n) - \tau_{k+1}(n)}{\tau_k(n) - \tau_k(n-1)} \times \pi \tag{6}$$

<Instantaneous Frequency>

$$f_k(n) = \frac{1}{2 \times (\tau_k(n) - \tau_k(n-1))}, \tag{7}$$

where $\tau_k(n)$ is the time at which the voltage of OSCk crosses 0 [V] at n-th time.

Figure 2 shows, typical examples of observed phase waves, which originate from the propagation of the phase difference between adjacent oscillators, and the phase-inversion waves which originate from the propagation of the phase states (in-phase synchronization or anti-phase synchronization) between adjacent oscillators. These results are obtained for the same parameter values by changing the initial conditions as follows:

1. Set the initial conditions of all oscillators as the same.
2. Input an arbitrary phase difference Φ_{in} to the first oscillator

of OSC_2 and $\Phi_{1,2}$ is the phase of OSC_2 based on the phase of OSC_1. The current (x) and the voltage (y) are obtained from the following conditional equations:

$$\sin\Phi_{in} \cong -\frac{y}{\sqrt{x^2+y^2}}, \qquad \cos\Phi_{in} \cong \frac{x}{\sqrt{x^2+y^2}}. \tag{8}$$

In the diagrams of Fig. 2, the vertical axis is the sum of the voltages of adjacent oscillators, and the horizontal axis is time. Hence, the diagrams show how the phase differences between adjacent oscillators change with time. Figure 2(a) shows a pair of phase waves changing from in-phase synchronization to a state with phase difference and changing from the state with phase difference to in-phase synchronization. Figure 2(b)

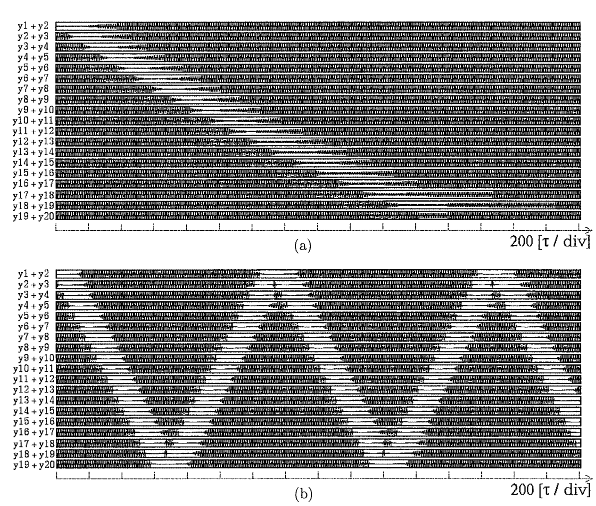


Fig. 2 Typical examples of waves. $\alpha = 0.050$ and $\varepsilon = 0.30$. (a) Phase waves $\Phi_{in} = 45°$. (b) Phase-inversion waves $\Phi_{in} = 180°$.

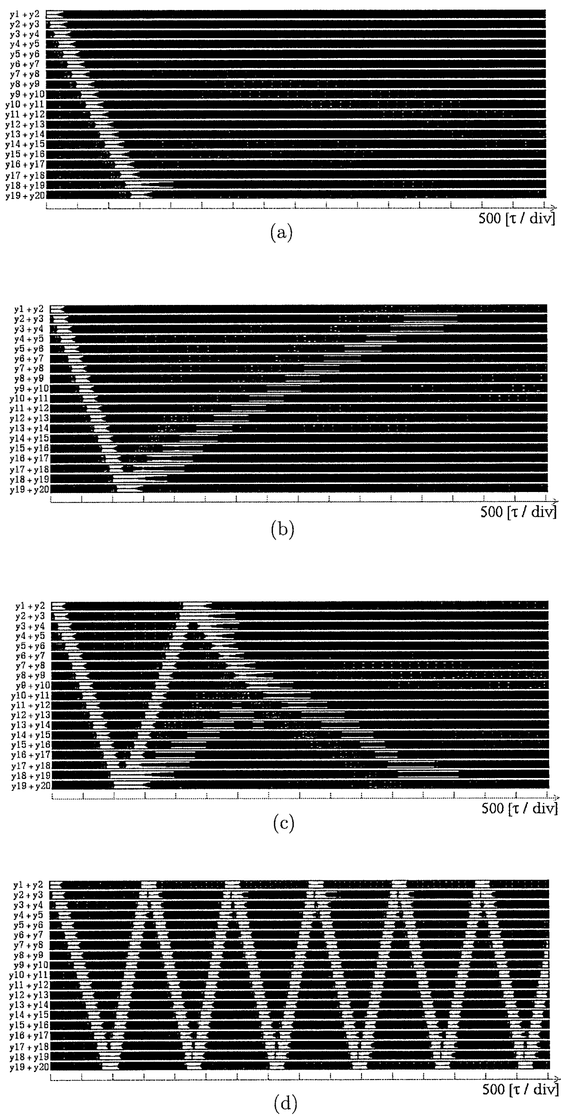


Fig. 3 Phase waves. $\alpha = 0.050$ and $\varepsilon = 0.30$. (a) $\Phi_{in} = +60°$. (b) $\Phi_{in} = +68°$. (c) $\Phi_{in} = +71°$. (d) $\Phi_{in} = +80°$.

shows a pair of phase-inversion waves changing from in-phase synchronization to anti-phase synchronization and changing from anti-phase synchronization to in-phase synchronization. The propagation speed of the phase waves becomes higher as Φ_{in} increases. And the phase-inversion waves propagate at the highest speed.

3.2 Classification

Observed phenomena are classified into four patterns by the input phase difference as follows:

(a) **No reflection**
$-67° \leqq \Phi_{in} \leqq +64°$: The phase difference decreases while the phase waves are propagating. The phase waves disappear at the edge of the array (see Fig. 3(a) and Fig. 4(a)).

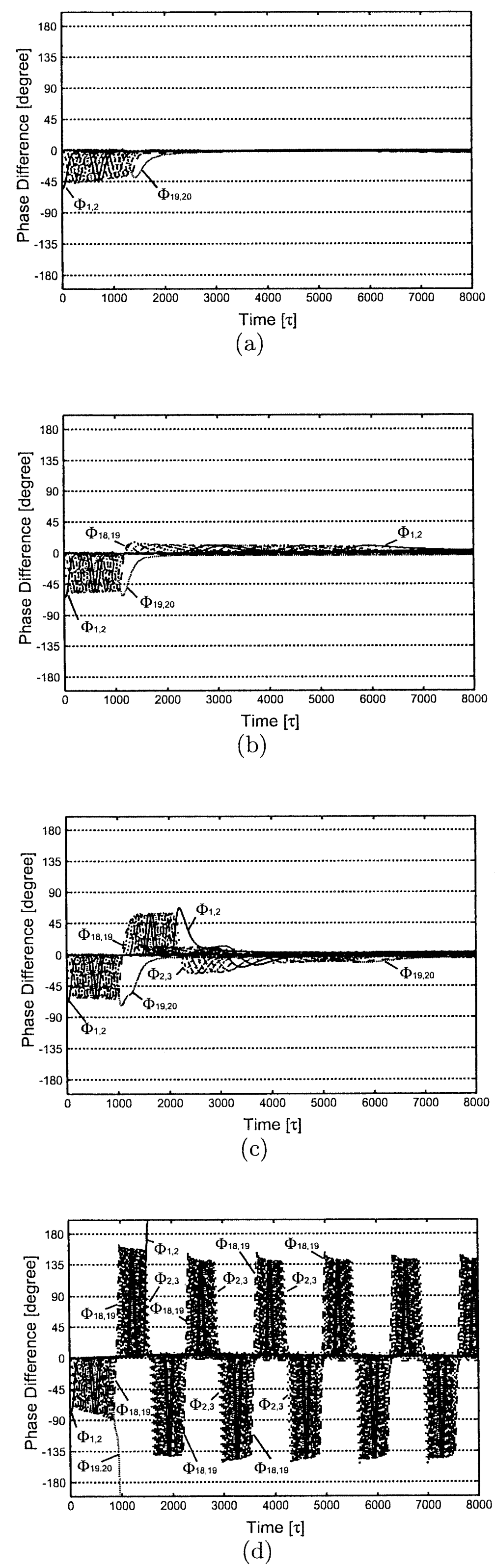


Fig. 4 Phase waves (Phase Difference – Time). $\alpha = 0.050$ and $\varepsilon = 0.30$. (a) $\Phi_{in} = +60°$. (b) $\Phi_{in} = +68°$. (c) $\Phi_{in} = +71°$. (d) $\Phi_{in} = +80°$.

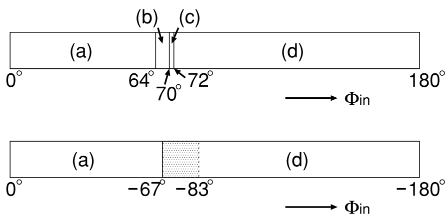


Fig. 5 Domain of observed phenomena generated by the phase difference of positive or negative value. $\alpha = 0.050$ and $\varepsilon = 0.30$.

(b) **One-time reflection**
$+65° \leqq \Phi_{in} \leqq +70°$: The phase waves propagate and reflect at the edge of the array. The phase difference decreases while the phase waves are propagating after the reflection. The phase waves disappear at the edge of the array (see Fig. 3(b) and Fig. 4(b)).

(c) **Two-times reflection**
$\Phi_{in} \cong +71°$: The phase waves propagate and reflect two times at the edges of the array. The phase difference decreases while the phase waves are propagating. The phase waves disappear at the edge of the array (see Fig. 3(c) and Fig. 4(c)).

(d) **Change to phase-inversion waves**
$+72° \leqq \Phi_{in} \leqq +180°$ and $-68° \leqq \Phi_{in} \leqq -180°$: The phase difference increases while the phase waves are propagating. The phase waves change to the phase-inversion waves when the phase waves are reflected at the edge of the array or in the middle of the array (see Fig. 3(d) and Fig. 4(d)).

Figure 5 shows the domain of the observed phenomena. We may be able to observe a phenomenon that the phase waves reflect three times or more and disappear by changing N. When N is very large, we can observe a phenomenon that the phase waves disappear in the middle of the array.

3.3 Two Pairs of Phase Waves

Two pairs of waves (two pairs of phase waves or a pair of phase waves and a pair of phase-inversion waves) are generated in the domain of the shaded area of Fig. 5 (see Fig. 6). For $\Phi_{in} = -68°$, two pairs of phase waves are generated as shown in Fig. 6(a). For $\Phi_{in} = -70°$, a pair of phase waves and a pair of phase-inversion waves are generated as shown in Fig. 6(b). For $\Phi_{in} = -69°$, two pairs of phase-inversion waves exist as shown in Fig. 6(c). However, when waves are generated, a pair of phase-inversion waves is first generated as phase waves and they waves change to phase-inversion waves. Therefore, Fig. 6(c) exhibits the same mechanism as Fig. 6(b) when waves are generated.

Furthermore, two pairs of waves can be generated when waves reflect at the first time. When Φ_{in} has a positive value, $\Phi_{k,k+1}$ has a negative value. If we consider that the phase waves are generated at OSC_{20}, Φ_{in} is $\Phi_{19,20}$. In other words, a phase difference of negative value is input as Φ_{in}. Thus, this phenomenon is the same as those generated by a negative value of Φ_{in}. Therefore, the mechanism of this phenomenon is the same as the mechanism of generating two pairs of waves.

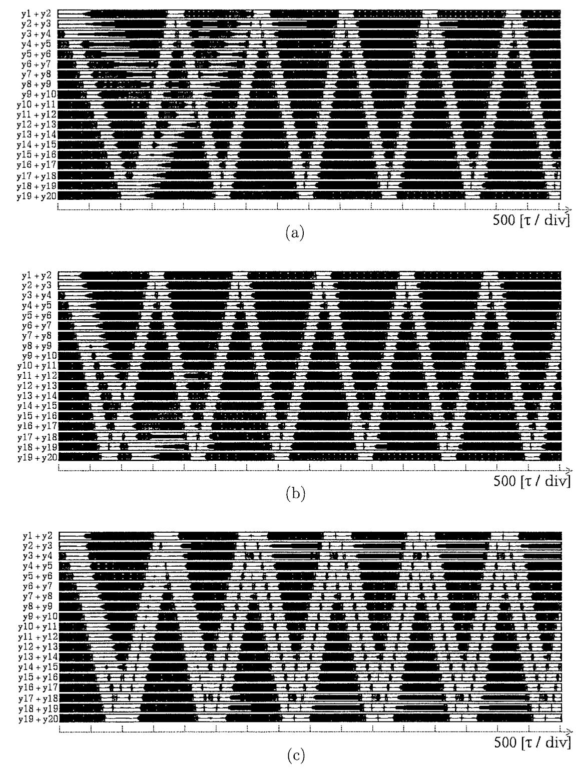


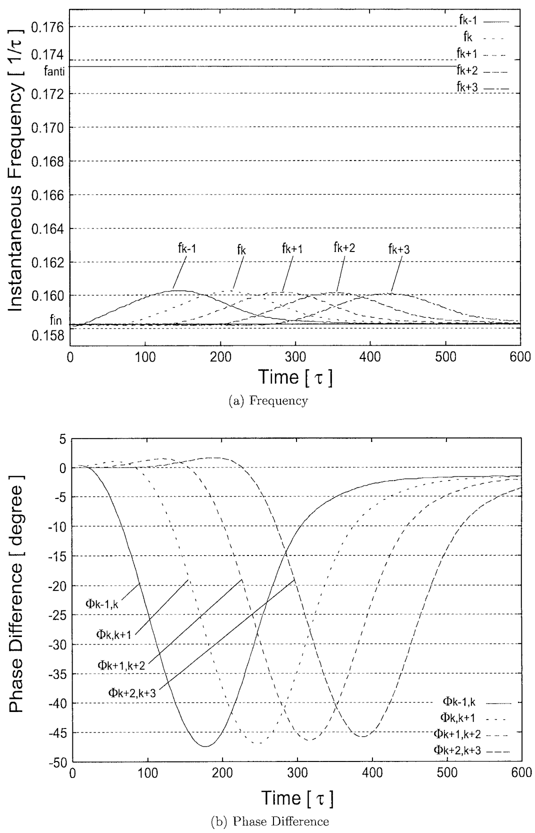
Fig. 6 Two pairs of phase waves. $\alpha = 0.050$ and $\varepsilon = 0.30$. (a) $\Phi_{in} = -68°$. (b) $\Phi_{in} = -70°$. (c) $\Phi_{in} = -69°$.

4. Mechanisms of Phase Waves

4.1 Mechanism of Propagation

Figure 7 shows how the instantaneous frequencies and the phase differences change. We define f_{in} as the oscillation frequency of the two coupled oscillators being in-phase. Also, we define f_{anti} as the oscillation frequency of the two coupled oscillators being anti-phase. For the parameter values used in this study, the two frequencies are numerically obtained as $f_{in} = 0.1584$ and $f_{anti} = 0.1736$.

Phase waves are propagated by this mechanism (see Table 1). When the propagating phase difference is sufficiently large to be attracted to anti-phase synchronization, the phase waves change to the phase-inversion waves.



(a) Frequency

(b) Phase Difference

Fig. 7 Mechanism of propagation of phase waves. Example of generated phase waves by phase difference of positive value ($\Phi_{in} = +60°$).

Table 1 Mechanism of propagation.

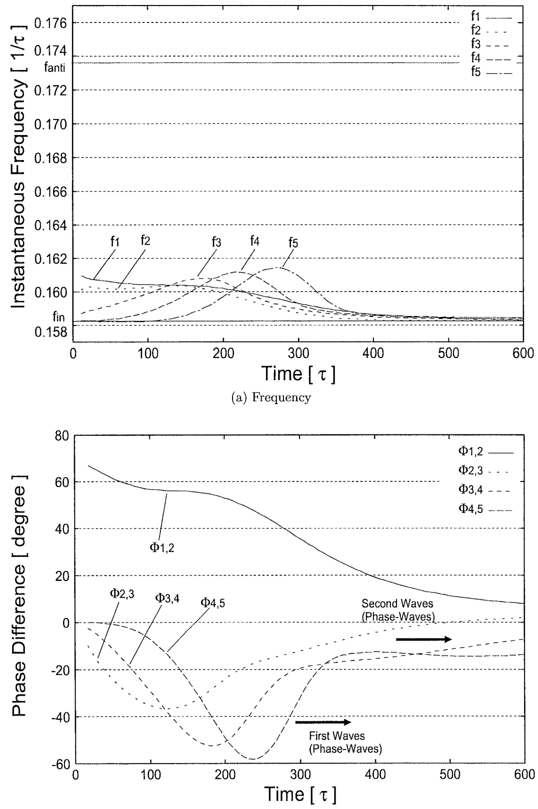
time $[\tau]$	**sequence of generated phenomena**
Let us assume that all oscillators are in in-phase synchronization. The phase waves are going to reach the $(k-1)$-th oscillator from the first oscillator.	
around 10	Instantaneous frequencies of the $(k-1)$-th and the k-th oscillator (f_{k-1} and f_k) become higher than the in-phase frequency (f_{in}), because, the anti-phase frequency (f_{anti}) is higher than f_{in}.
around 90	The phase difference between the k-th and the $(k+1)$-th oscillator ($\Phi_{k,k+1}$) changes from 0 toward $-\pi$, because f_k is higher than f_{k+1}.
around 130	f_{k+1} changes from f_{in} toward f_{anti}.
around 225	Because $\Phi_{k-1,k}$ is relatively small, the phase difference tends to decrease in in-phase synchronization. Therefore, f_k changes toward f_{in}.

Table 2 Mechanism of Fig. 6(a).

time $[\tau]$	**sequence of generated phenomena**	
When all oscillators are in in-phase synchronization, a negative phase difference is given to the first oscillator.		
around 0	Because $\Phi_{1,2}$ is not equal to 0, f_1 and f_2 are larger than f_{in}. However, f_3 is equal to f_{in}, because $\Phi_{2,3}$ and $\Phi_{3,4}$ are equal to 0.	
around 20	$\Phi_{1,2}$ is attracted to 0 in a moment, because $\Phi_{1,2}$ is not sufficiently large to be attracted to anti-phase synchronization. Therefore, f_1 and f_2 change toward f_{in}.	
	$\Phi_{2,3}$ changes from 0, because f_2 is higher than f_{in}. Therefore, f_3 starts to change toward f_{anti}.	
	first waves	**second waves**
around 50	Propagation of the first phase-waves begins.	$\Phi_{2,3}$ is decreasing during $f_2 > f_3$.
around 80		f_2 starts to change from decreasing to f_{in} to increasing to f_{anti} again.
around 120		f_1 changes toward f_{in} and becomes nearly equal to f_2, because the transition of f_2 is very slow due to the above reason. Therefore, $\Phi_{1,2}$ maintains almost the same value during propagation of the first pair of phase waves.
around 125		$\Phi_{2,3}$ changes toward 0, because $f_2 < f_3$ by the first waves.
around 150		f_2 starts to change to decrease to f_{in} again. f_1 and f_2 decrease to f_{in} together, because $\Phi_{1,2}$ is not sufficiently large to be attracted to anti-phase synchronization. Therefore, f_1 decreases to f_{in}. Because f_1 and f_2 become nearly equal, the decrease of $\Phi_{1,2}$ becomes very slow. The second pair of phase waves is generated at this time.
around 170		f_3 starts to change toward f_{in} by the first waves.
around 200		Because the decrease of $\Phi_{1,2}$ is very slow, rates of decrease of f_1 and f_2 are small values.
around 225		The rate of decrease of $\Phi_{2,3}$ changes to a small value, because f_3 becomes nearly equal to f_2 which decrease slowly. The second pair of phase waves is propagated by the above mechanism.

4.2 Difference between Positive and Negative Values

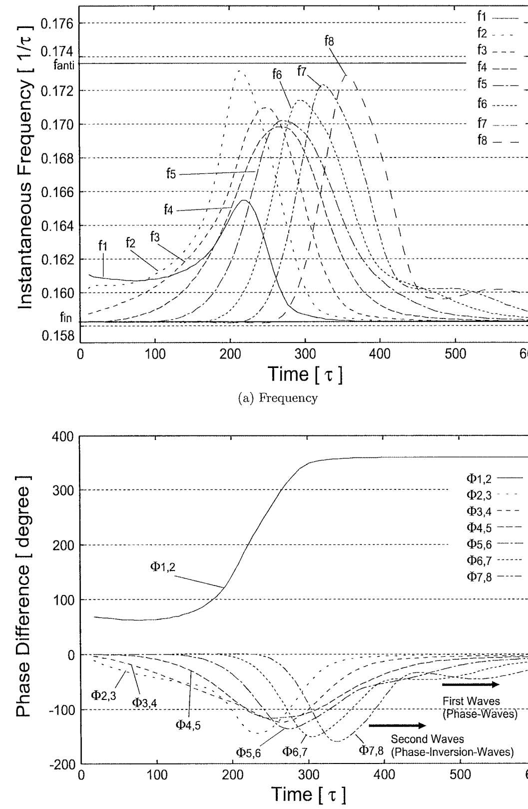
We observed that the behavior of the phase waves generated by giving a phase difference of plus value is dif-



(a) Frequency

(b) Phase Difference

Fig. 8 Mechanism of simultaneous generation of two pairs of the phase waves. Example of generated phase-waves by phase difference of negative value ($\Phi_{in} = -68°$).



(a) Frequency

(b) Phase Difference

Fig. 9 Mechanism of simultaneous generation of a pair of the phase waves and a pair of the phase-inversion-waves. Example of generated phase waves by phase difference of negative value ($\Phi_{in} = -70°$).

ferent from that generated by giving a phase difference of minus value as shown in Fig. 5.

If the phase waves are generated by the phase difference of +45°, instantaneous frequency of the first oscillator (f_1) changes from the in-phase frequency (f_{in}) toward the anti-phase frequency (f_{anti}), and instantaneous frequency of the second oscillator (f_2) changes from f_{in} toward f_{anti} after the change of f_1. However, f_1 starts to return to f_{in}, because a phase difference between the first and the second oscillators ($|\Phi_{1,2}|$) is relatively small. f_2 also starts to return to f_{in} after the change of f_1. Therefore, a phase of the second oscillator catches up with a phase of the first oscillator.

If the phase waves are generated by the phase difference of −45°, f_1 changes from f_{in} toward f_{anti}. Therefore a phase of the first oscillator is rapidly catching up with a phase of the second oscillator.

4.3 Generation of Two Pairs of Phase Waves

We can observe two types of two pairs of phase waves (see Sect. 3.3):

1. Two pairs of phase waves (Fig. 6(a))
2. A pair of phase waves and a pair of phase-inversion waves (Fig. 6(b) and Fig. 6(c)).

4.3.1 Mechanism of Fig. 6(a)

Two pairs of phase waves are generated by this mechanism (see Table 2).

Figure 8 shows how the instantaneous frequencies and the phase differences change when two pairs of phase waves are generated.

4.3.2 Mechanism of Fig. 6(b) and Fig. 6(c)

The generation mechanism of the first phase waves is the same as in Fig. 6(a). However, the rate of increase of f_2 is larger than that when two pairs of phase waves are generated, because the minimum value of $\Phi_{2,3}$ by the first pair of phase waves is larger than that when two pairs of phase waves are generated (around $\tau = 100$ in Fig. 8(b) and Fig. 9(b)). Two pairs of phase waves are generated by this mechanism (see Table 3).

Because the transition of $\Phi_{1,2}$ is slow, the first pair of phase waves and the second pair of phase waves can exist separately. The second pair of phase waves comprises the same waves which are generated when $\Phi_{in} = 180°$. Therefore, a pair of phase waves changes to a pair of phase-inversion waves in a moment. This

Table 3 Mechanism of Figs. 6(b) and 6(c).

time [τ]	**sequence of generated phenomena**	
When all oscillators are in in-phase synchronization, a negative phase difference is given to the first oscillator.		
around 0	Because $\Phi_{1,2}$ is not equal to 0, f_1 and f_2 are larger than f_{in}. However, f_3 is equal to f_{in}, because $\Phi_{2,3}$ and $\Phi_{3,4}$ are equal to 0.	
around 20	$\Phi_{1,2}$ is attracted to 0 in a moment, because $\Phi_{1,2}$ is not sufficiently large to be attracted to anti-phase synchronization. Therefore, f_1 and f_2 change toward f_{in}.	
	$\Phi_{2,3}$ changes from 0, because f_2 is higher than f_{in}. Therefore, f_3 starts to change toward f_{anti}.	
	first waves	**second waves**
around 80	Propagation of the first phase-waves begins.	$\Phi_{1,2}$ changes from decreasing to increasing, because f_2 begins to increase by $\Phi_{2,3}$ which decreases.
around 90		f_1 changes toward f_{anti} again.
around 100		The transition of $\Phi_{1,2}$ is very slow, because f_1 changes from decreasing to increasing and f_2 changes from decreasing to increasing.
around 215		f_1 increases to the middle of f_{in} and f_{anti} and decreases to f_{in} again, because the edge of the array does not become stable in anti-phase synchronization. $\Phi_{1,2}$ exceeds 180° and reaches 360° (in-phase synchronization). Therefore, the second pair of phase waves is generated.

mechanism is the same mechanism of reflection of the phase-inversion wave [9].

Figure 8 shows how the instantaneous frequencies and the phase differences change when a pair of phase waves and a pair of phase-inversion waves are generated.

5. Circuit Experiments

In this section, we confirm the generation of the phase-inversion waves and the phase waves by circuit experiments. The results for $N = 9$ are shown in Fig. 10. The corresponding numerically calculated results are shown in Fig. 11. We observed that the circuit experimental results and the numerically calculated results agree very well. The small difference between the two results may come from the resistance in real inductors or difficulty of setting the initial phase difference in real circuit experiments.

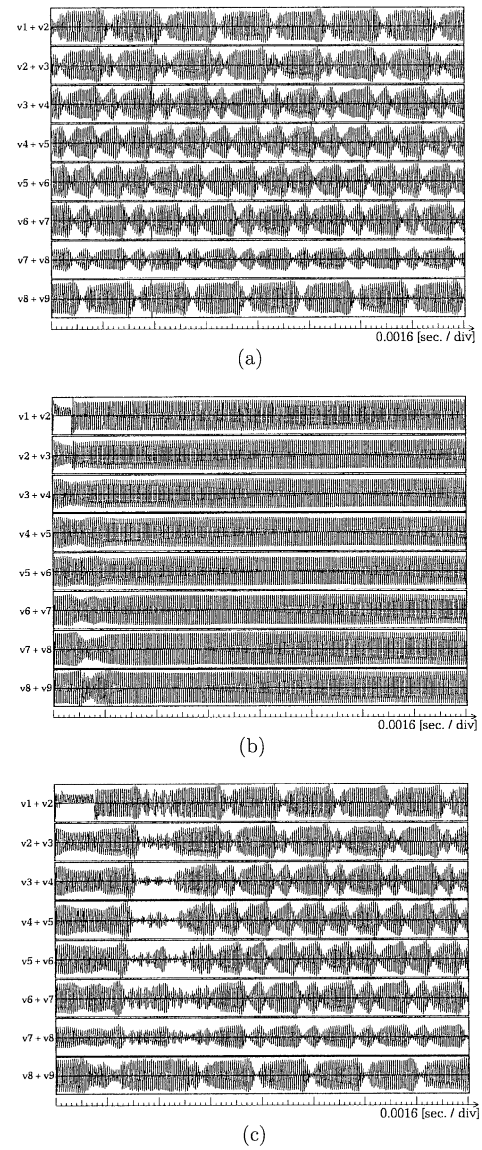


Fig. 10 Circuit experimental results for $N = 9$. $L_0 = 1100$[mH], $L_1 = 200$[mH], $C = 47$[nF] and $r = 1$[kΩ]. (a) The phase-inversion waves. (b) Disappearing phase waves. (c) Changing from phase waves into phase-inversion waves.

6. Conclusions

In this study, phase waves were investigated by numerical calculation for the case of 20 oscillators coupled as a ladder. The observed phase waves were classified into four patterns and the differences were analyzed in detail. We observed that the behavior of the phase waves generated by giving a phase difference of positive value was different from the behavior of those generated by giving a phase difference of negative value. We were able to observe the generation of two pairs of phase waves. The propagation mechanism of the phase waves was analyzed. Furthermore, the mechanism of the complicated phenomena of the phase waves was

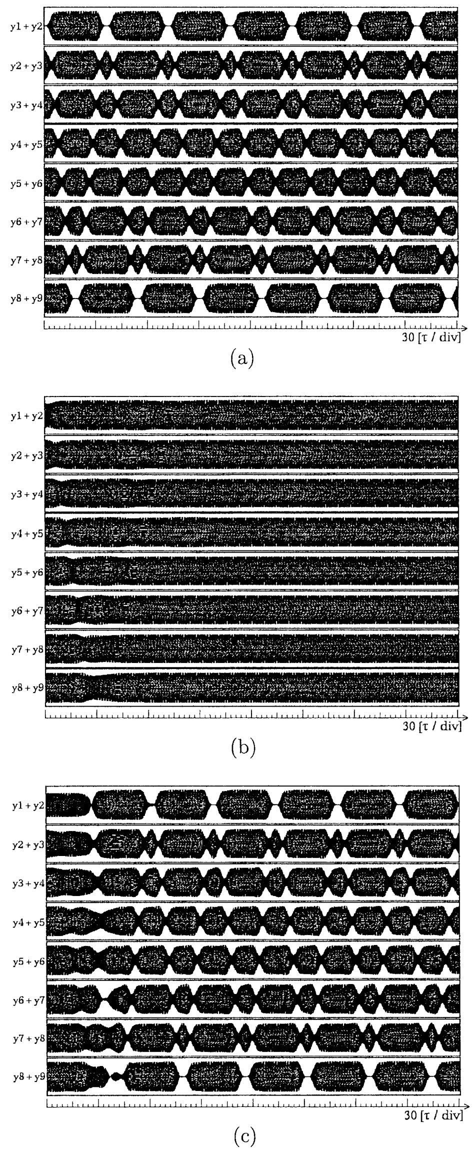


Fig. 11 Corresponding numerically calculated results for $N = 9$. $\alpha = 0.050$ and $\varepsilon = 0.30$. (a) The phase-inversion waves. (b) Disappearing phase waves. (c) Changing from phase waves into phase-inversion waves.

also explained. Finally, for the case of nine oscillators, we carried out both computer calculations and circuit experiments. Circuit experimental results agreed well with computer calculated results qualitatively.

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Akio Ushida received the B.E. and M.E. degrees in Electrical Engineering from Tokushima University in 1961 and 1966, respectively, and the Ph.D. degree in Electrical Engineering from University of Osaka Prefecture in 1974. He was an Associate Professor from 1973 to 1980 at Tokushima University. Since 1980 he has been a Professor in the Department of Electrical and Electronic Engineering at Tokushima University. From 1974 to 1975 he spent one year as a visiting scholar at the Department of Electrical Engineering and Computer Sciences at the University of California, Berkeley. His current research interests include numerical methods and computer-aided analysis of nonlinear systems. Dr. Ushida is a member of IEEE.