PAPER

Synchronization and Its Analysis in Chaotic Systems Coupled by Transmission Line

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SUMMARY In this study, synchronization phenomena in chaotic oscillators coupled by a transmission line are investigated. In particular investigation using real circuits is done for the first time. It is confirmed that the chaotic subsystems synchronize, although signals propagating along the transmission line are affected by the time delay. Further the period-doubling bifurcation with varying the time delay and anti-phase synchronization phenomena are observed in our circuit model. Also the voltage distribution of transmission line is simulated in order to investigate whether the current flowing through the transmission line is constant or not. It is found that the subsystems synchronize although the current through the transmission line keeps on varying.

key words: *chaos synchronization, transmission line, characteristic impedance, time delay*

1. Introduction

Since synchronization of chaos has been reported by Pecora and Carroll [1], it has received a great deal of attention. It is extremely interesting that chaotic systems or signals synchronize although small error of initial values in a chaotic system is expanded as time goes. For this background, a number of papers on chaos synchronization, for example [2]–[8] and so on, have been published. Further communication schemes making use of chaos synchronization have been proposed in [9]–[13] and the effects of non-ideal communication channels on chaos synchronization have been investigated in [14]–[18].

However, almost studies on chaos synchronization reported so far (such as [2]–[8]), have dealt with synchronization phenomena observed from chaotic oscillators coupled by *lumped elements*. In future, chaotic systems will be integrated and their operation speed will become higher for some engineering applications. In such systems, the effect of time delay can not be ignored. Thus we think that the investigation of systems with time-delayed signal is very important.

As far as we know, studies on such systems have been hardly reported. In [19] two chaotic systems coupled by two delay lines are investigated and various computer simulated results on synchronization are reported. In the system a state voltage passes through a delay line, an amplifier and further a transformer, and reaches to another subsystem. So the system seems to be complex and unnatural. Due to making use of ideal delay line, simulation with varying the time delay have only carried out. Further the results have not yet been confirmed by circuit experiments. It should go without saying that more detailed investigations must be required for engineering applications of chaos.

So, in this paper, we investigate a chaotic system coupled by transmission line. Remember that a transmission line introduces some time delay. In our models, two Chua's circuits are coupled by a transmission line instead of a lumped element. The circuit models, i.e. v_{C1}-coupled and v_{C2}-coupled systems, are shown in Fig. 1.

First the models are investigated experimentally. In our laboratory experiments, delay lines and coaxial cables have been used as transmission line. The delay line is an element that transmits input signal waveforms with fidelity but delays time. It has been confirmed that the chaotic subsystems in both models synchronize even if the time delay exists. Moreover, for v_{C1}-coupled system, interesting phenomena that the subsystems stop oscillations when the time delay becomes large, are observed.

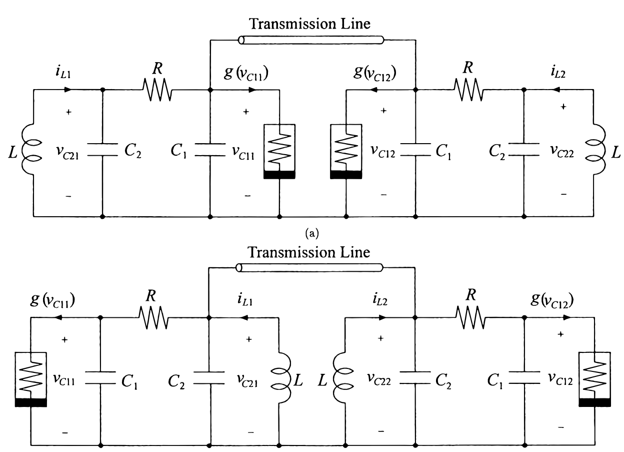

Fig. 1 The circuit model. (a) v_{C1}-coupled system. (b) v_{C2}-coupled system.

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To make these phenomena more reliable, computer simulations are further carried out. These models can be simulated by using the method of characteristics [20], which has been proposed for transient simulation of transmission lines. It is found that the experimentally confirmed phenomena can be also observed in computer simulations.

In order to investigate synchronization phenomena in detail, the bifurcation diagrams with respect to both the time delay and the characteristic impedance are shown. The amazing phenomena including period-doubling bifurcation and anti-phase synchronization are observed from v_{C1}-coupled system. The time evolution of voltage distribution along the transmission line is also given to examine whether the current through the transmission line is varying or not. It is found that the current through the transmission line is not constant and keeps on varying, which is different from the case of lumped element coupling.

2. Circuit Model

Figure 1 shows the circuit models used in this paper. In these models, the Chua's circuit is used as each chaotic subcircuit and two subcircuits are coupled by a transmission line. The Chua's circuit is a extremely simple system but it exhibits complex dynamics of bifurcation and chaos [21], [22].

In our simulations, the transmission line is modeled by the characteristic model [20], as shown in Fig. 2.

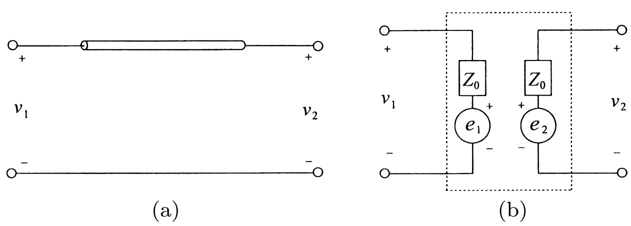

Fig. 2 (a) Transmission line and (b) its characteristic model.

Where Z_0 is the characteristic impedance. e_k is the waveform generator to simulate the reflection and these values are calculated by the following equations.

$$\begin{aligned} e_1(t) &= 2v_2(t) - e_2(t-\tau) \\ e_2(t) &= 2v_1(t) - e_1(t-\tau) \end{aligned} \tag{1}$$

Also τ denotes the time delay of the transmission line.

Applying the characteristic model to our circuit models, we can get the equivalent circuits shown in Fig. 3 and the corresponding circuit equations.

v_{C1}-coupled system:

$$\begin{aligned} C_1\frac{dv_{C1k}}{dt} &= G(v_{C2k} - v_{C1k}) - g(v_{C1k}) \\ &\quad + Y_0(e_k(t-\tau) - v_{C1k}), \\ C_2\frac{dv_{C2k}}{dt} &= G(v_{C1k} - v_{C2k}) + i_{Lk}, \\ L\frac{di_{Lk}}{dt} &= -v_{C2k} \end{aligned} \tag{2}$$

v_{C2}-coupled system:

$$\begin{aligned} C_1\frac{dv_{C1k}}{dt} &= G(v_{C2k} - v_{C1k}) - g(v_{C1k}), \\ C_2\frac{dv_{C2k}}{dt} &= G(v_{C1k} - v_{C2k}) + i_{Lk} \\ &\quad + Y_0(e_k(t-\tau) - v_{C2k}), \\ L\frac{di_{Lk}}{dt} &= -v_{C2k} \end{aligned} \tag{3}$$

Where $G = 1/R$, $Y_0 = 1/Z_0$ and $g(\cdot)$ is a piecewise linear resistor defined by

$$g(v) = m_1 v + \frac{1}{2}(m_0 - m_1)[|v + B_p| - |v - B_p|].$$

After normalizing, we have the following dimensionless equations.

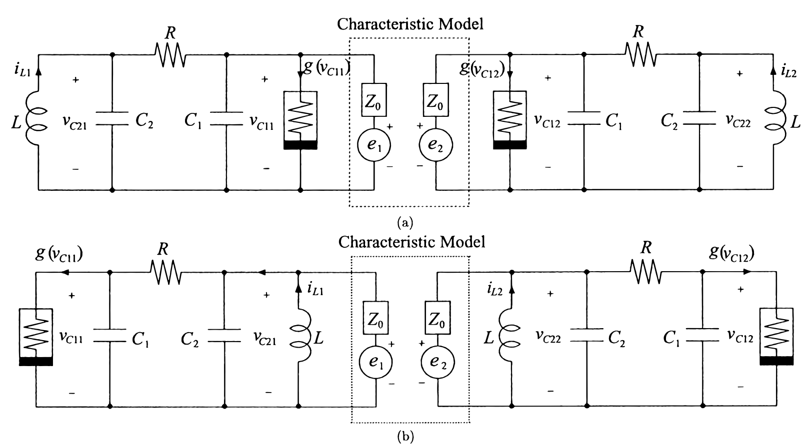

Fig. 3 The equivalent circuits for (a) v_{C1}-coupled system and (b) v_{C2}-coupled system.

v_{C1}-coupled system:

$$\begin{aligned}\dot{x}_k &= \alpha(y_k - x_k - f(x_k) + \gamma(w_k - x_k)),\\ \dot{y}_k &= x_k - y_k + z_k,\\ \dot{z}_k &= -\beta y_k\end{aligned} \tag{4}$$

v_{C2}-coupled system:

$$\begin{aligned}\dot{x}_k &= \alpha(y_k - x_k - f(x_k)),\\ \dot{y}_k &= x_k - y_k + z_k + \gamma(w_k - y_k),\\ \dot{z}_k &= -\beta y_k\end{aligned} \tag{5}$$

Where

$$f(x) = bx + \frac{1}{2}(a-b)[|x+1| - |x-1|],$$

and

$$\begin{aligned}&\alpha = C_2/C_1, \quad \beta = C_2/LG^2, \quad \gamma = Y_0/G,\\ &\hat{t} = Gt/C_2, \quad \hat{\tau} = G\tau/C_2, \quad \text{“}\cdot\text{”} = d/d\hat{t},\\ &x_k = v_{C1k}/B_p, \quad y_k = v_{C2k}/B_p, \quad z_k = i_{Lk}/GB_p,\\ &w_k = e_k(\hat{t} - \hat{\tau})/B_p, \quad a = m_0/G, \quad b = m_1/G.\end{aligned}$$

3. Synchronization Phenomena

3.1 Circuit Experiments

The resistor coupled Chua's circuits have been investigated numerically and experimentally in [3]. In this study, the resistor is replaced by an element with time delay; i.e. transmission line. In our experiments, the Chua's circuits are built referring to [23]. The circuit parameters are adjusted so that the circuits exhibit the double scroll attractor, and so that they synchronize when they are directly coupled by a resistor as reported in [3]. The measured values of synchronizing system are as follows:

$$\begin{aligned}&R = 1.435\ \text{k}\Omega, \quad C_1 = 9.50\ \text{nF}, \quad C_2 = 103.8\ \text{nF},\\ &L = 19.98\ \text{mH}, \quad r_L = 40.25\,\Omega,\\ &m_0 = -0.75\ \text{mS}, \quad m_1 = -0.41\ \text{mS},\\ &B_p = 1.06\ \text{V}.\end{aligned}$$

These parameters will be fixed throughout this paper.

As transmission line in our circuit models, a delay line or a coaxial cable is used. The delay line is an element that transmits input signal waveforms with fidelity but delays time only when it is matched with suitable impedance at both ends. Namely how to use the delay lines is the same as the usage of coaxial cables. Note that both the delay lines and the coaxial cables are not matched in our all experiments. Table 1 shows the characteristics of the delay lines and coaxial cables we used.

v_{C1}-coupled system:
The experimental results are illustrated in Fig. 4. As found from the figures, though the signals from one subsystem to another one through the transmission line are affected by the time delay, the chaotic subsystems synchronize completely when the delay lines are used.

Table 1 The characteristics of real transmission lines.

Transmission Line	Z_0	τ
Delay Line #1 FSL-5n	$50\,\Omega$	5 nsec
Delay Line #2 FN-500B	$500\,\Omega$	500 nsec
Coaxial Cable #1 3D2V 100 m	$50\,\Omega$	500 nsec
Coaxial Cable #2 3D2V 2 km	$50\,\Omega$	$10\,\mu$sec

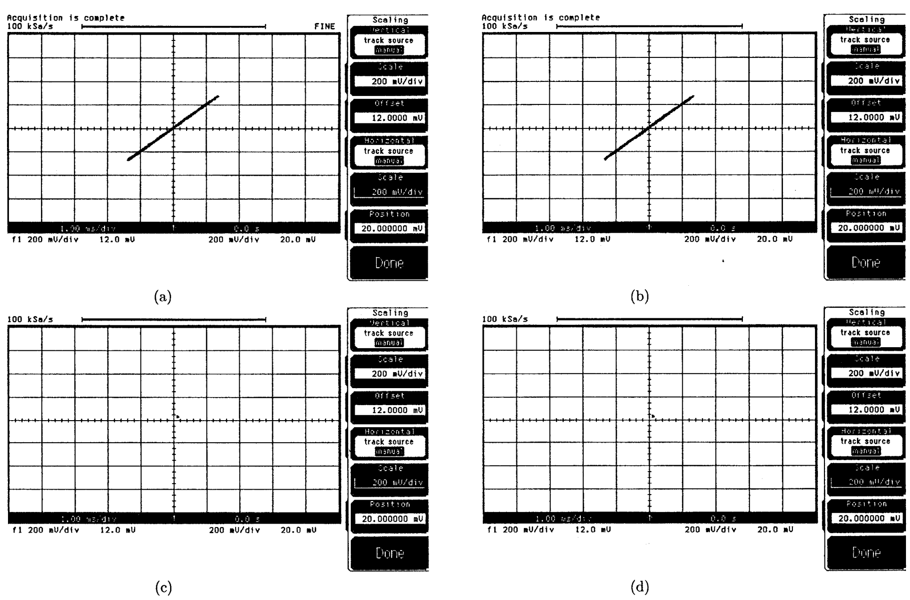

Fig. 4 Synchronization in v_{C1}-coupled system (circuit experiments). (a) Delay Line #1. (b) Delay Line #2. (c) Coaxial Cable #1. (d) Coaxial Cable #2.

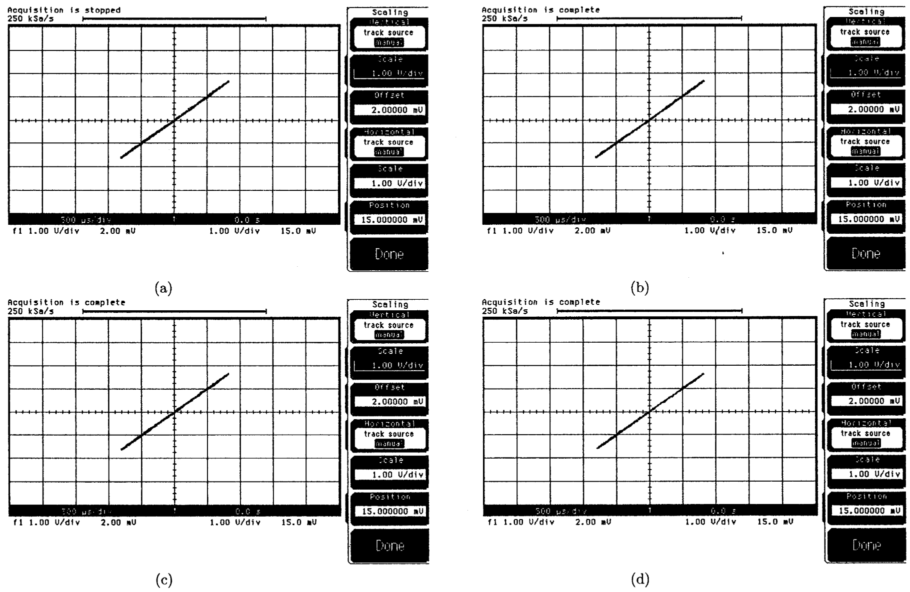

Fig. 5 Synchronization in v_{C2}-coupled system (circuit experiments). (a) Delay Line #1. (b) Delay Line #2. (c) Coaxial Cable #1. (d) Coaxial Cable #2.

However the coaxial cables are used, then the subsystems stopped oscillation. From Figs. 4(a) and (c), it seems that oscillation stops if only τ changes from 5 nsec to 500 nsec and Z_0 remains 50 Ω. Further, from Figs. 4(b) and (c), the subsystems seem to synchronize again, when Z_0 is varied from 50 Ω to 500 Ω even if τ is still 500 nsec.

v_{C2}-coupled system:

The circuit experiments for v_{C2}-coupled system is also done. The experimental results are illustrated in Fig. 5. The subsystems synchronize for all cases. For the case that 2 km long of coaxial cable was used, surprisingly enough, two Chua's circuits synchronized. Unfortunately we do not have any transmission lines with larger delay, thus we could not confirm whether what kinds of phenomena in real system will happen for larger delay.

3.2 Computer Simulations

In this subsection, we confirm whether chaos synchronization observed in real circuits are achieved numerically. In our simulations, Eqs. (4) and (5) are solved by the fourth order Runge-Kutta method. The parameter values and the initial conditions of chaotic subsystems are

$$\alpha = 10, \quad a = -1.2, \quad b = -0.75, \quad B_p = 1,$$
$$x_1 = 1.0, \quad y_1 = 0.02, \quad z_1 = 0.0,$$
$$x_2 = -0.7, \quad y_2 = 0.4, \quad z_2 = 0.08.$$

Taking a suitable value of β the chaotic subsystem exhibits the well-known double scroll attractor.

Moreover, the initial conditions of the waveform generators in the equivalent circuits are set to zero, i.e.,

$$e_i(t) = 0, (t \in [-\tau, 0]).$$

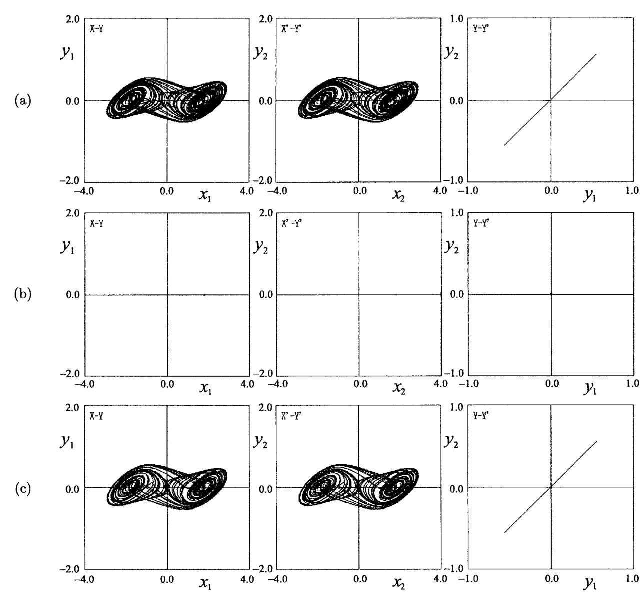

Fig. 6 Synchronization in v_{C1}-coupled system (computer simulations). (a) $\gamma = 10$ and $\hat{\tau} = 0.001$. (b) $\gamma = 10$ and $\hat{\tau} = 0.01$. (c) $\gamma = 1$ and $\hat{\tau} = 0.01$.

But we have confirmed that the same phenomena can be observed for the random initial conditions.

v_{C1}-coupled system:

By varying the values of γ and $\hat{\tau}$, we simulate the v_{C1}-coupled system for $\beta = 15$. Figure 6 shows the results. It can be observed that the chaotic subsystems synchronize completely for $\gamma = 10$ and $\hat{\tau} = 0.001$ (i.e. $Z_0 = 143.5\,\Omega$ and $\tau = 149$ nsec). When only the time delay increases as $\hat{\tau} = 0.01$ (i.e. $\tau = 1490$ nsec), each chaotic subsystems stop oscillation. But the value of γ is also varied as $\gamma = 1$ (i.e. $Z_0 = 1435\,\Omega$), the subsys-

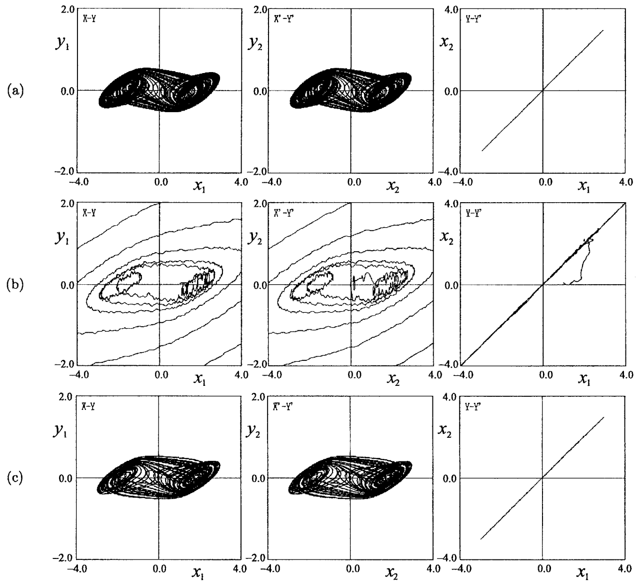

Fig. 7 Synchronization in v_{C2}-coupled system (computer simulations). (a) $\gamma = 30$ and $\hat{\tau} = 0.05$. (b) $\gamma = 30$ and $\hat{\tau} = 0.2$. (c) $\gamma = 15$ and $\hat{\tau} = 0.2$.

tems become to synchronize again even for $\hat{\tau} = 0.01$. By taking the relationship between Z_0, τ and γ, $\hat{\tau}$ into account, these results are the same as the case of experiments.

v_{C2}-coupled system:
In the similar way to v_{C1}-coupled system, numerical investigations are carried out. The results for $\beta = 18$ are shown in Fig. 7. The chaotic subsystems in v_{C2}-coupled system also synchronize perfectly for $\gamma = 30$ and $\hat{\tau} = 0.01$ (i.e. $Z_0 = 47.8\,\Omega$ and $\tau = 1490\,$nsec). If only the time delay increases to $\hat{\tau} = 0.2$ (i.e. $\tau = 29.8\,\mu$sec), however, the orbits of chaotic subsystems diverged as shown in Fig. 7(b). It seems that the orbits of subsystems diverge with increasing the time delay. This could not be observed in circuit experiments, because the time delay probably is small, even for 2 km coaxial cable. Further we found that similar to v_{C1}-coupled system, the subsystems get to synchronize by varying the value of γ to 15 (i.e. $95.7\,\Omega$).

4. Detailed Analysis

4.1 Bifurcation Diagram

The results in previous section let us find the following two issues.

- When the characteristic impedance is fixed, oscillations of the subsystems stop or orbits of the subsystems diverge as the time delay increases.
- By varying the value of characteristic impedance, the subsystems become to synchronize even if time delay increases.

Consequently it is natural to think that the systems

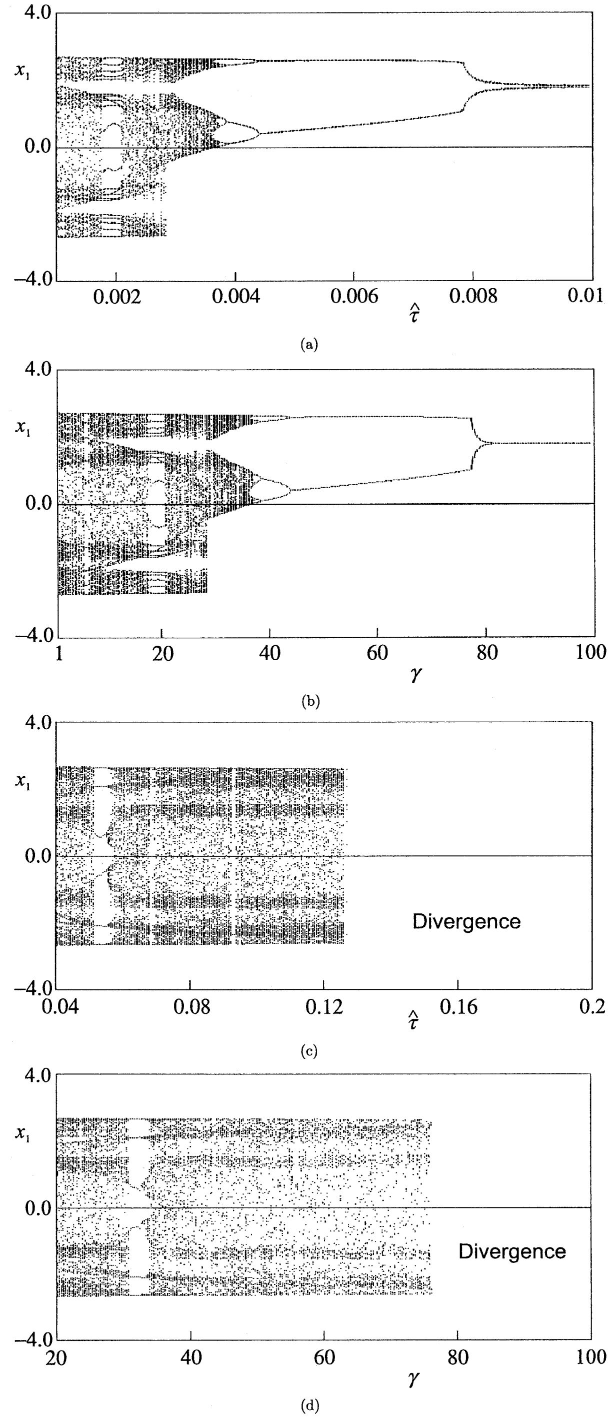

Fig. 8 One parameter bifurcation diagrams (computer simulations). (a) v_{C1}-coupled system ($\beta = 15$ and $\gamma = 10$). (b) v_{C1}-coupled system ($\beta = 15$ and $\hat{\tau} = 0.001$). (c) v_{C2}-coupled system ($\beta = 18$ and $\gamma = 30$). (d) v_{C2}-coupled system ($\beta = 18$ and $\hat{\tau} = 0.05$).

may give rise to bifurcation. To confirm this expectation, we plot bifurcation diagrams with respect to the time delay and the characteristic impedance.

Figure 8 shows the bifurcation diagrams. Where the horizontal axes correspond to the normalized time

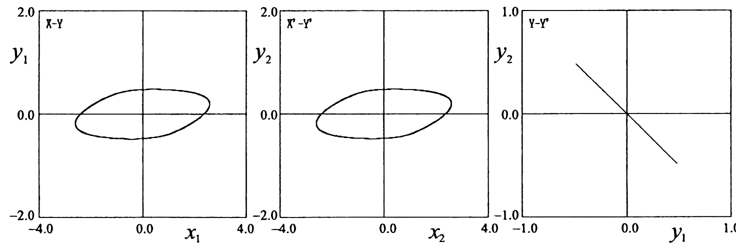

Fig. 9 Anti-phase synchronization for v_{C1}-coupled system (computer simulations). $\beta = 15$, $\gamma = 10$ and $\hat{\tau} = 0.4$.

delay $\hat{\tau}$ for Figs. 8(a) and (c), and correspond to γ which is related to the characteristic impedance, for Figs. 8(b) and (d). Vertical axis is the voltage v_{C1}. Surprisingly enough, for v_{C1}-coupled system, rich bifurcation phenomena including the period-doubling route to chaos and several periodic windows are observed with varying $\hat{\tau}$ or γ. Note that the two subsystems are synchronizing completely for the parameter values in these figures (except the range of divergence). The bifurcation diagrams for different values of γ or $\hat{\tau}$ are omitted, because almost similar diagrams were obtained.

In [19] it has been also reported that their circuit model exhibits the period-doubling bifurcation under certain conditions. However, their subsystems do not synchronize completely during the bifurcation route.

Further the time delay increases, anti-phase synchronization is observed as shown in Fig. 9. This is very interesting phenomena, because anti-phase synchronization can not be observed from resistor coupled systems. More detailed analysis of the synchronization is our future research.

On the other hand, in v_{C2}-coupled system, chaos synchronization can be observed for relatively wide range of time delay, since periodic orbits do not emerge even if the time delay becomes fairly large. By increasing the time delay further, the trajectories eventually diverge. Note that in v_{C2}-coupled system the equilibrium points are always unstable unlike v_{C1}-coupled system. Moreover, many types of periodic window take place according as values of $\hat{\tau}$.

Next, we make detailed laboratory experiments for various different length of time delay in order to confirm whether the bifurcation phenomena observed from our simulations are also generated in real system.

Experiments were carried out by cascading some of delay lines listed in Table 2, for example we connect 12 of Delay Line AT-50 to realize $\tau = 600\,\text{nsec}$ for Fig. 10(a). Note that any transmission lines with a same Z_0 can be cascaded to get larger time delay. The results for v_{C1}-coupled system are shown in Fig. 10. It is found that the period-doubling bifurcation phenomena can be also observed in real system. Unfortunately, we have not confirmed anti-phase synchronization experimentally, which is due to the shortage of time delay[†].

Table 2 The delay lines with $Z_0 = 100\,\Omega$.

Transmission Line	τ
Delay Line AT-01	1 nsec
Delay Line AT-05	5 nsec
Delay Line AT-10	10 nsec
Delay Line AT-50	50 nsec

4.2 Voltage Distribution

At first thought, we expected that the current through the transmission line may be zero or constant, because in systems coupled by a lumped element the current flowing the coupling element is zero or constant. Thus we investigate the current distribution of the transmission line.

Let the current and voltage of transmission line be $i(x,t)$ and $v(x,t)$, respectively. Since $i(x,t) = v(x,t)/Z_0$, we can investigate the voltage distribution alternatively. In our simulations, the transmission line is divided to 50 segments of transmission lines, and then each of them are replaced with the characteristic models.

The results with the subsystems synchronizing are shown in Figs. 11 and 12. In turn from left, the attractor in the left subcircuit, the attractor in the right one, the state of synchronization, and the voltage distribution are given. In the figure of voltage distribution, the axis toward the right direction is the distance from the left end of transmission line, the upward axis is the voltage v_{C1} or v_{C2}, and axis toward the depth corresponds to time.

As you can see from this figure, the voltage distribution is not constant for both systems. So we can conclude that though the current flowing through the transmission line is varying, the chaotic subsystems synchronize. This is so wonder, because the current through the coupling element in synchronizing systems coupled with lumped element is constant.

[†]The maximum time delay in our experiments is 1100 nsec (20 of AT-50 and 10 of AT-10 are used).

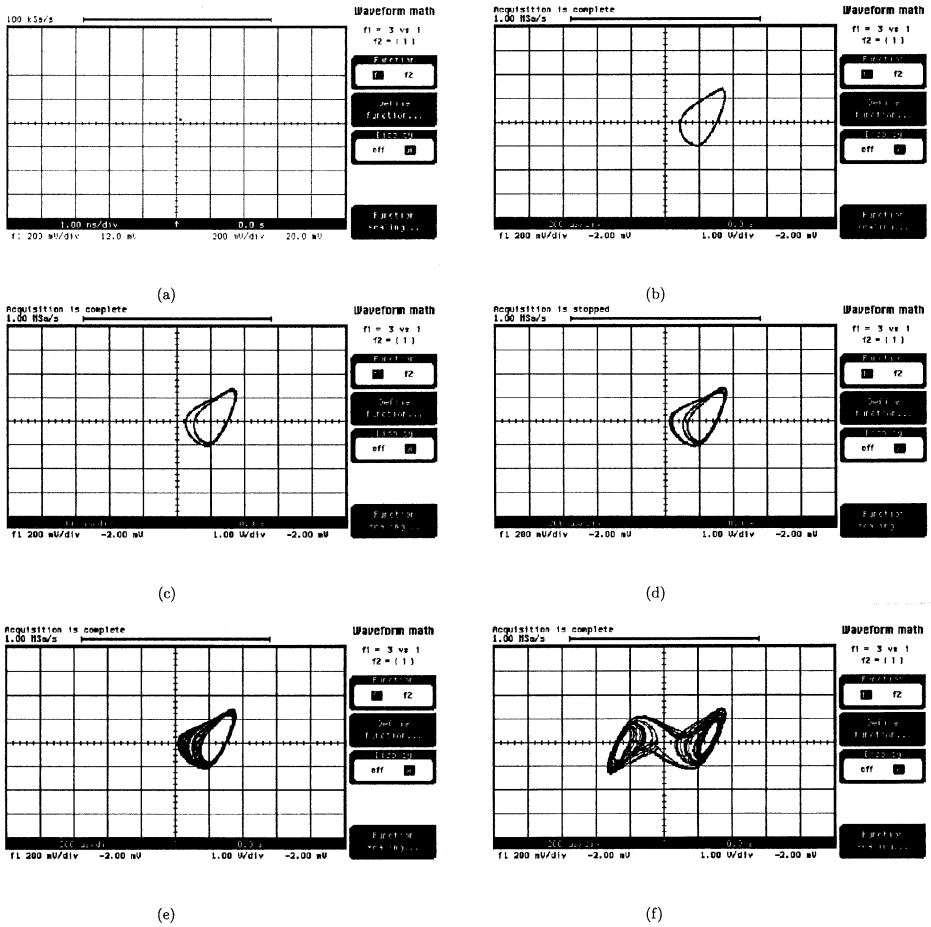

Fig. 10 Phase portraits of period-doubling bifurcation for v_{C1}-coupled system (circuit experiments). (a) fixed point for $\tau = 600$ nsec. (b) period-1 attractor for $\tau = 250$ nsec. (c) period-2 attractor for $\tau = 200$ nsec. (d) period-4 attractor for $\tau = 165$ nsec. (e) spiral attractor for $\tau = 150$ nsec. (f) double scroll attractor for $\tau = 50$ nsec.

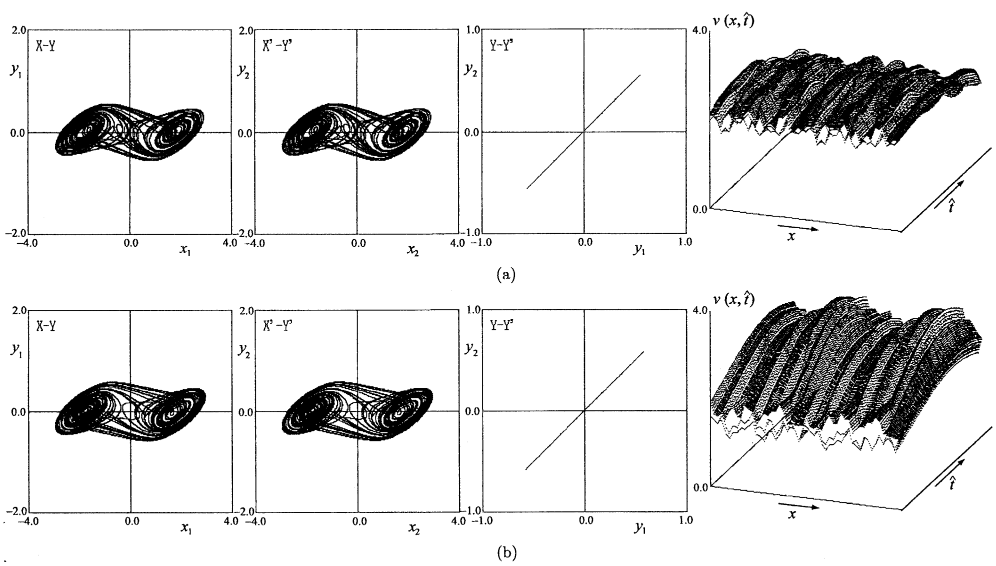

Fig. 11 Voltage distribution for v_{C1}-coupled system (computer simulations). $\beta = 15$. (a) $\gamma = 10$ and $\hat{\tau} = 0.001$. (b) $\gamma = 1$ and $\hat{\tau} = 0.01$.

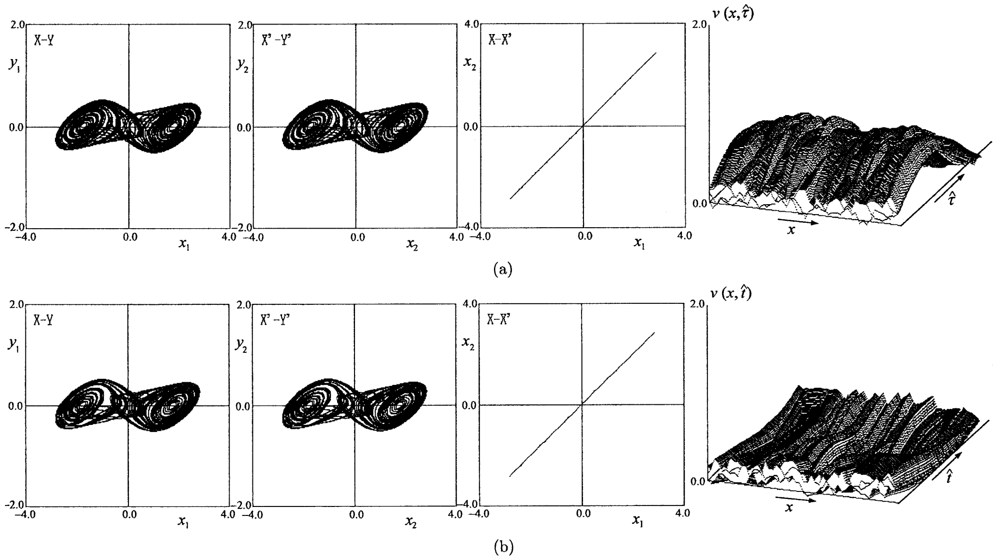

Fig. 12 Voltage distribution for v_{C2}-coupled system (computer simulations). $\beta = 18$. (a) $\gamma = 30$ and $\hat{\tau} = 0.01$. (b) $\gamma = 15$ and $\hat{\tau} = 0.1$.

5. Conclusion

In this study, we have reported very interesting phenomena in a chaotic system coupled by a transmission line. Chaos synchronization is also achieved in such system as well as systems coupled with lumped elements. It is found that in our models bifurcation and complex phenomena occur with variation of the time delay, which can not be observed in resistor coupled systems. Also the current through the coupling transmission line is not constant, which is different from the case of lumped element coupling. The systems may have more exciting phenomena according to the characteristic impedance and time delay.

Our goal is to make clear synchronization mechanisms and various nonlinear phenomena in our models, but unfortunately we have not yet reached a level which we can consider about them. Thus, for the purpose, we primary have to investigate our systems more fully and then begin qualitative analysis. Moreover we will think over some applications using our models.

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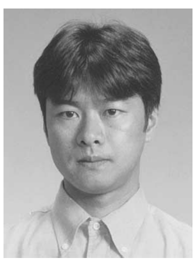

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