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Steady-State Response of Nonlinear Circuits Containing Parasitic Elements

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SUMMARY We propose here a time-domain shooting algorithm for calculating the steady-state responses of nonlinear RF circuits containing parasitic elements that is based on both a modified Newton and a secant methods. Bipolar transistors and MOSFETs in ICs have small parasitic capacitors among their terminals. We can not neglect them because they will gives large effects to the shooting algorithm at the high frequency. Since our purpose is to develop a user friendly simulator, we mainly take into account the relatively large normal capacitors such as coupling and/or by-pass capacitors and so on, because the parasitic capacitors are usually smaller and contained in the device models. We have developed a very simple simulator only using the fundamental tools of SPICE, which can be applied to relatively large scale ICs, efficiently.

key words: *steady-state analysis, RF circuits, time-domain secant method, parasitic capacitors, SPICE*

1. Introduction

It is very important to analyze the steady-state responses for designing communication circuits such as modulators, mixers, etc. When the attenuation of transient response is sufficiently large, we can easily calculate the response with the *brute-force method*. However, it is sometimes happened that the transient response continues for a long period due to the small attenuation. In this case, there are two basic approaches; i.e., the frequency-domain method [1]–[3] and the time-domain method [4]–[7]. The former is based on the harmonic balance method, which is usefully applied to weakly nonlinear circuits. However, the computational efficiency rapidly decreases in the cases when the number of nonlinear elements increases, and the nonlinearities become stronger, because the *determining equation* for calculating the Fourier coefficients becomes very large scale. Fortunately, some algorithms for solving large scale determining equation have been proposed [8], [9] which are efficiently applied to the system with the large scale sparse Jacobian matrix. Frequency-domain relaxation method [14] is also efficiently applied to the analysis of relatively large scale weakly nonlinear RF circuits.

On the other hand, the time-domain method is based on the transient analysis, where the initial guess giving rise to the steady-state response is calculated by the Newton type shooting method. The first one is based on the Newton Raphson method whose Jacobian matrix is estimated by the analysis of the time-varying sensitivity circuits equal to the number of state-variables [4], [5]. Note that it will take a large computer memory for a large scale circuit containing many parasitic elements. Fortunately, the transient terms due to the parasitic elements will be quickly reduced in the sensitivity analysis, so that the Jacobian matrix corresponding to the elements will be approximately replaced by the unit matrix in the Newton iteration [6]. Thus, it is possible to reduce the size of the Jacobian matrix. Although the extrapolation method is not theoretically guaranteed the convergency, it is a very simple algorithm, and seems to be suitable for the implementation with the SPICE simulator [7].

We propose here an algorithm for calculating the steady-state responses based on both the modified Newton and secant methods [10], [11]. Since the secant method is a type of Newton method whose Jacobian matrix is successively modified in the iterations, it can be applied to calculate both the stable and unstable steady-state responses if the circuit does not contain parasitic elements. Otherwise, our method can be only applied to caluculate the stable steady-state response.

There have been published many secant methods [19], and we already applied one of them to calculate the steady-state responses of nonlinear circuits [20] which is based on a discrete Newton method. The algorithm becomes sometimes unstable near at the solution point. In this paper, we apply much more stable algorithm using an orthogonal procedure [10]. Note that although the convergence ratios of the secant methods are smaller than the Newton method, they can be usefully applied to get the solution of a nonlinear system when the Jacobian matrix can not be explicitly obtained.

Thus, the method is suitable for the development of a simple SPICE simulator, because we only use the state variables at every periodic point in the transient analysis, and need not to use the time-varying sensitivity circuits in the iteration.

At the first step in our algorithm, the initial guess is estimated by the modified Newton method in the meaning that the Jacobian matrix is calculated at the dc operating point, and after then, the matrix is succes-

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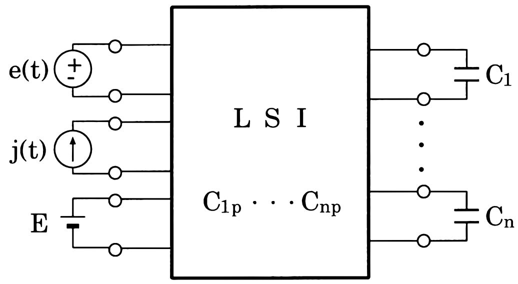

Fig. 1 LSI circuit with the normal capacitors $C_1, ..., C_n$.

sively modified by a secant method in the iterations. In our algorithm, the initial guess giving rise to the steady-state response is calculated for the normal capacitor voltages after the transient response due to the parasitic capacitors is reduced. The transient period is approximately estimated from the frequency-domain driving point characteristic for the modified circuit where the normal capacitors are replaced by the bias voltage sources. Thus, the characteristic curve can be easily obtained by the ac-sweep of SPICE, and the poles are estimated from the approximate rational polynomial function. We have developed a user friendly simulator using the fundamental tools of SPICE, which can be applied to the relatively large scale ICs, efficiently.

2. Modified Newton and Secant Methods

2.1 Effect of Parasitic Elements

To focus on the main idea of our algorithm, consider a LSI circuit shown in Fig. 1, where $C_1, ..., C_n$ are the *normal capacitors* such as coupling and/or by-pass capacitors and so on, which are considered as sufficiently large compared to the *parasitic capacitors* $C_{1p}, ..., C_{np}$, and they will give large effects to the period of transient response. We call the normal capacitors simply *capacitors*, in the following. For example, bipolar transistors have typically $10^{-12}F$ order nonlinear capacitors among the emitter-base, the base-collector and the emitter-collector [15], [16]. Depending on circuit configurations, we may not neglect their effects in the high frequency such as over 100 MHz. Since MOSFETs also have small parasitic capacitors such as the gate-drain C_{GD}, the gate-source C_{GS}, and the bulk-gate, bulk-drain and bulk-source, C_{BG}, C_{BD}, and C_{BS}, respectively [16]. Although they may give an effect to the transient response in the high frequency region, the effect will be much smaller than that from the normal capacitors. Furthermore, it is troublesome to consider them in the circuit simulation, so that we mainly take into account the normal capacitors in our simulation algorithm.

Assume that the circuit is driven by multiple inputs $e(t)$ and $j(t)$, whose frequency components are $f_1, ..., f_n$. If they are the integer relations, the steady-state response has a period called the *total period* [3] that is defined by

$$T = \frac{1}{G.C.M. \{f_1, \cdots, f_n\}}, \quad \nu = 2\pi/T \tag{1}$$

where $G.C.M.$ means the greatest common measure. Hence, we can estimate the fundamental frequency component for the multiple frequencies and the period.

Now, let us derive the circuit equation in the form of algebraic-differential equation. At first, let us choose a *normal tree* for a given circuit such that it must contain the maximum possible number of the capacitors, and after then, as many as the parasitic capacitors, and lastly, the resistors. Next, we define the variables such as $\boldsymbol{v}$ for the capacitor voltages, $\boldsymbol{v}_p$ for the parasitic capacitors and $\boldsymbol{v}_R$ for the resistors in the normal tree.

Then, we can describe the circuit equation in the following forms using the fundamental cutset equations and the some loop equations [17];

$$\boldsymbol{C}(\boldsymbol{v})\dot{\boldsymbol{v}} = \boldsymbol{f}_1(\boldsymbol{v}, \boldsymbol{v}_R, \boldsymbol{v}_p, \nu t) \tag{2a}$$

$$\varepsilon \boldsymbol{C}_p(\boldsymbol{v}_p)\dot{\boldsymbol{v}}_p = \boldsymbol{f}_2(\boldsymbol{v}, \boldsymbol{v}_R, \boldsymbol{v}_p, \nu t) \tag{2b}$$

$$\boldsymbol{f}_3(\boldsymbol{v}, \boldsymbol{v}_R, \boldsymbol{v}_p, \nu t) = \boldsymbol{0} \tag{2c}$$

where Eqs. (2a) and (2b) are the cutset equations corresponding to the normal tree capacitors and the parasitic capacitors, respectively. Equation (2c) is the loop equations containing the normal tree resistors and/or C-E loops [17]. We assume that ε for the parasitic capacitors shows a sufficiently small constant.

Our time-domain shooting algorithm finds out the capacitor voltages $\boldsymbol{v}(T_p)$ giving rise to the steady-state response, where T_p is the time such that the effect of the parasitic capacitors becomes negligible in the transient response. To estimate T_p in a qualitative point of view, we consider the variational equation at the steady-state response $\{\boldsymbol{v}_0, \boldsymbol{v}_{R0}, \boldsymbol{v}_{p0}\}$;

$$\boldsymbol{v} = \boldsymbol{v}_0 + \Delta\boldsymbol{v}, \quad \boldsymbol{v}_R = \boldsymbol{v}_{R0} + \Delta\boldsymbol{v}_R, \quad \boldsymbol{v}_p = \boldsymbol{v}_{p0} + \Delta\boldsymbol{v}_p \tag{3}$$

Substituting Eq. (3) into Eq. (2), we have

$$\begin{pmatrix} \boldsymbol{C}(\boldsymbol{v}_0)\Delta\dot{\boldsymbol{v}} \\ \varepsilon\boldsymbol{C}_p(\boldsymbol{v}_{p0})\Delta\dot{\boldsymbol{v}}_p \\ \boldsymbol{0} \end{pmatrix} = \begin{pmatrix} \frac{\partial \boldsymbol{f}_1}{\partial \boldsymbol{v}} & \frac{\partial \boldsymbol{f}_1}{\partial \boldsymbol{v}_R} & \frac{\partial \boldsymbol{f}_1}{\partial \boldsymbol{v}_p} \\ \frac{\partial \boldsymbol{f}_2}{\partial \boldsymbol{v}} & \frac{\partial \boldsymbol{f}_2}{\partial \boldsymbol{v}_R} & \frac{\partial \boldsymbol{f}_2}{\partial \boldsymbol{v}_p} \\ \frac{\partial \boldsymbol{f}_3}{\partial \boldsymbol{v}} & \frac{\partial \boldsymbol{f}_3}{\partial \boldsymbol{v}_R} & \frac{\partial \boldsymbol{f}_3}{\partial \boldsymbol{v}_p} \end{pmatrix} \times \begin{pmatrix} \Delta\boldsymbol{v} \\ \Delta\boldsymbol{v}_R \\ \Delta\boldsymbol{v}_p \end{pmatrix} - \begin{pmatrix} \frac{\partial \boldsymbol{C}(\boldsymbol{v})}{\partial \boldsymbol{v}}\dot{\boldsymbol{v}}_0\Delta\boldsymbol{v} \\ \frac{\varepsilon\partial \boldsymbol{C}_p(\boldsymbol{v}_p)}{\partial \boldsymbol{v}_p}\dot{\boldsymbol{v}}_{p0}\Delta\boldsymbol{v}_p \\ \boldsymbol{0} \end{pmatrix} \tag{4}$$

where the Jacobian matrix is estimated at the steady-state response. Equation (4) is a time-varying system with the period T. From the third row, we have

$$\Delta\boldsymbol{v}_R = -\left(\frac{\partial \boldsymbol{f}_3}{\partial \boldsymbol{v}_R}\right)^{-1}\left(\frac{\partial \boldsymbol{f}_3}{\partial \boldsymbol{v}}\ \frac{\partial \boldsymbol{f}_3}{\partial \boldsymbol{v}_p}\right)\begin{pmatrix} \Delta\boldsymbol{v} \\ \Delta\boldsymbol{v}_p \end{pmatrix} \tag{5}$$

Now, assume that the matrices $\boldsymbol{C}(\boldsymbol{v}_0)$ and $\boldsymbol{C}_p(\boldsymbol{v}_{p0})$ are positive and definite at the steady-state response. Then, they have the inverse matrices, and substituting Eq. (5) into Eq. (4), we have the following form;

$$\begin{pmatrix} \Delta\dot{\boldsymbol{v}} \\ \varepsilon\Delta\dot{\boldsymbol{v}}_p \end{pmatrix} = \begin{pmatrix} \boldsymbol{A}_{11}(\nu t) & \boldsymbol{A}_{12}(\nu t) \\ \boldsymbol{A}_{21}(\nu t) & \boldsymbol{A}_{22}(\nu t) \end{pmatrix} \begin{pmatrix} \Delta\boldsymbol{v} \\ \Delta\boldsymbol{v}_p \end{pmatrix} \tag{6}$$

In order to investigate the effect of the parasitic capacitors, we consider the second equation of Eq. (6);

$$\varepsilon\Delta\dot{\boldsymbol{v}}_p = \boldsymbol{A}_{21}(\nu t)\Delta\boldsymbol{v} + \boldsymbol{A}_{22}(\nu t)\Delta\boldsymbol{v}_p \tag{7}$$

First term in the right hand side is a variational value from the capacitor voltages which behaves like as a forced input in Eq. (7). It is known that the solution of a linear differential equation consists of two solutions; one is the *particular solution* $\boldsymbol{v}_{pp}(t)$, and another is the *general solution* $\boldsymbol{v}_{pt}(t)$ satisfying

$$\varepsilon\Delta\dot{\boldsymbol{v}}_{pt} = \boldsymbol{A}_{22}(\nu t)\Delta\boldsymbol{v}_{pt} \tag{8}$$

Then, it is known from the Floquet theorem [12] that the general solution has a following property;

$$\Delta\boldsymbol{v}_{pt}(t+T) = \exp(\lambda_p T)\Delta\boldsymbol{v}_{pt}(t), \quad \text{for} \quad \lambda_p = \lambda_2/\varepsilon \tag{9}$$

where λ_2 is a *characteristic multiplier* of the time-varying differential Eq. (8) whose real part will be negative. Observe that if $|\mathrm{Re}\lambda_p|$ is sufficiently large, the transient terms due to the parasitic capacitors will be quickly reduced, and the solution of Eq. (7) will quickly approach to the particular solution $\boldsymbol{v}_{pp}(t)$. It means that the response $\boldsymbol{v}_p(t)$ only depends on $\boldsymbol{v}(t)$ after the transient term due to the parasitic capacitors is reduced, and the variable $\boldsymbol{v}_p(t)$ *no more* behaves like as the state variables. Therefore, after the period T_p, it is possible to apply our shooting method only to the normal capacitor voltages $\boldsymbol{v}(t)$, and we can find the initial guess giving rise to the steady-state response.

Now, let us estimate the *approximate transient period* T_p in the case that the cirsuit is at the dc poerating point. Namely, we calculate the transient response for the modified circuit whose the capacitor Cs are replaced by the voltage source $\boldsymbol{v}(0)$ equal to the operating points. Then, the resultant circuit only contains the parasitic capacitors, and T_p can be estimate by the transient response. It is also possible to estimate T_p from the frequency response. Namely, let us calculate the driving point admittance in the frequency domain at any capacitor port†. In practice, the frequency response can be calculated by the ac-sweep of SPICE at one of the capacitor ports. The response curve is approximately described by a rational polynomial as follows;

$$Y(s) = \frac{a_0 + a_1 s + \ldots + a_m s^m}{b_0 + b_1 s + \ldots + b_n s^n} \tag{10}$$

In order to estimate the approximate transient period T_p, we need to know the smallest pole in the negative of the left hand complex plane, so that it is enough to apply the lower order of a rational polynomial function to the approximation [21].

Remark that if the effect from some of the normal capacitors to the transient response is negligible, we can also consider them as parasitic capacitors.

2.2 Our Secant Method

Our purpose is to develop a user friendly simulator using the fundamental tools of SPICE, where we only consider the capacitor voltages $\boldsymbol{v}(t)$ for determining the steady-state response. For a large scale circuit, it is troublesome to choose the parasitic capacitor voltages as the state variables, because the number becomes enormous when the circuit scale increases. Furthermore, they are contained in the device models.

Now, let the period of transient response due to the parasitic elements be T_p. Then, we need to calculate the initial guesses $\boldsymbol{v}(T_p)$ giving rise to the steady-state response, which will satisfy the following *determining equation*††:

$$\boldsymbol{F}\left(\boldsymbol{v}\left(T_p\right)\right) = \boldsymbol{v}\left(T_p + T\right) - \boldsymbol{v}\left(T_p\right) = \boldsymbol{0}, \ \text{for}\ T = 2\pi/\nu \tag{11}$$

Let us solve the determining Eq. (11) by the iterational method. It can be efficiently calculated by the Newton method [4] as follows;

$$\boldsymbol{v}^{j+1}\left(T_p\right) = \boldsymbol{v}^{j}\left(T_p\right) - \left(\left. \frac{\partial \boldsymbol{F}\left(\boldsymbol{v}\left(T_p\right)\right)}{\partial \boldsymbol{v}\left(T_p\right)} \right|_{\boldsymbol{v}(T_p)=\boldsymbol{v}^j(T_p)} \right)^{-1} \times \boldsymbol{F}\left(\boldsymbol{v}^j\left(T_p\right)\right), \quad j = 0, 1, \ldots \tag{12}$$

The Jacobian matrix can be calculated by the analysis of time-varying sensitivity circuits for all of the state variables [4], [5] in the period $[T_p, T + T_p]$. Therefore, the computer efficiency will be decreased as the circuit scale becomes lager and the total period T becomes longer.

On the other hand, there is a simple algorithm based on the modified Newton method, where the Jacobian matrix is estimated at the dc operating point. Namely, each column of the matrix is calculated by the transient response to a sufficiently small impulsive input for corresponding capacitor; $\Delta\boldsymbol{v}(T)/\Delta\boldsymbol{v}(0)$ for all the capacitor Cs. If we use the same Jacobian matrix

$$\frac{\partial \boldsymbol{F}(\boldsymbol{v}(T))}{\partial \boldsymbol{v}(T)} = \left(I - \frac{\Delta\boldsymbol{v}(T)}{\Delta\boldsymbol{v}(0)} \right) \tag{13}$$

†It is known that although there are the same number of different driving point admittances as the capacitor Cs, their poles for all of the admittances are located at the same points in the complex plane.

††It is shown in Sect. 2.1 that we can get the exact steady-state response with our secant method after the transient phenomena due to the effect of parasitic capacitors is completely finished.

at all of the iterations, the algorithm is called the modified Newton method. Although the algorithm can be efficiently applied to the weakly nonlinear circuits, the convergence is not guaranteed for the strong nonlinearities.

Hence, we propose here a secant method [10] such that the Jacobian matrix [11] is successively improved in the latter iterations. Let $\boldsymbol{J}^0$ be the initial Jacobian matrix estimated by the above method. Then, we have at the first iteration

$$\boldsymbol{J}^0 \delta \boldsymbol{v}^0(T_p) = -\boldsymbol{F}\left(\boldsymbol{v}^0(T_p)\right) \quad \text{for} \;\; \boldsymbol{v}^1(T_p) = \boldsymbol{v}^0(T_p) + \delta \boldsymbol{v}^0(T_p) \tag{14}$$

Thus, we need to estimate the modified Jacobian matrix $\boldsymbol{J}^1$. Suppose it satisfies the following relation;

$$\boldsymbol{F}\left(\boldsymbol{v}^1(T_p)\right) = \boldsymbol{F}\left(\boldsymbol{v}^0(T_p)\right) + \boldsymbol{J}^1 \delta \boldsymbol{v}^0(T_p) \quad \text{where} \;\; \boldsymbol{J}^1 = \boldsymbol{J}^0 + \boldsymbol{D}^0 \tag{15}$$

$\boldsymbol{D}^0$ corresponds to the variational Jacobian matrix to be determined. From Eqs. (14) and (15), we have

$$\boldsymbol{F}\left(\boldsymbol{v}^1(T_p)\right) = \boldsymbol{D}^0 \delta \boldsymbol{v}^0(T_p) \tag{16}$$

where $\boldsymbol{D}^0$ can be solved as follows;

$$\boldsymbol{D}^0 = \frac{\boldsymbol{F}\left(\boldsymbol{v}^1(T_p)\right)\left(\boldsymbol{x}^0\right)^T}{\left(\boldsymbol{x}^0\right)^T \delta \boldsymbol{v}^0(T_p)} \tag{17}$$

where $\boldsymbol{x}^0$ is an arbitrary vector satisfying $\left(\boldsymbol{x}^0\right)^T \delta \boldsymbol{v}^0(T_p) \neq 0$. In the same manner, we have the following iteration;

$$\boldsymbol{\delta v}^j(T_p) = -\left(\boldsymbol{J}^j\right)^{-1} \boldsymbol{F}(\boldsymbol{v}^j(T_p)) \tag{18}$$

$$\boldsymbol{v}^{j+1}(T_p) = \boldsymbol{v}^j(T_p) + \boldsymbol{\delta v}^j(T_p) \tag{19}$$

$$\boldsymbol{D}^j = \frac{\boldsymbol{F}\left(\boldsymbol{v}^{j+1}(T_p)\right)\left(\boldsymbol{x}^j\right)^T}{\left(\boldsymbol{x}^j\right)^T \delta \boldsymbol{v}^j(T_p)} \tag{20}$$

$$\boldsymbol{J}^{j+1} \delta \boldsymbol{v}^j(T_p) = \boldsymbol{F}(\boldsymbol{v}^{j+1}(T_p)) - \boldsymbol{F}(\boldsymbol{v}^j(T_p)) \quad \text{for} \;\; \boldsymbol{J}^{j+1} = \boldsymbol{J}^j + \boldsymbol{D}^j, \; j = 1, 2, \ldots \tag{21}$$

where if $j > n$, $\boldsymbol{x}^j$ is chosen orthogonal to the previous $n-1$ steps [18], $\delta \boldsymbol{v}^{j-n+1}(T_p), \ldots, \delta \boldsymbol{v}^{j-1}(T_p)$, and if $j < n$, we can only demand that $\boldsymbol{x}^j$ is orthogonal to the available $j-1$ steps, $\delta \boldsymbol{v}^1(T_p), \ldots, \delta \boldsymbol{v}^{j-1}(T_p)$. Observe that our secant method only uses the data $\boldsymbol{v}(t)$ at $t = T_p$ and $t = T + T_p$ in the transient response, and it is obtained with the SPICE. Thus, we can easily develop a user friendly simulator only using fundamental tools of SPICE, where we need not to derive any troublesome circuit equations. It makes our scant method much more powerful.

Our secant algorithm

S.0 Set $j = 0$, and set the initial guess $\boldsymbol{v}^0(T_p)$ equal to the bias voltages. The initial Jacobian matrix $\boldsymbol{J}^0$ is estimated by Eq. (13) at the dc bias points. Applying the rational approximation to the driving point admittance obtained by the ac-sweep of SPICE, we estimate the approximate transient period T_p due to the parasitic elements. Set $\boldsymbol{x}^0 = [1, 0, \ldots, 0]$, and stopping condition ε.

S.1 Calculate the transient response in the period of $[0, T + T_p]$ from the initial guess $\boldsymbol{v}(0) = \boldsymbol{v}^j(T_p)$. Let us estimate the variation by

$$\delta \boldsymbol{v}^j(T_p) = -\left[\boldsymbol{J}^j\right]^{-1} \boldsymbol{F}\left(\boldsymbol{v}^j(T_p)\right)$$

S.2 Calculate the solution at $j+1$st iteration

$$\boldsymbol{v}^{j+1}(T_p) = \boldsymbol{v}^j(T_p) + \delta \boldsymbol{v}^j(T_p)$$

S.3 If $\left\|\boldsymbol{F}\left(\boldsymbol{v}^{j+1}(T_p)\right)\right\| < \varepsilon$ for a given small ε, then stop. Otherwise, calculate the variational value of the Jacobian matrix

$$\boldsymbol{D}^j = \frac{\boldsymbol{F}\left(\boldsymbol{v}^{j+1}(T_p)\right)\left(\boldsymbol{x}^j\right)^T}{\left(\boldsymbol{x}^j\right)^T \delta \boldsymbol{v}^j(T_p)}$$

S.4 Set $\boldsymbol{J}^{j+1} = \boldsymbol{J}^j + \boldsymbol{D}^j$.

S.5 Set $j = j + 1$ and go to Step 1.

In this algorithm, the vector $\boldsymbol{x}^j$ must be chosen orthogonal to the previous vectors $\delta \boldsymbol{v}^i, i = j-n+1, \ldots, j-1$, which is efficiently executed with the Schmidt orthonormalization procedure [18]. Observe that, to implement our algorithm, we only need to execute the transient analysis in the period $[0, T_p + T]$ and get $\boldsymbol{v}(T_p)$ and $\boldsymbol{v}(T + T_p)$.

The flow chart of our secant method is shown in Fig. 2. In our algorithm, the data $\boldsymbol{v}^j(T_p)$ and $\boldsymbol{v}^j(T_p + T)$ are obtained from the transient analysis of SPICE and they are transferred to C-language program, where the initial guess $\boldsymbol{v}^{j+1}(T_p)$ is calculated with the secant method. After then, the data from C-language program is again transferred to SPICE, and so on.

Remark 1: It is known that if one of the variables in $\delta \boldsymbol{v}^j(T_p) = \boldsymbol{v}^{j+1}(T_p) - \boldsymbol{v}^j(T_p)$ approaches to zero, the $\boldsymbol{x}^j$ obtained by the orthonormalization procedure [18] may have serious error. In this case, the iteration sometimes happens to become unstable. Thus, we will recommend to remove the variable v_i from $\boldsymbol{v}$ if

$$|\delta v_i^j| < \delta, \quad \text{for some } i$$

for a sufficiently small δ, where δ depends on both the digit of computer and the truncation error in the numerical integration method [13]. Thus, the dimension of $\boldsymbol{J}^j$ and $\boldsymbol{D}^j$ should be reduced by one, and the orthogonal vector $\boldsymbol{x}^j$ should be estimated for the remaining variables.

Remark 2: If we define the convergence ratio by p in the following relation

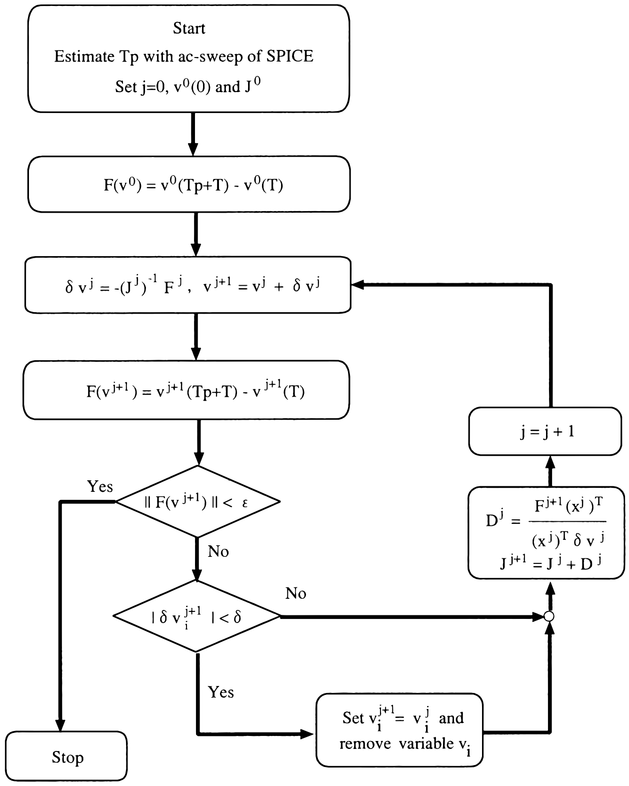

Fig. 2 Flow chart of the secant method.

$$||\boldsymbol{v}^{j+1}(T_p) - \hat{\boldsymbol{v}}(T_p)|| = k||\boldsymbol{v}^j(T_p) - \hat{\boldsymbol{v}}(T_p)||^p$$

where $\hat{\boldsymbol{v}}$ means the exact solution, then, the convergence rate of the secant method is $p = 1.62$ [11]. Although the ratio is smaller than the Newton method $p = 2$, our secant algorithm is a very simple because we need not solve the sensitivity circuit. Therefore, our method seems to be suitable for developing a user friendly simulator using SPICE.

3. Illstrative Examples

3.1 RLC Circuit with Nonlinear Resistance

In order to investigate the effect of parasitic capacitors in the steady-state solution, we consider a simple RLC circuit as shown in Fig. 3(a), where the parasitic capacitor C_p is chosen 10% of the normal capacitor C. Since $\lambda_p = -20$ due to the parasitic capacitor is very large in negative, the effect in the transient response will be negligible after 1 [sec].

Although the number of state variables is 3 in this example, the transient phenomena will behave like as 2 state-variables circuit after the transient period due to C_p is reduced. Thus, we can find out the initial guess $\boldsymbol{v}(T_p)$ giving rise to the steady-state solution after $T_p = 1$ [sec].

In Table 1 where the *exact solution* is obtained by the Newton method [4]. The second result is obtained by solving the determining equation $\boldsymbol{F}(\boldsymbol{v}(0)) = \boldsymbol{v}(0) - \boldsymbol{v}(T)$. In this case, even if the convergence algorithm has a solution, it will be the *fault* solution in the $(v_C(0), i_L(0), v_{C_p}(0))$-plane†. The third result is obtained with our secant method, where we have chosen $T_p = T$ for simplicity. After the transient due to C_p is reduced, the circuit will behave like as in the 2 dimensional plane. Therefore, the result is almost equal to the exact solution. The convergence ratio is sufficient large as shown in Fig. 3(b).

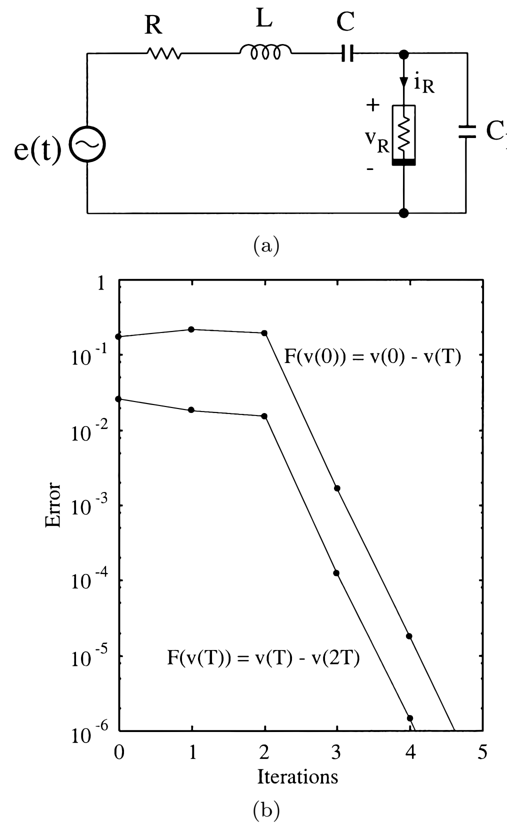

Fig. 3 (a) RLC circuit with nonlinear resistance. $R = 1$, $L = 1$, $C = 1$, $C_p = 0.1$, $i_R = v_R + v_R^3$ $e(t) = \sin 2\pi t$. (b) Convergence rate. Error $= \|\boldsymbol{F}(\boldsymbol{v}^j)\|$.

Table 1 Comparisons of the initial guesses.

	$v_C(T_p)$	$i_L(T_p)$	$v_{Cp}(T_p)$
Exact solution	-7.7357×10^{-3}	-0.16130	1.4537
$\mathbf{F}(\mathbf{v}(0)) = \mathbf{0}$	-5.8436×10^{-3}	-0.16196	-
$\mathbf{F}(\mathbf{v}(T_p)) = \mathbf{0}$	-7.7355×10^{-3}	-0.16130	-

3.2 RC Amplifier

Now, consider a simple RC amplifier circuit shown in Fig. 4(a). The transistor has the small parasitic capacitors among the emitter, base and collector [15], which

†Note that there will exist an infinite number of solutions satisfying $\boldsymbol{F}(\boldsymbol{v}(0)) = \boldsymbol{v}(0) - \boldsymbol{v}(T)$ in 3 dimensional $(v_C(0), i_L(0), v_{C_p}(0))$-plane because the algorithm using $\boldsymbol{F}(\boldsymbol{v}(0))$ is in 2-dimensional plane.

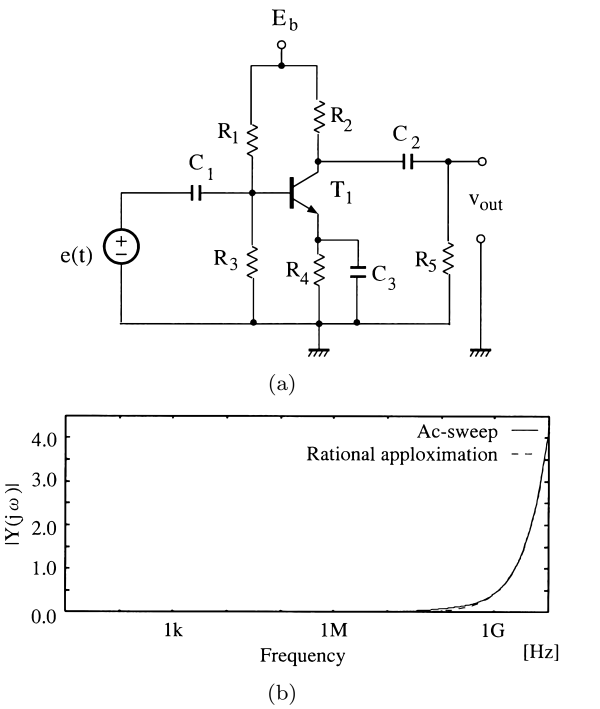

Fig. 4 (a) RC amplifier. $R_1 = 490\,\mathrm{k}\Omega$, $R_2 = 2\,\mathrm{k}\Omega$, $R_3 = 100\,\mathrm{k}\Omega$, $R_4 = 200\,\Omega$, $R_5 = 600\,\Omega$, $C_1 = 10\,\mathrm{nF}$, $C_2 = 10\,\mathrm{nF}$, $C_3 = 10\,\mathrm{nF}$, $e(t) = 0.1 \sin 2\pi 10^8 t\,[\mathrm{V}]$. (b) The driving point characteristic curve at port C_1.

cannot be neglected at higher than 100 [MHz].

At first, we choose the coupling capacitors C_1, C_2 and the by-pass capacitor C_3 as the normal capacitors. To investigate the effect of the parasitic capacitors, the normal capacitors are replaced with the operating voltage sources $v_1(0) = 1.007\,[\mathrm{V}]$, $v_2(0) = 6.641\,[\mathrm{V}]$ and $v_3(0) = 0.338\,[\mathrm{V}]$, which are obtained by the dc analysis of SPICE. Now, we apply the ac-sweep of SPICE, and get the driving point characteristic curve as shown in Fig. 4(b). We approximate it with the second order rational function, because we enough to know only the smallest pole in negative [21]; i.e.,

$$Y(s) = \frac{3.27\times10^{-4} + 2.81\times10^{-11}s + 9.48\times10^{-20}s^2}{1. + 1.415\times10^{-9}s}$$

It is well-known from the circuit theory that although frequency responses from the other ports have the different characteristics, their poles are located in almost same point;

$$\lambda_p = -7.1 \times 10^8$$

Therefore, we can hope to get the exact solution with $T_p = 1./7.1 \times 10^{-8} \ll 10^{-8}$ [sec], and

$$\boldsymbol{F}(\boldsymbol{v}(T_p)) = \boldsymbol{v}(T_p) - \boldsymbol{v}(2T_p), \quad \text{for} \quad T_p = T$$

Remark that we can get the solution in 7 iterations. On the other hand, the time-domain secant method $\boldsymbol{F}(\boldsymbol{v}(0))$ neglecting the effect of the parasitic capacitors never converges to any solution. Note that our algorithm using $\boldsymbol{F}(\boldsymbol{v}(T_p))$ can find out the steady-state solution within 28 [sec], and the transient analysis with the SPICE gets the solution within 241 [sec].

3.3 Four Phase Mixer

This is an example of a relatively large scale four phase mixer circuit that consists of 122 bipolar transistors, and some capacitors, as shown in Fig. 5(a). In this case, the time-domain shooting method based on the Newton method [4] will be very time-consuming, because the number of the parasitic is more than 250 in this circuit and the total period for two inputs is large.

In designing of the mixer circuit, it is very important to know the intermodulated frequency components in the output waveform. Assume that the circuit is driven by two signals as follows;

$$v_1(t) = 282 \cos(2\pi \times 13 \times 10^6 t)\ [\mathrm{mV}] : \text{Local oscillator}$$

$$v_2(t) = 9.12 \sin(2\pi \times 14 \times 10^6 t)\ [\mathrm{mV}] : \text{Input signal}$$

The fundamental frequency of the mixer outputs are 1 [MHz] which are obtained at the $v_{1out}, \ldots, v_{4out}$ terminals in Fig. 5(a). In this case, C_1, C_2 and C_3 are coupling capacitors between the sub-circuits which will give large effect on the transient phenomena. On the other hand, the capacitor Cs used as filter circuits in the output stage do not give the effect to the right hand sub-circuits, because they are separated by the buffer amplifiers. Therefore, we have chosen the voltages of C_1, C_2 and C_3 as the state variables in our secant method.

Furthermore, since the lower side sub-circuits in this circuit are also separated by a buffer amplifier from the upper one, we can assume that the voltages at C_1, C_2 and C_3 will only contain the fundamental frequency of $\omega_1 = 2\pi \times 13 \times 10^6$ and its higher harmonics. This makes the analysis much easier, because we can define the total period $T = 1/(13 \times 10^6)$ [sec].

Now, let us apply our algorithm to the circuit. Since the transient period due to the parasitic capacitors is very short compared to the input frequency 13 [MHz], we can get the same result for both $\boldsymbol{F}(\boldsymbol{v}(0)) = \boldsymbol{0}$ and $\boldsymbol{F}(\boldsymbol{v}(T)) = \boldsymbol{0}$ in our algorithm. The convergence rate for $\boldsymbol{F}(\boldsymbol{v}(T)) = \boldsymbol{0}$ is shown in Fig. 5(b). We found from the result that the convergence ratios in the first 3 iterations are smaller compared to the following iterations. The result can be explained as follows that the Jacobian matrix has 3×3 elements and the initial matrix $\boldsymbol{J}^0$ estimated at the dc operating point does not seem to be a good approximation. However, the Jacobian is successively improved in the iterations. Thus, the convergence ratio after the first 3 iteration becomes larger.

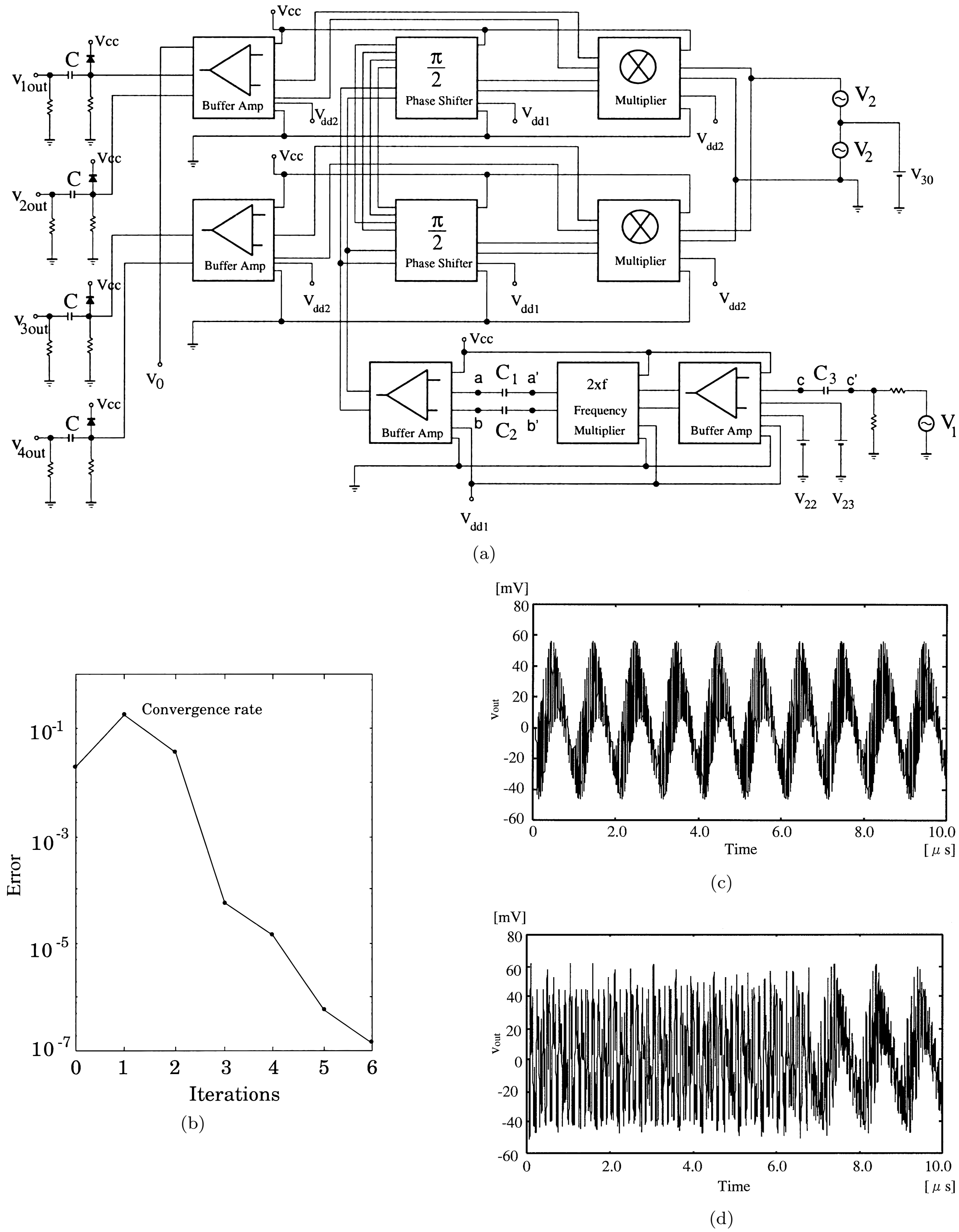

Fig. 5 (a) Four phase mixer circuit. $C_1 = C_2 = 20\,\text{pF}$, $C_3 = 2.5\,\text{nF}$, $C = 1\,\mu\text{F}$. (b) Convergence rate. Error $= \left\|\boldsymbol{F}(\boldsymbol{v}^j)\right\|$. (c) Steady-state waveform with our method. (d) Transient waveform obtained from SPICE.

We compared the computational times for other different methods.

In Table 2 where "PSS" means the periodic steady-state analysis based on the time-domain shooting method, whose SpectreRF is widely used in the RF circuit simulation. We can conclude that the SpectreRF (PSS) in this example is inefficient compared with our method. Thus, we found that our secant method can be applied to the steady-state analysis of relatively large scale ICs, efficiently. The output waveforms obtained from our method and transient analysis of SPICE are shown in Figs. 5(c), (d), respectively.

Table 2 Comparison of the computational times.

	Computation time
PSPICE (Transient)	704.21 [sec]
SpectreRF (PSS)	140.54 [sec]
Relaxation method [14]	54.99 [sec]
Our secant method	24.48 [sec]

Remark that the total period of the circuit is defined by the difference between the local oscillator and the input signal frequencies. Thus, when the difference is very small, the period becomes very longer, and the time-domain shooting method using SpectreRF becomes time-consuming. Note that we can not get the steady-state response for $f_1 = 13$ [MHz] and $f_2 = 13.1$ [MHz] with our 160 Mbytes computer because of the memory over [14]. The computational efficiency of our method for this example does not be changed even if the total period becomes longer.

4. Conclusions and Remarks

In this paper, we have shown an efficient time-domain secant method for calculating the steady-state response. Although the convergence rate is smaller than the Newton method, the algorithm is very simple and suitable for the development of the user friendly simulator. In our simulator, the initial guess from the SPICE is improved by the secant method written by the C-language program, and the initial guess is again returned to the SPICE, and so on. We continue the same iteration until the exact solution can be obtained. Thus, our algorithm is very simple, and need not derive any troublesome circuit equations and the sensitivity circuit analysis. Our simulator will be efficiently applied to relatively large scale RF circuits such as modulators and mixers. In this paper, we assumed that the effect of parasitic capacitors is smaller than the normal capacitors. However, the efficiency of our method may be decrease if the transient response T_p due to the parasitic elements becomes longer.

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References

[1] K. Kundert, J. White, and A. Sangiovanni-Vincentelli, Steady-State Methods for Simulating Analog and Microwave Circuits, Kluwer Academic Publishers, Boston, 1990.
[2] H.G. Brachtendorf, G. Welsch, and R. Laur, "Fast simulation of the steady-state of circuits by the harmonic balance technique," Proc. IEEE Int. Symp. on Circuits & Systems, pp.1388–1391, 1995.
[3] A. Ushida, T. Adachi, and L.O. Chua, "Steady-state analysis nonlinear circuits based on hybrid method," IEEE Trans. Circuits & Syst.-I: Fundamental Theory and Applications, vol.39, no.8, pp.649–661, 1992.
[4] T.J. Aprille, Jr. and T.N. Trick, "Steady-state analysis of nonlinear cirsuits with periodic input," Proc. IEEE, vol.60, pp.108–114, 1972.
[5] L.O. Chua and A. Ushida, "Algorithm for computing almost periodic steady-state response of nonlinear systems to multiple input frequencies," IEEE Trans. Circuits & Syst., vol.CAS-28, pp.953–971, 1981.
[6] K. Kakizaki and T. Sugawara, "A modified Newton method for the steady-state analysis," IEEE Trans. Comput.-Aided Des. Intergrated Circuits & Syst., vol.CAD-4, pp.354–360, 1985.
[7] S. Skelboe, "Computation of the periodic steady-state response of nonlinear networks by extrapolation methods," IEEE Trans. Circuits & Syst., vol.CAS-27, pp.161–175, 1980.
[8] R. Telichevesky, K. Kundert, and J. White, "Efficient steady-state analysis based on matrix-tree Krylovsubspace method," Proc. Int. Conf. on Computer-Aided Design, San Francisco, DAC'95, pp.480–484, 1995.
[9] R. Telichevesky, K. Kundert, and J. White, "Efficient AC and noise analysis of two-tone RF circuits," Proc. Int. Conf. on Computer-Aided-Design, Las Vegas, pp.292–297, 1996.
[10] J.G.P. Barnes, "An algorithm for solving nonlinear equation based on the secant method," Computer Journal, pp.66–72, 1965.
[11] J.F. Traub, Iterative methods for the solution of equations, Prentice-Hall, Englewood Cliffs, N.J., 1964.
[12] J.K. Hale, Oscillations in Nonlinear Systems, McGraw-Hill, 1963.
[13] L.O. Chua and P.-M. Lin, Computer Aided Analysis of Electronic Circuits: Algorithms and Computational Techniques, Prentice-Hall, Englewood Cliffs, N.J., 1975.
[14] Y. Yamagami, Y. Nishio, A. Ushida, M. Takahashi, and K. Ogawa, "Analysis of communication circuits based on multidimensional Fourier transformation," IEEE Trans. Comput.-Aided Des. Intergrated Circuits & Syst., vol.18, pp.1165–1177, 1999.
[15] H.K. Gummel and H.C. Poon, "An integral charge control model of bipolar transistors," Bell Syst. Tech. J., vol.94, pp.827–852, 1970.
[16] R.L. Geiger, P.E. Allen, and N.R. Strader, VLSI Design Techniques for Analog and Digital Circuits, McGraw-Hill, New York, 1990.
[17] R.A. Rohhrer, Circuit Theory: An Introduction to the State Variable Approach, McGraw-Hill, 1970.
[18] L.A. Zadeh and C.A. Desoer, Linear System Theory: The State Space Approach, McGraw-Hill, 1963.
[19] J.M. Ortega and W.C. Rheinboldt, Iterative Solution of Nonlinear Equations in Several Variables, Academic Press, 1970.
[20] A. Ushida, "A steady-state analysis of nonlinear dynamic systems," IECE Trans., vol.J61-A, no.3, pp.231–238, 1978.
[21] E. Chiprout and M.S. Nakhla, "Analysis of interconnect networks using complex frequency hopping (CFH)," IEEE Trans. Comput.-Aided Des. Integrated Circuits & Syst., vol.14, no.2, pp.186–200, 1995.

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