Steady-State Response of Nonlinear Circuits Containing Parasitic Elements

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SUMMARY We propose here a time-domain shooting algorithm for calculating the steady-state responses of nonlinear RF circuits containing parasitic elements that is based on both a modified Newton and a secant methods. Bipolar transistors and MOSFETs in ICs have small parasitic capacitors among their terminals. We can not neglect them because they will give large effects to the shooting algorithm. Since our purpose is to develop a user friendly simulator, we mainly take into account the relatively large normal capacitors such as coupling and/or by-pass capacitors and so on, because the parasitic capacitors are usually smaller and contained in the device models. We have developed a very simple simulator only using the fundamental tools of SPICE, which can be applied to relatively large scale ICs, efficiently.

Key words: steady-state analysis, RF circuits, time-domain secant method, parasitic capacitors, SPICE

1. Introduction

It is very important to analyze the steady-state responses for designing communication circuits such as modulators, mixers, etc. When the attenuation of transient response is sufficiently large, we can easily calculate the response with the brute-force method. However, it is sometimes happened that the transient response continues for a long period due to the small attenuation. In this case, there are two basic approaches; i.e., the frequency-domain method [1]–[3] and the time-domain method [4], [5]. The former is based on the harmonic balance method, which is usefully applied to weakly nonlinear circuits. However, the computational efficiency rapidly decreases in the cases when the number of nonlinear elements increases, and the nonlinearities become stronger, because the determining equation for calculating the Fourier coefficients becomes very large scale. Fortunately, some algorithms for solving large scale determining equation have been proposed [8], [9] which are efficiently applied to the system with the large scale sparse Jacobian matrix. Frequency-domain relaxation method [14] is also efficiently applied to the analysis of relatively large scale weakly nonlinear RF circuits.

On the other hand, the time-domain method is based on the transient analysis, where the initial guess giving rise to the steady-state response is calculated by the Newton type shooting method. The first one is based on the Newton Raphson method whose Jacobian matrix is estimated by the analysis of the time-varying sensitivity circuits equal to the number of state-variables [4], [5]. Note that it will take a large computer memory for a large scale circuit containing many parasitic elements. Fortunately, the transient terms due to the parasitic elements will be quickly reduced in the sensitivity analysis, so that the Jacobian matrix corresponding to the elements will be approximately replaced by the unit matrix in the Newton iteration [6]. Thus, it is possible to reduce the size of the Jacobian matrix. Although the extrapolation method is not theoretically guaranteed the convergency, it is a very simple algorithm, and seems to be suitable for the implementation with the SPICE simulator [7].

We propose here an algorithm for calculating the steady-state responses based on both the modified Newton and secant methods [10], [11]. Since the secant method is a type of Newton method whose Jacobian matrix is successively modified in the iterations, it can be applied to calculate both the stable and unstable steady-state responses if the circuit does not contain parasitic elements. Otherwise, our method can be only applied to calculate the stable steady-state response. There have been published many secant methods [19], and we already applied one of them to calculate the steady-state responses of nonlinear circuits [20] which is based on a discrete Newton method. The algorithm becomes sometimes unstable near at the solution point. In this paper, we apply much more stable algorithm using an orthogonal procedure [10]. Note that although the convergence ratios of the secant methods are smaller than the Newton method, they can be usefully applied to get the solution of a nonlinear system when the Jacobian matrix can not be explicitly obtained.

Thus, the method is suitable for the development of a simple SPICE simulator, because we only use the state variables at every periodic point in the transient analysis, and need not to use the time-varying sensitivity circuits in the iteration.

At the first step in our algorithm, the initial guess is estimated by the modified Newton method in the meaning that the Jacobian matrix is calculated at the dc operating point, and after then, the matrix is succes-
2. Modified Newton and Secant Methods

2.1 Effect of Parasitic Elements

To focus on the main idea of our algorithm, consider a LSI circuit shown in Fig. 1, where \( C_1, \ldots, C_n \) are the normal capacitors. If they are the integer relations, the steady-state response has a period called the total period [3] that is defined by

\[
T = \frac{1}{G.C.M.\{f_1, \ldots, f_n\}}, \quad \nu = 2\pi/T
\]

where G.C.M. means the greatest common measure. Hence, we can estimate the fundamental frequency component for the multiple frequencies and the period.

Now, let us derive the circuit equation in the form of algebraic-differential equation. At first, let us choose a normal tree for a given circuit such that it must contain the maximum possible number of the capacitors, and after then, as many as the parasitic capacitors, and lastly, the resistors. Next, we define the variables such as \( v \) for the capacitor voltages, \( v_R \) for the parasitic capacitors, and \( v_p \) for the resistors in the normal tree.

Then, we can describe the circuit equation in the following forms using the fundamental cutset equations and some loop equations [17];

\[
C(v)\dot{v} = f_1(v, v_R, v_p, \nu t)
\]

\[
\varepsilon C_p(v_p)\dot{v}_p = f_2(v, v_R, v_p, \nu t)
\]

\[
f_3(v, v_R, v_p, \nu t) = 0
\]

where Eqs. (2a) and (2b) are the cutset equations corresponding to the normal tree capacitors and the parasitic capacitors, respectively. Equation (2c) is the loop equations containing the normal tree resistors and/or C-E loops [17]. We assume that \( \varepsilon \) for the parasitic capacitors shows a sufficiently small constant.

Our time-domain shooting algorithm finds out the capacitor voltages \( v(T_p) \) giving rise to the steady-state response, where \( T_p \) is the time such that the effect of the parasitic capacitors becomes negligible in the transient response. To estimate \( T_p \) in a qualitative point of view, we consider the variational equation at the steady-state response \( \{v_0, v_{R0}, v_{p0}\} \);

\[
v = v_0 + \Delta v, \quad v_R = v_{R0} + \Delta v_R, \quad v_p = v_{p0} + \Delta v_p
\]

Substituting Eq. (3) into Eq. (2), we have

\[
\begin{pmatrix}
C(v_0)\Delta \dot{v} \\
\varepsilon C_p(v_{p0})\Delta \dot{v}_p \\
0
\end{pmatrix}
= \begin{pmatrix}
\frac{\partial f_1}{\partial v} & \frac{\partial f_1}{\partial v_R} & \frac{\partial f_1}{\partial v_p} \\
\frac{\partial f_2}{\partial v} & \frac{\partial f_2}{\partial v_R} & \frac{\partial f_2}{\partial v_p} \\
\frac{\partial f_3}{\partial v} & \frac{\partial f_3}{\partial v_R} & \frac{\partial f_3}{\partial v_p}
\end{pmatrix}
\times
\begin{pmatrix}
\Delta v \\
\Delta v_R \\
\Delta v_p
\end{pmatrix}
\]

where the Jacobian matrix is estimated at the steady-state response. Equation (4) is a time-varying system with the period \( T \). From the third row, we have

\[
\Delta v_R = - \left( \frac{\partial f_3}{\partial v_R} \right)^{-1} \left( \frac{\partial f_3}{\partial v} \frac{\partial f_3}{\partial v_p} \right) \left( \Delta v - \Delta v_p \right)
\]
Now, assume that the matrices \( C(\upsilon_{0}) \) and \( C_{p}(\upsilon_{0}) \) are positive and definite at the steady-state response. Then, they have the inverse matrices, and substituting Eq. (5) into Eq. (4), we have the following form:

\[
\begin{pmatrix}
\Delta \upsilon \\
\varepsilon \Delta \upsilon_{p}
\end{pmatrix}
= \begin{pmatrix}
A_{11}(\upsilon t) & A_{12}(\upsilon t) \\
A_{21}(\upsilon t) & A_{22}(\upsilon t)
\end{pmatrix}
\begin{pmatrix}
\Delta \upsilon \\
\Delta \upsilon_{p}
\end{pmatrix}
\tag{6}
\]

In order to investigate the effect of the parasitic capacitors, we consider the second equation of Eq. (6):

\[
\varepsilon \Delta \dot{\upsilon}_{pt} = A_{22}(\upsilon t) \Delta \upsilon_{pt}
\tag{7}
\]

First term in the right hand side is a variational value from the capacitor voltages which behaves like as a forced input in Eq. (7). It is known that the solution of a linear differential equation consists of two solutions; one is the particular solution \( \upsilon_{pp}(t) \), and another is the general solution \( \upsilon_{pt}(t) \) satisfying

\[
\varepsilon \Delta \dot{\upsilon}_{pt} = A_{22}(\upsilon t) \Delta \upsilon_{pt}
\tag{8}
\]

Then, it is known from the Floquet theorem [12] that the general solution has a following property;

\[
\Delta \upsilon_{pt}(t + T) = \exp(\lambda_{p} T) \Delta \upsilon_{pt}(t), \quad \text{for} \quad \lambda_{p} = \lambda_{2}/\varepsilon \tag{9}
\]

where \( \lambda_{2} \) is a characteristic multiplier of the time-varying differential Eq. (8) whose real part will be negative. Observe that if \( |\text{Re}\lambda_{p}| \) is sufficiently large, the transient terms due to the parasitic capacitors will be quickly reduced, and the solution of Eq. (7) will quickly approach to the particular solution \( \upsilon_{pp}(t) \). It means that the response \( \upsilon_{p}(t) \) only depends on \( \upsilon(t) \) after the transient term due to the parasitic capacitors is reduced, and the variable \( \upsilon_{p}(t) \) no more behaves like as the state variables. Therefore, after the period \( T_{p} \), it is possible to apply our shooting method only to the normal capacitor voltages \( \upsilon(t) \), and we can find the initial guess giving rise to the steady-state response.

Now, let us estimate the approximate transient period \( T_{p} \) in the case that the circuit is at the dc operating point. Namely, we calculate the transient response for the modified circuit whose the capacitor \( C_{s} \)s are replaced by the voltage source \( \upsilon(0) \) equal to the operating points. Then, the resultant circuit only contains the parasitic capacitors, and \( T_{p} \) can be estimate by the transient response. It is also possible to estimate \( T_{p} \) from the frequency response. Namely, let us calculate the driving point admittance in the frequency domain at any capacitor port\(^{\dagger}\). In practice, the frequency response can be calculated by the ac-sweep of SPICE at one of the capacitor ports. The response curve is approximately described by a rational polynomial as follows;

\[
Y(s) = \frac{a_{0} + a_{1}s + \ldots + a_{n}s^{n}}{b_{0} + b_{1}s + \ldots + b_{n}s^{n}} \tag{10}
\]

In order to estimate the approximate transient period \( T_{p} \), we need to know the smallest pole in the negative of the left hand complex plane, so that it is enough to apply the lower order of a rational polynomial function to the approximation [21].

**Remark** that if the effect from some of the normal capacitors to the transient response is negligible, we can also consider them as parasitic capacitors.

### 2.2 Our Secant Method

Our purpose is to develop a user friendly simulator using the fundamental tools of SPICE, where we only consider the capacitor voltages \( \upsilon(t) \) for determining the steady-state response. For a large scale circuit, it is troublesome to choose the parasitic capacitor voltages as the state variables, because the number becomes enormous when the circuit scale increases. Furthermore, they are contained in the device models.

Now, let the period of transient response due to the parasitic elements be \( T_{p} \). Then, we need to calculate the initial guesses \( \upsilon(T_{p}) \) giving rise to the steady-state response, which will satisfy the following *determining equation*\(^{\dagger\dagger}\):

\[
F(\upsilon(T_{p})) = \upsilon(T_{p} + T) - \upsilon(T_{p}) = 0, \quad \text{for} \quad T = \frac{2\pi}{\nu} \tag{11}
\]

Let us solve the determining Eq. (11) by the iterational method. It can be efficiently calculated by the Newton method [4] as follows;

\[
\upsilon^{j+1}(T_{p}) = \upsilon^{j}(T_{p}) - \left( \frac{\partial F(\upsilon(T_{p}))}{\partial \upsilon(T_{p})} \bigg|_{\upsilon(T_{p})=\upsilon^{j}(T_{p})} \right)^{-1}
\times F(\upsilon^{j}(T_{p})), \quad j = 0, 1, \ldots \tag{12}
\]

The Jacobian matrix can be calculated by the analysis of time-varying sensitivity circuits for all of the state variables [4], [5] in the period \( [T_{p}, T + T_{p}] \). Therefore, the computer efficiency will be decreased as the circuit scale becomes larger and the total period \( T \) becomes longer.

On the other hand, there is a simple algorithm based on the modified Newton method, where the Jacobian matrix is estimated at the dc operating point. Namely, each column of the matrix is calculated by the transient response to a sufficiently small impulsive input for corresponding capacitor; \( \Delta \upsilon(T)/\Delta \upsilon(0) \) for all the capacitor \( C_{s} \)s. If we use the same Jacobian matrix

\[
\frac{\partial F(\upsilon(T))}{\partial \upsilon(T)} = \begin{pmatrix}
I - \frac{\Delta \upsilon(T)}{\Delta \upsilon(0)}
\end{pmatrix} \tag{13}
\]

\(^{\dagger}\)It is known that although there are the same number of different driving point admittances as the capacitor \( C_{s} \)s, their poles for all of the admittances are located at the same points in the complex plane.

\(^{\dagger\dagger}\)It is shown in Sect. 2.1 that we can get the exact steady-state response with our secant method after the transient phenomena due to the effect of parasitic capacitors is completely finished.
at all of the iterations, the algorithm is called the modified Newton method. Although the algorithm can be efficiently applied to the weakly nonlinear circuits, the convergence is not guaranteed for the strong nonlinearities.

Hence, we propose here a secant method [10] such that the Jacobian matrix [11] is successively improved in the latter iterations. Let $J^0$ be the initial Jacobian matrix estimated by the above method. Then, we have at the first iteration

$$J^0 \delta v^0(T_p) = -F(v^0(T_p))$$

for $v^1(T_p) = v^0(T_p) + \delta v^0(T_p)$

Thus, we need to estimate the modified Jacobian matrix $J^1$. Suppose it satisfies the following relation;

$$F(v^1(T_p)) = F(v^0(T_p)) + J^1 \delta v^0(T_p)$$

where $J^1 = J^0 + D^0$

$D^0$ corresponds to the variational Jacobian matrix to be determined. From Eqs. (14) and (15), we have

$$F(v^1(T_p)) = D^0 \delta v^0(T_p)$$

where $D^0$ can be solved as follows;

$$D^0 = \frac{F(v^1(T_p)) (x^0)^T}{(x^0)^T \delta v^0(T_p)}$$

where $x^0$ is an arbitrary vector satisfying $(x^0)^T \delta v^0(T_p) \neq 0$. In the same manner, we have the following iteration;

$$\delta v^j(T_p) = -(J^j)^{-1} F(v^j(T_p))$$

$$v^{j+1}(T_p) = v^j(T_p) + \delta v^j(T_p)$$

$$D^j = \frac{F(v^{j+1}(T_p)) (x^j)^T}{(x^j)^T \delta v^j(T_p)}$$

$$J^{j+1} \delta v^j(T_p) = F(v^{j+1}(T_p)) - F(v^j(T_p))$$

for $J^{j+1} = J^j + D^j, \ j = 1, 2, \ldots$

where if $j > n$, $x^j$ is chosen orthogonal to the previous $n-1$ steps [18], $\delta v^{j-n+1}(T_p), \ldots, \delta v^{j-1}(T_p)$, and if $j < n$, we can only demand that $x^j$ is orthogonal to the available $j-1$ steps, $\delta v^1(T_p), \ldots, \delta v^{j-1}(T_p)$. Observe that our secant method only uses the data $v(t)$ at $t = T_p$ and $t = T + T_p$ in the transient response, and it is obtained with the SPICE. Thus, we can easily develop a user friendly simulator only using fundamental tools of SPICE, where we need not to derive any troublesome circuit equations. It makes our scant method much more powerful.

Our secant algorithm

S.0 Set $j = 0$, and set the initial guess $v^0(T_p)$ equal to the bias voltages. The initial Jacobian matrix $J^0$ is estimated by Eq. (13) at the dc bias points.

S.1 Calculate the transient response in the period of $[0, T + T_p]$ from the initial guess $v(0) = v^0(T_p)$.

S.2 Calculate the solution at $j + 1$ iteration

$$v^{j+1}(T_p) = v^j(T_p) + \delta v^j(T_p)$$

S.3 If $\|F(v^{j+1}(T_p))\| < \varepsilon$ for a given small $\varepsilon$, then stop. Otherwise, calculate the variational value of the Jacobian matrix

$$D^j = \frac{F(v^{j+1}(T_p)) (x^j)^T}{(x^j)^T \delta v^j(T_p)}$$

S.4 Set $J^{j+1} = J^j + D^j$.

S.5 Set $j = j + 1$ and go to Step 1.

In this algorithm, the vector $x^j$ must be chosen orthogonal to the previous vectors $\delta v^i, i = j-n+1, \ldots, j-1$, which is efficiently executed with the Schmidt orthonormalization procedure [18]. Observe that, to implement our algorithm, we only need to execute the transient analysis in the period $[0, T_p + T]$ and get $v(T_p)$ and $v(T + T_p)$.

The flow chart of our secant method is shown in Fig. 2. In our algorithm, the data $v^j(T_p)$ and $v(T_p + T)$ are obtained from the transient analysis of SPICE and they are transferred to C-language program, where the initial guess $v^{j+1}(T_p)$ is calculated with the secant method. After then, the data from C-language program is again transferred to SPICE, and so on.

**Remark 1:** It is known that if one of the variables in $\delta v^j(T_p) = v^{j+1}(T_p) - v^j(T_p)$ approaches to zero, the $x^j$ obtained by the orthonormalization procedure [18] may have serious error. In this case, the iteration sometimes happens to become unstable. Thus, we will recommend to remove the variable $v_i$ from $v$ if $|\delta v_i| < \delta$, for some $i$

for a sufficiently small $\delta$, where $\delta$ depends on both the digit of computer and the truncation error in the numerical integration method [13]. Thus, the dimension of $J^j$ and $D^j$ should be reduced by one, and the orthogonal vector $x^j$ should be estimated for the remaining variables.

**Remark 2:** If we define the convergence ratio by $p$ in the following relation
where ̂v means the exact solution, then, the convergence rate of the secant method is \( p = 1 \).

Although the ratio is smaller than the Newton method \( p = 2 \), our secant algorithm is a very simple because we need not solve the sensitivity circuit. Therefore, our method seems to be suitable for developing a user friendly simulator using SPICE.

3. Illustrative Examples

3.1 RLC Circuit with Nonlinear Resistance

In order to investigate the effect of parasitic capacitors in the steady-state solution, we consider a simple RLC circuit as shown in Fig. 3(a), where the parasitic capacitor \( C_p \) is chosen 10% of the normal capacitor \( C \). Since \( \lambda_p = -20 \) due to the parasitic capacitor is very large in negative, the effect in the transient response will be negligible after 1 sec.

Although the number of state variables is 3 in this example, the transient phenomena will behave like as 2 state-variables circuit after the transient period due to \( C_p \) is reduced. Thus, we can find out the initial guess \( v(T_p) \) giving rise to the steady-state solution after \( T_p = 1 \) sec.

In Table 1 where the exact solution is obtained by the Newton method [4]. The second result is obtained by solving the determining equation \( F(v(0)) = v(0) - v(T) \).

In this case, even if the convergence algorithm has a solution, it will be the fault solution in the \( (v_C(0), i_L(0), v_C(0)) \)-plane. The third result is obtained with our secant method, where we have chosen \( T_p = T \) for simplicity. After the transient due to \( C_p \) is reduced, the circuit will behave like as in the 2-dimensional plane. Therefore, the result is almost equal to the exact solution. The convergence ratio is sufficient large as shown in Fig. 3(b).

3.2 RC Amplifier

Now, consider a simple RC amplifier circuit shown in Fig. 4(a). The transistor has the small parasitic capacitors among the emitter, base and collector [15], which

\( v(0) - v(T) \). In this case, even if the convergence algorithm has a solution, it will be in the \( (v_C(0), i_L(0), v_C(0)) \)-plane. The third result is obtained with our secant method, where we have chosen \( T_p = T \) for simplicity. After the transient due to \( C_p \) is reduced, the circuit will behave like as in the 2-dimensional plane.
cannot be neglected at higher than 100 [MHz].

At first, we choose the coupling capacitors $C_1$, $C_2$ and the by-pass capacitor $C_3$ as the normal capacitors. To investigate the effect of the parasitic capacitors, the normal capacitors are replaced with the operating voltage sources $v_1(0) = 1.007 [V]$, $v_2(0) = 6.641 [V]$ and $v_3(0) = 0.338 [V]$, which are obtained by the dc analysis of SPICE. Now, we apply the ac-sweep of SPICE, and get the driving point characteristic curve as shown in Fig. 4(b). We approximate it with the second order rational function, because we enough to know only the frequency responses from the other ports have the different characteristics, their poles are located in almost same point:

$$Y(s) = \frac{3.27 \times 10^{-4} + 2.81 \times 10^{-11}s + 9.48 \times 10^{-20}s^2}{1 + 1.415 \times 10^{-9}s}$$

It is well-known from the circuit theory that although frequency responses from the other ports have the different characteristics, their poles are located in almost same point:

$$\lambda_p = -7.1 \times 10^8$$

Therefore, we can hope to get the exact solution with $T_p = 1. / 7.1 \times 10^{-8} \ll 10^{-8} [sec]$, and

$$F(v(T_p)) = v(T_p) - v(2T_p), \text{ for } T_p = T$$

Remark that we can get the solution in 7 iterations. On the other hand, the time-domain secant method $F(v(0))$ neglecting the effect of the parasitic capacitors never converges to any solution. Note that our algorithm using $F(v(T_p))$ can find out the steady-state solution within 28 [sec], and the transient analysis with the SPICE gets the solution within 241 [sec].

3.3 Four Phase Mixer

This is an example of a relatively large scale four phase mixer circuit that consists of 122 bipolar transistors, and some capacitors, as shown in Fig. 5(a). In this case, the time-domain shooting method based on the Newton method [4] will be very time-consuming, because the number of the parasitic is more than 250 in this circuit and the total period for two inputs is large.

In designing of the mixer circuit, it is very important to know the intermodulated frequency components in the output waveform. Assume that the circuit is driven by two signals as follows:

$$v_1(t) = 282 \cos(2\pi \times 13 \times 10^6 t) [mV] : \text{Local oscillator}$$

$$v_2(t) = 9.12 \sin(2\pi \times 14 \times 10^6 t) [mV] : \text{Input signal}$$

The fundamental frequency of the mixer outputs are 1 [MHz] which are obtained at the $v_{out_1}, \ldots, v_{out_4}$ terminals in Fig. 5(a). In this case, $C_1$, $C_2$ and $C_3$ are coupling capacitors between the sub-circuits which will give large effect on the transient phenomena. On the other hand, the capacitor C’s used as filter circuits in the output stage do not give the effect to the right hand sub-circuits, because they are separated by the buffer amplifiers. Therefore, we have chosen the voltages of $C_1$, $C_2$ and $C_3$ as the state variables in our secant method.

Furthermore, since the lower side sub-circuits in this circuit are also separated by a buffer amplifier from the upper one, we can assume that the voltages at $v_{out_1}, \ldots, v_{out_4}$ will only contain the fundamental frequency of $\omega_1 = 2\pi \times 13 \times 10^6$ and its higher harmonics. This makes the analysis much easier, because we can define the total period $T = 1/(13 \times 10^6) [sec]$. Now, let us apply our algorithm to the circuit. Since the transient period due to the parasitic capacitors is very short compared to the input frequency 13 [MHz], we can get the same result for both $F(v(0)) = 0$ and $F(v(T)) = 0$ in our algorithm. The convergence rate for $F(v(T)) = 0$ is shown in Fig. 5(b). We found from the result that the convergence ratios in the first 3 iterations are smaller compared to the following iterations. The result can be explained as follows that the Jacobian matrix has $3 \times 3$ elements and the initial matrix $J^0$ estimated at the dc operating point does not seem to be a good approximation. However, the Jacobian is successively improved in the iterations. Thus, the convergence ratio after the first 3 iteration becomes larger.
We compared the computational times for other different methods. In Table 2 where “PSS” means the periodic steady-state analysis based on the time-domain shooting method, whose SpectreRF is widely used in the RF circuit simulation. We can conclude that the SpectreRF (PSS) in this example is inefficient compared with our method. Thus, we found that our secant method can be applied to the steady-state analysis of relatively large scale ICs, efficiently. The output waveforms obtained from our method and transient analysis of SPICE are shown in Figs. 5(c), (d), respectively.
Remark that the total period of the circuit is defined by the difference between the local oscillator and the input signal frequencies. Thus, when the difference is very small, the period becomes very longer, and the time-domain shooting method using SpectreRF becomes time-consuming. Note that we can not get the steady-state response for $f_1 = 13$ [MHz] and $f_2 = 13.1$ [MHz] with our 160 Mbytes computer because of the memory over [14]. The computational efficiency of our method for this example does not be changed even if the total period becomes longer.

4. Conclusions and Remarks

In this paper, we have shown an efficient time-domain secant method for calculating the steady-state response. Although the convergence rate is smaller than the Newton method, the algorithm is very simple and suitable for the development of the user friendly simulator. In our simulator, the initial guess from the SPICE is improved by the secant method written by the C-language program, and the initial guess is again returned to the SPICE, and so on. We continue the same iteration until the exact solution can be obtained. Thus, our algorithm is very simple, and need not derive any troublesome circuit equations and the sensitivity circuit analysis. Our simulator will be efficiently applied to relatively large scale RF circuits such as modulators and mixers. In this paper, we assumed that the effect of parasitic capacitors is smaller than the normal capacitors. However, the efficiency of our method may be decrease if the transient response $T_p$ due to the parasitic elements becomes longer.

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References

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