

PAPER

# Performance Comparison of Communication Systems Using Chaos Synchronization

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**SUMMARY** In this paper, the performance of some communication systems using chaos synchronization is evaluated and compared. A new channel model taking the attenuation, impedance mismatch and noise into account, is proposed for the performance evaluation. The evaluation of bit error rate is done for both ideal and non-ideal conditions using the channel model. It is confirmed that some chaos-based communication systems have a good performance compared with conventional analog communication schemes.

**key words:** *chaos-based communication system, chaos synchronization, transmission line, channel model, bit error rate*

## 1. Introduction

Recently chaos and its possible application to communications have received a great deal of attention. At the beginning vigorous efforts for communication schemes utilizing chaos synchronization [1]–[5] have been paid, while in the most recently studies on digital communications employing the property of wide-band and uncorrelated chaotic signals [6]–[9] have attracted attention. For realization of chaos-based communication systems, in either case, it is of significance to find or develop communication systems with a practically usable performance. In this sense performance evaluation of chaos-based communication systems and those comparison are very important. However performance comparison of chaos-based communication systems has been reported only in [6], [9]. Further it has been pointed out that chaos synchronization can not be used for communications because of the effect of a noisy and band-limited channel [6], [7]. However this has not been confirmed for all communication systems using chaos synchronization.

Therefore in this study we evaluate the performance of some communication systems using chaos synchronization and compare them in order to confirm

whether the assertion in [6], [7] is true or not and which system has best performance. As representative examples of communication systems using chaos synchronization, we select the chaotic masking [1], the chaotic coding [2] and the chaotic modulation [4] systems from many systems proposed so far.

On the other hand, the effect of noise on chaos-based communication systems has been investigated recently. However, there are other effects such as attenuation, impedance mismatch, band limitation, non-linearity of transmission property and so on, in real communication channels. Because we can expect that chaos-based communication systems are considerably affected by such effects than conventional communication systems, thus we propose a channel model using a transmission line in order to consider attenuation, impedance mismatch and noise, in this paper. In this model, the transmission line is matched with its matching impedance, where an additive white Gaussian noise is added to the transmission signal similarly to [7], [9]. The signal affected by attenuation is then compensated by an amplifier proposed in our previous study [10]. In [9] the author has assumed that attenuation of channel can be compensated by an automatic gain control circuit and has not considered it. (In [7] channel conditions in simulations have not been explained.) It should be noted that in our channel model propagation signals are attenuated really and then compensated actually and that the performance of the systems is evaluated under the conditions. In this paper the effect of impedance mismatch is left out of consideration for brevity.

For the performance evaluation the bit error rate (BER) is simulated for not only lossy and noisy communication channel but ideal one, because the BER of chaos-based communication systems is not always zero even for noise-free case. The BER evaluation of three systems is done and it is found that the performance of chaotic coding and modulation systems have a good performance.

## 2. Chaotic Communication Systems

For the performance evaluation of chaotic communication systems, the chaotic masking [1], chaotic coding [2] and chaotic modulation [4] systems are taken up in this

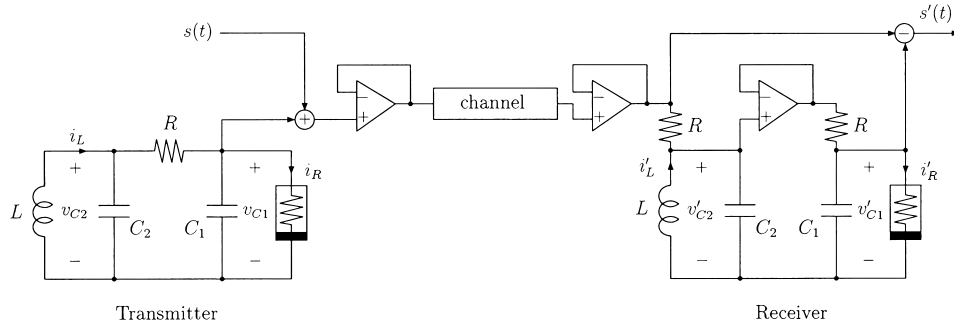
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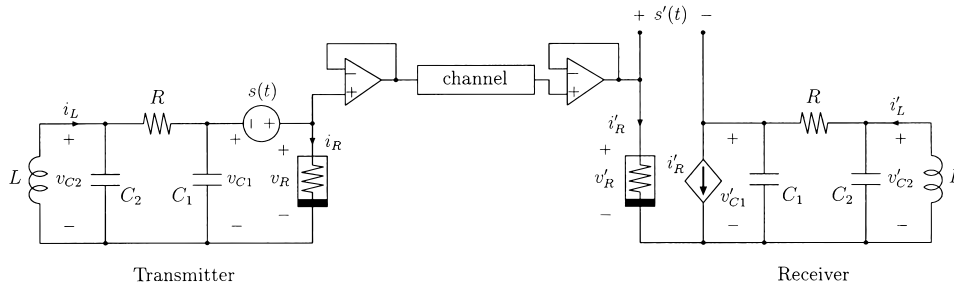
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**Fig. 1** Chaotic masking system.



**Fig. 2** Chaotic coding system. The diamond shaped current source is a current-controlled dependent source.

paper. In this section, we summarize briefly the communication systems. The transmitter and receiver in all systems are based on the well-known Chua's circuit.

### 2.1 Chaotic Masking System

Chaotic masking system [1] is shown in Fig. 1. The circuit equations of the transmitter are described as follows:

$$\begin{aligned} C_1 \frac{dv_{C1}}{dt} &= \frac{v_{C2} - v_{C1}}{R} - g(v_{C1}) \\ C_2 \frac{dv_{C2}}{dt} &= \frac{v_{C1} - v_{C2}}{R} + i_L \\ L \frac{di_L}{dt} &= -v_{C2} \end{aligned} \quad (1)$$

where a function  $g(\cdot)$  is a piecewise linear function and is defined by

$$g(v) = m_1 v + \frac{1}{2}(m_0 - m_1)[|v + B_p| - |v - B_p|]. \quad (2)$$

An information signal  $s(t)$  is simply added to the chaotic signal  $v_{C1}$  generated by the transmitter and then transmitted. Based on the drive-response concept, the receiver is configured and it is then governed by the following equations:

$$\begin{aligned} C_2 \frac{dv'_{C2}}{dt} &= \frac{\hat{v}_{C1} - v'_{C2}}{R} + i'_L \\ L \frac{di'_L}{dt} &= -v'_{C2} \end{aligned}$$

$$C_1 \frac{dv'_{C1}}{dt} = \frac{v'_{C2} - v'_{C1}}{R} - g(v'_{C1}). \quad (3)$$

where  $\hat{v}_{C1}$  is the received signal through the channel. If the transmitter and receiver synchronize, the information signal  $s'(t)$  can be recovered as

$$s'(t) = \hat{v}_{C1} - v'_{C1}. \quad (4)$$

### 2.2 Chaotic Coding System

The chaotic communication system proposed in [2] may be regarded as a kind of the chaotic modulation system, so we call the system chaotic coding system to discriminate from the chaotic modulation systems [3], [4].

An information signal  $s(t)$  is coded by the chaotic signal  $v_{C1}$  using a coding function. The system using the coding function  $c = v_{C1} + s(t)$  is shown in Fig. 2. In this system the amplitude of  $s(t)$  must be small in order to ensure that the transmitter remains chaotic.

The circuit equations of the transmitter are given by

$$\begin{aligned} C_1 \frac{dv_{C1}}{dt} &= \frac{v_{C2} - v_{C1}}{R} - g(v_{C1} + s(t)) \\ C_2 \frac{dv_{C2}}{dt} &= \frac{v_{C1} - v_{C2}}{R} + i_L \\ L \frac{di_L}{dt} &= -v_{C2}. \end{aligned} \quad (5)$$

The modulated signal  $v_R(t)$  is transmitted to the receiver. The dynamics of the receiver is

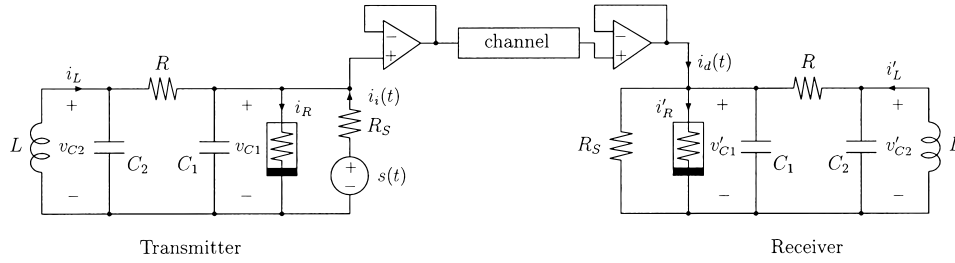


Fig. 3 Chaotic modulation system.

$$\begin{aligned}
 C_1 \frac{dv'_{C1}}{dt} &= \frac{v'_{C2} - v'_{C1}}{R} - g(v'_R) \\
 C_2 \frac{dv'_{C2}}{dt} &= \frac{v'_{C1} - v'_{C2}}{R} + i'_L \\
 L \frac{di'_L}{dt} &= -v'_{C2}.
 \end{aligned} \quad (6)$$

The recovered information signal can be obtained as the potential

$$s'(t) = v'_R - v_{C1}. \quad (7)$$

### 2.3 Chaotic Modulation System

Two types of chaotic modulation systems have been reported in [3], [4]. We take the system [4] in this paper since the circuit configuration is simple. The chaotic modulation system [4] is shown in Fig. 3.

The circuit equations for the transmitter are given by

$$\begin{aligned}
 C_1 \frac{dv_{C1}}{dt} &= \frac{v_{C2} - v_{C1}}{R} - g(v_{C1}) + \frac{s(t) - v_{C1}}{R_S} \\
 C_2 \frac{dv_{C2}}{dt} &= \frac{v_{C1} - v_{C2}}{R} + i_L \\
 L \frac{di_L}{dt} &= -v_{C2}
 \end{aligned} \quad (8)$$

where a current signal  $i_i(t)$  which corresponds to the last term of the first equation in (8) is injected into the transmitter and modulates the chaotic signal  $v_{C1}$ . The modulated signal  $v_{C1}$  is then transmitted to the receiver which is described by

$$\begin{aligned}
 C_1 \frac{dv'_{C1}}{dt} &= \frac{v'_{C2} - v'_{C1}}{R} - g(v'_{C1}) + \frac{v'_{C1}}{R_S} \\
 C_2 \frac{dv'_{C2}}{dt} &= \frac{v'_{C1} - v'_{C2}}{R} + i'_L \\
 L \frac{di'_L}{dt} &= -v'_{C2}.
 \end{aligned} \quad (9)$$

The recovery of information can be achieved by detecting the current  $i_d(t)$  flowing into the receiver; that is,

$$\begin{aligned}
 s'(t) &= R_S i_d(t) \\
 &= R_S \left[ C_1 \frac{dv'_{C1}}{dt} - \frac{v'_{C2} - v'_{C1}}{R} \right].
 \end{aligned}$$

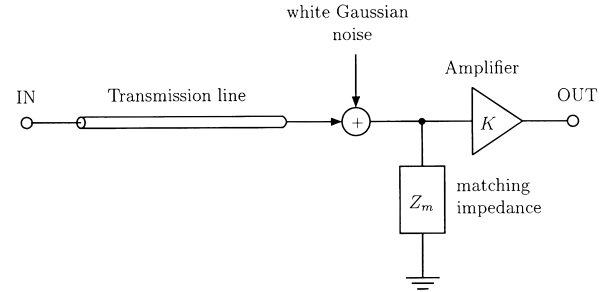


Fig. 4 Channel model using transmission line.

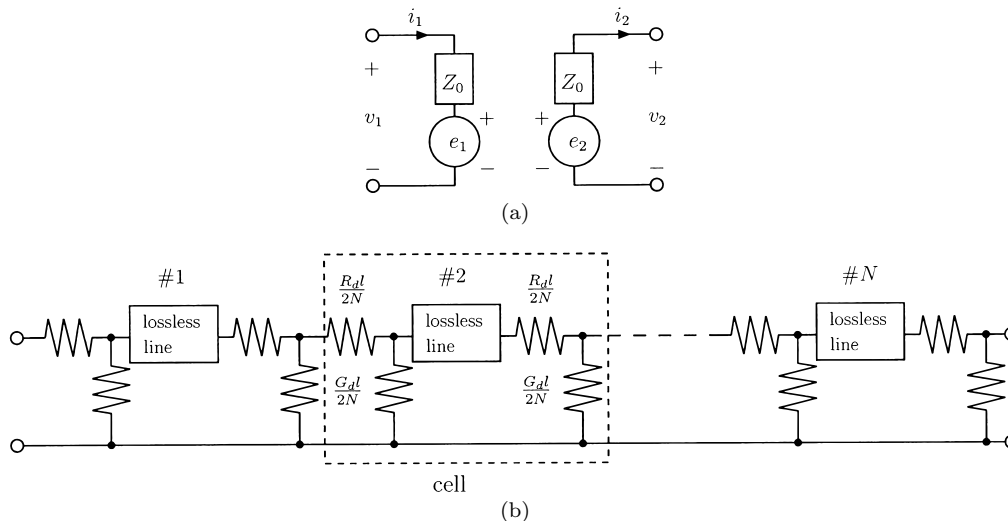
$$+g(v'_{C1}) - \frac{v'_{C1}}{R_S} \Big]. \quad (10)$$

Please note that the derivative  $C_1 dv'_{C1}/dt$  in (10) can be evaluated by difference approximation in numerical experiments.

### 3. Channel Model

The transmitter and receiver in all systems are connected with a communication channel. In the systems we can expect that if the channel is ideal (i.e. lossless, perfectly matched, noiseless and so on) then the transmitters and receivers will synchronize. But real communication channels are lossy, noisy, and often imperfectly matched, the performance of the systems must be evaluated under such conditions. For the purpose proper channel models are required. In [7], [9] AWGN (additive white Gaussian noise) channel models have been used.

We will propose and use a model as shown in Fig. 4. This model contains a transmission line and its matching impedance. The transmission line is further modeled by convenient models [11], [12] shown in Fig. 5 which are widely used for the transient simulation of transmission lines. An additive white Gaussian noise is added to the transmission signal similarly to [7], [9]. An amplifier is used to compensate the effect of attenuation and this compensation technique has been proposed in [10]. Note again that in this model transmission signals affected by attenuation are compensated really and this is different from the relevant studies reported so far.



**Fig. 5** (a) Characteristic model of lossless transmission line and (b) modeling of lossy transmission line.

If the transmission line is lossless, it is replaced by the characteristic model as shown in Fig. 5(a) [11].  $e_1(t)$  and  $e_2(t)$  are the waveform generators to simulate the reflection. Let  $Z_0$  and  $\tau$  be the characteristic impedance and the time delay respectively, then the waveform generators of the lossless line are calculated as follows:

$$\begin{aligned} e_1(t) &= 2v_2(t) - e_2(t - \tau) \\ e_2(t) &= 2v_1(t) - e_1(t - \tau) \end{aligned} \quad (11)$$

Remark that  $e_1(t)$  and  $e_2(t)$  must be stored over the period  $\tau$ .

On the other hand, if the transmission line is lossy, it can be modeled as shown in Fig. 5(b) [12]. The model consists of some segments called cell. Each cell is composed of four lumped resistors and a lossless line which is further replaced with its characteristic model. The number of cells, which depends on the modeling error, must be adjusted according to the magnitude of loss and frequency of transmission signal. This model is more accurate than other models even if a transmission line is not distortionless.

#### 4. Performance Comparison

In this section we investigate the bit error rate (BER) for the performance evaluation of the systems. The performance is evaluated for not only lossy and noisy communication channel but also ideal one, because the BER of chaos-based communication systems is not always zero even for noise-free case [7].

We must simulate the systems with the use of numerical integration methods, because they utilize analog chaotic signal generated by the Chua's circuit which is governed by nonlinear differential equations. The fourth order Runge-Kutta method is used in the perfor-

mance estimation. A non-return-to-zero binary waveform is generated from a random binary sequence, when the amplitude and bit rate are given. After the transmitter and receiver synchronize, the binary waveform is injected to the systems at time  $T_s$ . Decision of recovered data sequence is done by detecting the sign of the recovered signal  $s'(t)$ . The decision process is repeated at the moment  $t$  satisfying

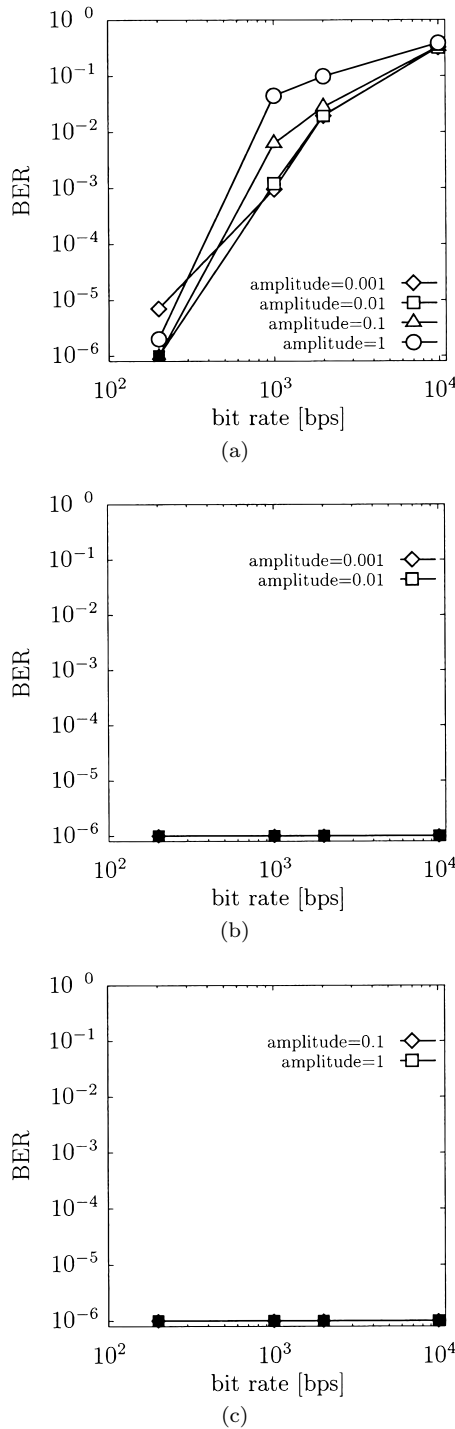
$$t - \tau - T_s = nT_b - h, \quad n = 1, 2, \dots, N$$

and then a recovered bit is compared with a corresponding transmitted bit. Where  $\tau$  is the time delay of transmission line,  $N$  is the number of transmitted bits,  $1/T_b$  is bit rate and  $h$  is step-size in the simulations. In our experiments,  $N = 10^6$  and  $h = 10^{-4}$ . The reason why detecting at just before of bit switching is as follows. In the chaotic masking system desynchronization of chaos often occurs due to change of information binary signal. Therefore, bit error probability is expected to be the smallest at the decision timing. In the decision process the output of a low-pass filter (LPF) may be used instead of the recovered signal to reduce unwanted effects described later.

First of all, in order to evaluate the performance fairly, we fix the parameters in all systems as follows:

$$\begin{aligned} R &= 1700[\Omega], \quad C_1 = 10[\text{nF}], \quad C_2 = 100[\text{nF}] \\ L &= 18[\text{mH}], \quad m_0 = -0.75[\text{mS}], \\ m_1 &= -0.41[\text{mS}], \quad B_p = 1[\text{V}]. \end{aligned} \quad (12)$$

Now we begin to estimate the ideal BER performance of the communication systems for ideal channel (i.e. the channel is lossless, perfectly matched and noiseless). The channel parameters are  $Z_0 = Z_m = 50[\Omega]$ ,  $\tau = 0.5[\text{msec}]$  and  $K = 1$ , where  $K$  is the gain of the amplifier in our channel model. The ideal performance is shown in Fig. 6 The chaotic masking system can cause undesirable transients at bit switching of



**Fig. 6** Ideal performance in the chaotic communication systems. (a) chaotic masking system, (b) chaotic coding system, (c) chaotic modulation system. For only chaotic masking system a LPF with the time constant 0.2[msec] is used. The dark shaded mark denotes a BER less than  $10^{-6}$ .

information digital signal, which often result in desynchronization (bit error). So a LPF is introduced in order to reduce the transients and its output is used for the BER evaluation. Note that it is not shown in Fig. 1. In the chaotic coding system the BER for smaller ampli-

tude is only shown because the system became unstable for larger one. In contrast to the chaotic coding system, only larger amplitude is used for the chaotic modulation system. In the system the quantity  $C_1 dv'_{C1}/dt$  in (10) must be estimated to detect the current  $i_d$  in the simulation. We can evaluate it only by applying difference approximation, but this caused “spike-like” error in the recovered signal when  $|dv'_{C1}/dt|$  becomes larger. Smaller amplitude is affected fairly by this error, thus relatively larger amplitude is used.

In the chaotic masking system the BER becomes better if bit rate is lower. On the other hand, in the chaotic coding and modulation systems, the BERs are less than  $10^{-6}$  and remain unchanged for the change of amplitude and bit rate. They have a good performance for ideal conditions.

Next noise performance for the carrier-to-noise ratio (CNR) is evaluated regarding the chaotic signal generated in the transmitters with no input signal as the carrier signal. In general the signal-to-noise ratio (SNR) is used for performance evaluation, but the average power of information signal in the three systems is different because of the above mentioned reason. Consequently the performance comparison using SNR is not fair, thus we think that it is better to use CNR than SNR. From our simulations the average power of the carrier in all systems is about 3.586 for the parameters of the transmitters (12). Using the value the noise level (namely the variance of Gaussian noise) according to a CNR is calculated. In BER evaluations the additive noise is summed up to transmission signals at intervals of step-size.

In the estimation of noise performance, a 1[km] lossy transmission line with the parameters

$$R_d = 20[\Omega/\text{km}], \quad L_d = 0.25[\text{mH}/\text{km}]$$

$$C_d = 100[\text{nF}/\text{km}], \quad G_d = 0[\text{S}/\text{km}]$$

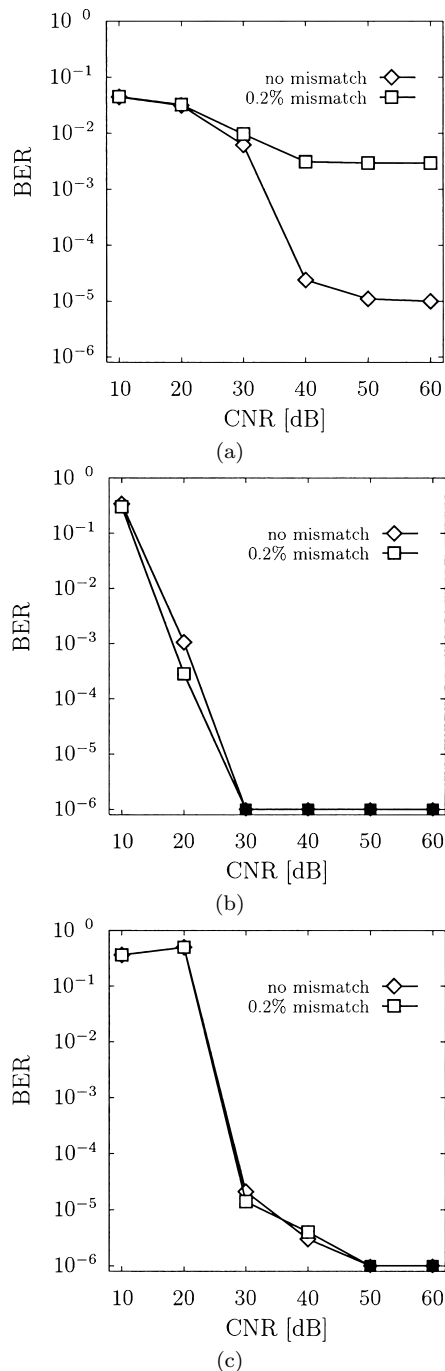
is used, where we decided the number of cells  $N$  as 25. The value of matching impedance is calculated by the following equation:

$$Z_m = |Z_0(j\omega_c)| = \left| \sqrt{\frac{R_d + j\omega_c L_d}{G_d + j\omega_c C_d}} \right|$$

where  $\omega_c$  is the natural angular frequency of the Chua's circuit. In this paper the effect of impedance mismatch is not taken account for simplicity. The gain  $K$  of the amplifier is set to 1.06 (see [10] for details).

Also it can be expected that the recovered signals are affected by noise and so on, thus a LPF is introduced in order to reduce such effects. The LPF with the time constant 0.2[msec] is used for the following BER estimation, which is not shown in Figs. 1–3.

Figure 7 shows the BER obtained. Further the performance for small parameter mismatch on linear resistor  $R$  is also investigated and illustrated, because



**Fig. 7** Noise performance in the chaotic communication systems. (a) chaotic masking system (amplitude=0.1[V], bit rate=200[bps]), (b) chaotic coding system (amplitude=0.01[V], bit rate=2000[bps]), (c) chaotic modulation system (amplitude=0.5[V], bit rate=2000[bps]). For all systems a LPF with the time constant 0.2[msec] is used. The dark shaded mark denotes a BER less than  $10^{-6}$ .

it is difficult to match perfectly the component values of the transmitter and the receiver in real circuits. For larger noise level (smaller CNR) the chaotic masking system has a best performance, while the performance of the chaotic coding and modulation systems

becomes better as noise decreases. In particular the BERs of the chaotic coding and modulation systems are less than  $10^{-6}$  for a region of CNR, similarly to ideal case. Moreover this is also applied to the case that there is a small parameter mismatch between the transmitter and receiver. It has been mentioned in [13] that the SNR required to get BER  $10^{-6}$  in analog communication is greater than 60[dB]. Therefore we can conclude that the chaotic coding and modulation systems have good performance compared with analog communication schemes. Also chaos synchronization can be used for communications if transmission conditions are not so poor. In the point of implementation the chaotic modulation system has advantages, because the chaotic coding system has the current-controlled dependent source and it is difficult to realize ideal one.

## 5. Conclusions

In this paper we have evaluated the performance of some communication systems using chaos synchronization and compared them. Further we have proposed a channel model taking loss, noise and impedance mismatch into account, and shown how to evaluate BER for chaos-based communication systems in detail. The chaotic coding and modulation system have a good performance even if their parameters have a small mismatch. Although these systems are utilizing chaos synchronization, they will offer a practically usable performance. The chaotic modulation system has advantages in the sense of facility for realization.

We will evaluate the performance of other chaos-based communication systems, e.g., the chaotic inverse system and consider to design a chaos-based communication system reliable under poor conditions.

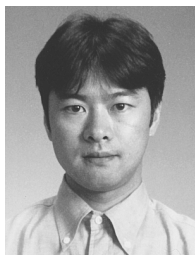
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