PAPER Section on VLSI Design and CAD Algorithms

Efficient Curve Fitting Technique for Analysis of Interconnect Networks with Frequency-Dependent Parameters

Yuichi TANJI†, *Student Member*, Yoshifumi NISHIO†, Takashi SHIMAMOTO†, *and* Akio USHIDA†, *Members*

SUMMARY Analysis of frequency-dependent lossy transmission lines is very important for designing the high-speed VLSI, MCM and PCB. The frequency-dependent parameters are always obtained as tabulated data. In this paper, a new curve fitting technique of the tabulated data for the moment matching technique in the interconnect analysis is presented. This method based on Chebyshev interpolation enhances the efficiency of the moment matching technique.
key words: *frequency-dependent lossy transmission lines, curve fitting technique, multi point Padé approximation, recursive convolution*

1. Introduction

The high-speed performance of microwave or digital circuit systems is limited by the interconnect effects rather than the switching speed of semiconductor devices. When the operating frequency increases, the current density of conductor tends to be great near the surface of the conductor. Due to the high packing density, the interconnects in VLSI, MCM and PCB are placed closely each other, and the current density is also great at the near side between the conductors. They are known as the skin and proximity effects [1], respectively, thus the interconnects of high-speed integrated circuits have frequency-dependent characteristics. The frequency-dependent parameters are always obtained by a numerical procedure and as tabulated data in real frequency. Therefore, the analysis of frequency-dependent lossy transmission lines with tabulated data is very important for accurate analysis of VLSI, MCM and PCB.

For frequency-dependent interconnect analysis, the frequency-domain method [2], [3] is very accurate. However, this method is not useful from computational point of view, because the system to be analyzed contains very large number of transmission lines, and the frequency-domain method needs large number of data points. The moment matching technique [4]–[6] is efficient and accurate for the interconnect analysis. Recently, these methods are extended to the frequency-dependent case [7], [8]. Since the moment matching technique is essentially Padé approximation of a Laplace function, if transfer function is described in power series of complex s, the moment matching technique can be applied to the analysis. Thus the key technique in Refs. [7], [8] is how tabulated data in real frequency is described in power series of complex s, and the Weighted Least Square Fitting (WLSF) method in [7] and the piecewise polynomial approximation in [8] are used.

In this paper, we provide a new curve fitting technique for describing tabulated data with a power series of complex s and apply the moment matching technique to the interconnect analysis. As preceding stage of the moment matching technique, state variables of circuit equation are expressed with power series of complex s which is known as moment generation [5]. Thus, the input-output relation of transmission lines must be described with matrix polynomial of complex s, and the frequency-dependent parameter is also required to be power series of complex s. Therefore, on applying the moment matching technique to the analysis of frequency-dependent lossy transmission lines with tabulated data, the accuracy depends on curve fitting technique of tabulated data. In the WLSF method [7], first, the tabulated data is approximated by rational function of complex s, and the rational function is converted into power series of complex s, because the input-output relation must be expressed by matrix polynomial of complex s. As suggested in Ref. [7], the rational function itself in the WLSF method gives good approximation result, but the power series converted from the rational function is not accurate. Hence, the WLSF method is not suitable for the moment matching technique, because the moment matching technique makes rational function of a specified output from its power series of complex s. On the other hand, the proposed method is based on Chebyshev interpolation technique. Chebyshev polynomial is an almost minimax approximate polynomial, hence, the proposed method gives a good fitted curve in the form of power series of complex s. The polynomial must be constructed as having real coefficients due to realistic impedance or admittance functions. The discrete orthogonal property of Chebyshev polynomial allows us to construct the continuous polynomial with real coefficients, different from the piecewise one in [8].

Manuscript received March 13, 1998.
Manuscript revised June 19, 1998.
†The authors are with the Faculty of Engineering, Tokushima University, Tokushima-shi, 770–0814 Japan.

In Sect. 2, we will modify the matrix exponential method for describing the input-output relation of frequency-dependent lossy transmission lines. In Sect. 3, the curve fitting technique of the tabulating frequency-dependent parameters is presented. In Sect. 4, the techniques for getting the time- and frequency-domain responses are provided by means of the multi point Padé approximation and the recursive convolution. In numerical examples, the efficiency of the proposed method is illustrated, comparing with the WLSF method. Moreover, the time- and frequency-domain responses are calculated, in order to show that the proposed curve fitting technique is suitable for the moment matching technique. It is confirmed that these results agree with the frequency-domain method [2], [3].

2. Frequency-Dependent Lossy Transmission Lines

The frequency-dependent transmission lines are described by the Telegrapher's equation in the Laplace-domain:

$$\frac{d}{dx}\begin{bmatrix} \mathbf{V}(s,x) \\ \mathbf{I}(s,x) \end{bmatrix} = \mathbf{F}(s)\begin{bmatrix} \mathbf{V}(s,x) \\ \mathbf{I}(s,x) \end{bmatrix} \qquad (1)$$

where

$$\mathbf{F}(s) = \begin{bmatrix} \mathbf{0} & -\mathbf{Z}(s) \\ -\mathbf{Y}(s) & \mathbf{0} \end{bmatrix}$$

$$\mathbf{Z}(s) = \mathbf{R}(s) + s\mathbf{L}(s), \quad \mathbf{Y}(s) = \mathbf{G}(s) + s\mathbf{C}(s),$$

x is the distance along the lines, s is the complex frequency, and $\mathbf{V}(s,x)$ and $\mathbf{I}(s,x)$ are port voltages and currents of the transmission lines, respectively. The parameters $\mathbf{R}(s)$, $\mathbf{L}(s)$, $\mathbf{C}(s)$ and $\mathbf{G}(s)$ are per-unit-length registance, inductance, capacitance and conductance matrices, respectively, and these matrices are arbitrary matrix function of complex s. Actually, each element of these matrices is not given as a function of complex s, but tabulated data to some points, $j\omega_i$'s on the imaginary axis.

In this paper, our aim is how to apply the moment matching technique [5], [6] for solving (1). If any transfer functions are described with power series of complex s, we can apply the moment matching technique to the analysis, because the moment matching technique is essentially Padé approximation. Hence, it is the key technique that the input ($x = 0$)-output ($x = d$) relation of transmission lines is described with matrix polynomial of complex s. Assuming that the parameter matrices $\mathbf{R}(s)$, $\mathbf{L}(s)$ $\mathbf{C}(s)$ and $\mathbf{G}(s)$ are matrix polynomial of complex s, it is easy to describe the input-output relation with matrix polynomial, by using the matrix exponential method [7], [8]. Here we briefly modify the matrix exponential method to increase the convergency.

Applying the matrix exponential method, the input-output relation of (1) is given by

$$\begin{bmatrix} \mathbf{V}(s,d) \\ \mathbf{I}(s,d) \end{bmatrix} = \exp(\mathbf{F}(s)d)\begin{bmatrix} \mathbf{V}(s,0) \\ \mathbf{I}(s,0) \end{bmatrix}$$

$$= \sum_{n=0}^{\infty}\frac{1}{n!}(\mathbf{F}(s)d)^n\begin{bmatrix} \mathbf{V}(s,0) \\ \mathbf{I}(s,0) \end{bmatrix}$$

$$= \sum_{n=0}^{\infty}\mathbf{T}_n s^n\begin{bmatrix} \mathbf{V}(s,0) \\ \mathbf{I}(s,0) \end{bmatrix} \qquad (2)$$

where

$$\mathbf{T}_n = \frac{1}{n!}(\mathbf{F}(s)d)^n.$$

In Ref. [5], it is illustrated that convergence of the infinite series (2) greatly depends on the length d of transmission lines. If the convergence is smooth, the transmission lines must be divided in some regions (This implies that $\exp(\mathbf{F}(s)d_1)$ converges more rapidly than $\exp(\mathbf{F}(s)d_2)$, if $d_1 < d_2$). In this case, the following relation is very useful in order to get the whole characteristics of transmission lines:

$$\exp(\mathbf{F}(s)d) = \exp\left(\mathbf{F}(s)\frac{d}{2}\right)\cdot\exp\left(\mathbf{F}(s)\frac{d}{2}\right). \qquad (3)$$

However, dividing the transmission lines requires for more computational cost, because this means that some equations are added to the circuit equation. Alternatively, $\exp(-\mathbf{F}(s)\frac{d}{2})$ is multiplied from left side of (2) instead of dividing, and we can get the following relation:

$$\exp\left(-\mathbf{F}(s)\frac{d}{2}\right)\begin{bmatrix} \mathbf{V}(s,d) \\ \mathbf{I}(s,d) \end{bmatrix} = \exp\left(\mathbf{F}(s)\frac{d}{2}\right)\begin{bmatrix} \mathbf{V}(s,0) \\ \mathbf{I}(s,0) \end{bmatrix}. \qquad (4)$$

The relation (4) represents the continuity of the voltages and currents at the center point of the transmission lines, whereas the relation (2) gives a relation of the output variables to the input. Thus, the relation (4) is more effective than (2), because the complexity depends on the length of the transmission lines.

Assuming $\mathbf{F}(s)$ as M degree matrix polynomial of complex s, $\mathbf{F}(s) = \sum_{i=0}^{M}\mathbf{F}_i s^i$, the coefficients of the matrix exponential (2) are obtained in recursive manner [7], [8]:

$$\mathbf{T}_n = \begin{cases} \displaystyle\sum_{m=-1}^{\infty}\mathbf{T}_{0,m} & (n=0) \\ \displaystyle\sum_{m=\mathrm{int}(\frac{n-1}{M})}^{\infty}\mathbf{T}_{n,m} & (n \neq 0) \end{cases} \qquad (5)$$

where

$$\mathbf{T}_{n,m} = \frac{d}{m+1}\sum_{k=0}^{\min(n,M)}\mathbf{F}_k\mathbf{T}_{n-k,m-1}$$

$$(n = 0, \ldots, mM, m \neq 0)$$

$$\mathbf{T}_{n,m} = \frac{d}{m+1}\sum_{k=n-mM}^{M}\mathbf{F}_k\mathbf{T}_{n-k,m-1}$$

$$(n = mM+1, \ldots, (m+1)M, m \neq 0)$$
$$\mathbf{T}_{n,0} = \mathbf{F}_n d, \quad (n = 0, \ldots, M)$$
$$\mathbf{T}_{0,-1} = \mathbf{I}$$

$\mathbf{I}$ is the identity matrix. $\exp(-\mathbf{F}(s)d)$ can be calculated by multiplying $\mathbf{T}_{n,m}$ by $(-1)^{m+1}$. Although the matrix exponential (2) converges after 40-50 terms [7], [8], $\exp(\mathbf{F}(s)d/2)$ converges more rapidly than $\exp(\mathbf{F}(s)d)$, because the length of transmission lines is half.

In order to get the port relation of the transmission lines, the matrix exponentials of (4) are rewritten in the block matrix form:

$$\exp\left(-\mathbf{F}(s)\frac{d}{2}\right) = \sum_{n=0}^{\infty} \begin{bmatrix} \mathbf{E}^1_{11,n} & \mathbf{E}^1_{12,n} \\ \mathbf{E}^1_{21,n} & \mathbf{E}^1_{22,n} \end{bmatrix} s^n$$
$$\exp\left(\mathbf{F}(s)\frac{d}{2}\right) = \sum_{n=0}^{\infty} \begin{bmatrix} \mathbf{E}^2_{11,n} & \mathbf{E}^2_{12,n} \\ \mathbf{E}^2_{21,n} & \mathbf{E}^2_{22,n} \end{bmatrix} s^n$$

where each element matrix has same order. Interchanging the elements of (4), we can get the port relation as follows:

$$\sum_{n=0}^{\infty} \mathbf{P}_n s^n \begin{bmatrix} \mathbf{V}(s,0) \\ \mathbf{V}(s,d) \end{bmatrix} + \sum_{n=0}^{\infty} \mathbf{Q}_n s^n \begin{bmatrix} \mathbf{I}(s,0) \\ -\mathbf{I}(s,d) \end{bmatrix} = \mathbf{0}. \tag{6}$$

where

$$\mathbf{P}_n = \begin{bmatrix} \mathbf{E}^2_{11,n} & -\mathbf{E}^1_{11,n} \\ \mathbf{E}^2_{21,n} & -\mathbf{E}^1_{21,n} \end{bmatrix}$$
$$\mathbf{Q}_n = \begin{bmatrix} \mathbf{E}^2_{12,n} & \mathbf{E}^1_{12,n} \\ \mathbf{E}^2_{22,n} & \mathbf{E}^1_{22,n} \end{bmatrix}.$$

3. Chebyshev Interpolation Scheme of Frequency-Dependent Parameters

In the previous section, the matrix exponential method is used for describing the input-output relation of frequency-dependent lossy transmission lines by matrix polynomial of complex s, where it is assumed that the parameters given as tabulated data to some points on imaginary axis are able to write in power series of complex s. In this section, the procedure for making the power series from the tabulated data is provided.

3.1 Curve Fitting Algorithm

Let $r(s)$, $l(s)$, $c(s)$ and $g(s)$ be an element of $\mathbf{R}(s)$, $\mathbf{L}(s)$, $\mathbf{C}(s)$ and $\mathbf{G}(s)$, respectively, where these values are given as tabulated data to some points, $j\omega_i$'s ($j = \sqrt{-1}$), on the imaginary axis. Elements $z(s) = r(s) + sl(s)$ of the series impedance matrix $\mathbf{Z}(\mathrm{s})$ and $y(s) = g(s) + sc(s)$ of the parallel admittance matrix $\mathbf{Y}(\mathbf{s})$ are approximated by $(N-1)$-degree polynomial with respect to $j\omega$ so that the values of $z(s)$ and $y(s)$ at some $s = j\omega_i$'s satisfy

$$z(j\omega_i) \approx \sum_{k=0}^{N-1} z_k (j\omega_i)^k, \quad y(j\omega_i) \approx \sum_{k=0}^{N-1} y_k (j\omega_i)^k \tag{7}$$

where the coefficients z_k and y_k are assumed as real numbers, which is a reasonable assumption due to realistic impedance or admittance functions.

Let us consider $z(s)$ only, and $y(s)$ can be obtained by the same procedure. In Ref. [7], eliminating the lossless part of $l(s)$ is introduced in order to approximate accurately, namely, the lossless part $l(\infty)$ is separated from $l(j\omega)$ such as

$$l'(j\omega) = l(j\omega) - l(\infty). \tag{8}$$

Then, $z'(j\omega_i) = r(j\omega_i) + j\omega_i l'(j\omega_i)$ is interpolated by the Chebyshev series. A transform $\xi = \omega/\omega_{max}$ is used to convert $\omega \in [0, \omega_{max}]$ into $\xi \in [0,1]$. Assuming $z'(-j\omega)$ is complex conjugate to $z'(j\omega)$, the interpolated polynomial in $\xi \in [-1,1]$ is obtained by

$$z'(j\omega_{max}\xi) = {\sum_{k=0}^{N-1}}' a_k D_k(\xi) \tag{9}$$

where the symbol $\sum'$ denotes the summation with the first component divided by 2 and $D_k(\xi) = \cos k\theta$ $(0 \leqq \theta \leqq \pi)$. From the discrete orthogonal property of Chebyshev polynomial, a_k $(k = 0, 1 \ldots, N-1)$ are given as follows:
if N is odd,

$$a_k = \begin{cases} \dfrac{4}{N}\displaystyle\sum_{n=0}^{\frac{N-3}{2}} r(j\omega_{max}\cos\theta_n)\cos k\theta_n + r(0)\cos\dfrac{k\pi}{2} & (k\text{: even}) \\ \dfrac{4}{N}\displaystyle\sum_{n=0}^{\frac{N-3}{2}} j\omega_{max}\cos\theta_n l'(j\omega_{max}\cos\theta_n)\cos k\theta_n & (k\text{: odd}) \end{cases} \tag{10a}$$

if N is even,

$$a_k = \begin{cases} \dfrac{4}{N}\displaystyle\sum_{n=0}^{\frac{N}{2}-1} r(j\omega_{max}\cos\theta_n)\cos k\theta_n & (k\text{: even}) \\ \dfrac{4}{N}\displaystyle\sum_{n=0}^{\frac{N}{2}-1} j\omega_{max}\cos\theta_n l'(j\omega_{max}\cos\theta_n)\cos k\theta_n & (k\text{: odd}) \end{cases} \tag{10b}$$

where $\theta_n = \frac{2n+1}{2N}\pi$ is the Chebyshev point, and $j\omega_{max}\cos k\theta_n$ is corresponding to $j\omega_i$ in (7). Note that if N is even number, the information at $\omega = 0$ does not reflect the fitted curve. Hence, N is prefer to be odd number.

The coefficients a_k are real part or imaginary part only, thus we can derive the power series of $j\omega_{max}\xi$

having real coefficients. First, $z'(j\omega_{max}\xi)$ of (9) is converted into a power series with respect to ξ. Using the recurrence formula of Chebyshev polynomial,

$$\begin{cases} D_0(\xi) = 1, \quad D_1(\xi) = \xi, \\ D_{k+1}(\xi) = 2\xi D_k(\xi) - D_{k-1}(\xi), \end{cases} \tag{11}$$

Chebyshev polynomials $D_k(\xi)$ $(k = 2, 3, \ldots)$ are obtained by

$$\begin{aligned} D_2(\xi) &= 2\xi^2 - 1, \\ D_3(\xi) &= 4\xi^3 - 3\xi, \\ D_4(\xi) &= 8\xi^4 - 8\xi^2 + 1, \\ D_5(\xi) &= 16\xi^5 - 20\xi^3 + 5\xi, \\ D_6(\xi) &= 32\xi^6 - 48\xi^4 + 18\xi^2 - 1, \\ &\vdots \end{aligned} \tag{12}$$

As a result, the finite Chebyshev series (9) is converted into a power series with respect to ξ:

$$z'(j\omega_{max}\xi) = \sum_{k=0}^{N-1} b_k \xi^k. \tag{13}$$

From (12), $D_{2m}(\xi)$ and $D_{2m+1}(\xi)$ are even and odd functions, respectively. Thus, b_{2m} and b_{2m+1} in (13) are respectively real and imaginary part only as a_{2m} and a_{2m+1} in (10a) and (10b). Consequently, $z'(j\omega_{max}\xi)$ is expressed in power series of $j\omega_{max}\xi$ with real coefficients:

$$z'(j\omega_{max}\xi) = \sum_{k=0}^{N-1} z'_k (j\omega_{max}\xi)^k, \tag{14}$$

where

$$z'_k = \begin{cases} (-1)^{\frac{k}{2}} \dfrac{b_k}{\omega_{max}^k} & (k\text{: even}) \\[2ex] (-1)^{\frac{k+1}{2}} \dfrac{jb_k}{\omega_{max}^k} & (k\text{: odd}) \end{cases}$$

From (8) and (14), an element of the series impedance matrix of transmission lines is described by

$$z(s) = z'_0 + (z'_1 + l(\infty))s + \sum_{k=2}^{N-1} z'_k s^k. \tag{15}$$

where all coefficients of s^k's are real numbers.

3.2 Shifted Coefficients of Power Series

When the multi point Padé approximation [5], [6] is used to get dominant poles, the shifted moments, that is, the coefficients of Taylor expansion at an arbitrary point s_k are needed. Thus, $\mathbf{F}(s)$ in (2) and the matrix exponential must be a matrix polynomial of complex $\sigma = s - s_k$.

Let $\mathbf{F}(s)$ be M degree matrix polynomial, $\mathbf{F}(s) = \sum_{i=0}^{M} \mathbf{F}_i s^i$, then $\mathbf{F}(s)$ is converted into a matrix polynomial of complex $\sigma = s - s_k$:

$$\mathbf{F}(s) \equiv \mathbf{F}(\sigma) = \sum_{m=0}^{M}\sum_{i=0}^{m} \binom{m}{i} \mathbf{F}_m s_k^{m-i} \sigma^i. \tag{16}$$

4. Multi Point Padé Approximation

The time- and frequency-responses are calculated by the multi point Padé approximation and the recursive convolution. Resently, Chiprout and Nakhla [5] have proposed Complex Frequency Hopping (CFH) which efficiently extracts the dominant poles in binary search nature from multi point Padé approximation. This method needs complex number operation (L/U decomposition, finding poles) so that it is somewhat time-consuming. So, we modify this method in Padé approximation at arbitrary expansion point by using Celik's method [6], and also propose the recursive convolution with piecewise linear assumption.

4.1 Padé Approximation at Arbitrary Expansion Point

In the Asymptotic Waveform Evaluation (AWE) method [4], impulse response of a specified output is expressed by rational function in the Laplace-domain. Here, Padé approximation is appropriately applied to constructing the rational function, where 8–10 dominant poles at most are obtained. We need more poles to characterize a specified output accurately. So, CFH is applied in order to take more dominant poles.

CFH extracts the dominant poles in binary search nature from the multi point Padé approximation. Thus, Padé approximation at a point s_k is needed. In Ref. [5], the solutions of the Modified Nodal Admittance (MNA) equation in the Laplace-domain are expanded at a point s_k and expressed in the rational function of complex $\sigma = s - s_k$. Here, complex L/U decomposition is used to determine the coefficients of denominator polynomial of the rational function, and a set of the poles are calculated by solving the roots of the denominator polynomial with complex coefficients. Since it is somewhat time-consuming, the Padé approximation proposed by Celik [6] are applied.

Assuming complex conjugate expansion points s_k and s_{-k}, the impulse response $H(s)$ of a specified output is described by $[(q-1)/q]$ rational function so that the shifted moments [6] are satisfied as

$$H(s) = \frac{b_0 + b_1 s + \ldots + b_{q-1}s^{q-1}}{1 + a_1 s + \ldots + a_q s^q} \tag{17}$$

$$= m_{k,0} + m_{k,1}\sigma + \ldots + m_{k,q-1}\sigma^{q-1} \tag{18}$$

$$= m_{-k,0} + m_{-k,1}\hat{\sigma} + \ldots + m_{-k,q-1}\hat{\sigma}^{q-1} \tag{19}$$

where $\sigma = s - s_k$, $\hat{\sigma} = s - s_{-k}$, and $m_{\pm k,i}$ $(i = 0, 1, \ldots, q-1)$ are shifted moments calculated in recursive manner [6].

Rearranging the rational function (17) in complex σ and matching the corresponding powers of σ of (18) give

$$\mathbf{M}_k \mathbf{c} = \mathbf{m}_k \tag{20}$$

where

$$\mathbf{c} = (b_0, b_1, \ldots, b_{q-1}, a_1, a_2, \ldots, a_q)^T,$$
$$\mathbf{m}_k = (m_{k,0}, m_{k,1}, \ldots, m_{k,q-1})^T.$$

In (20), $\mathbf{M}_k$ is $q \times 2q$ matrix and given by

$$\mathbf{M}_k = [\ \mathbf{C}_1 \ \vdots\ \mathbf{C}_2 \ \vdots\ -\mathbf{B}\mathbf{C}_2 \ \vdots\ -\mathbf{B}\mathbf{C}_3\ ] \tag{21}$$

where

$$[\ \mathbf{C}_1 \mid \mathbf{C}_2 \mid \mathbf{C}_3\ ] = \left[\begin{array}{c|ccccc|c} 1 & s_k & s_k^2 & \cdots & s_k^{q-1} & & s_k^q \\ & 1 & \binom{2}{1}s_k & \cdots & \binom{q-1}{1}s_k^{q-2} & & \binom{q}{1}s_k^{q-1} \\ & & 1 & \cdots & & & \\ & & & \ddots & & & \\ & & & & 1 & & \binom{q}{q-1}s_k \end{array}\right]$$

$$\mathbf{B} = \begin{bmatrix} m_{k,0} & & & \\ m_{k,1} & m_{k,0} & & \\ \vdots & \vdots & \ddots & \\ m_{k,q-1} & m_{k,q-2} & \cdots & m_{k,0} \end{bmatrix}.$$

Since m_{-k} is complex conjugate of m_k, the determining equation corresponding (20) with the sifted moment m_{-k} [6] is obtained by

$$\mathbf{M}_k^* \mathbf{c} = \mathbf{m}_k^* \tag{22}$$

where the symbol $*$ means complex conjugate.

As a result, the set of coefficients (17) is obtained by solving $2q \times 2q$ linear equation:

$$\begin{bmatrix} \mathbf{M}_k \\ \mathbf{M}_k^* \end{bmatrix} \begin{bmatrix} \mathbf{b} \\ \mathbf{a} \end{bmatrix} = \begin{bmatrix} \mathbf{m}_k \\ \mathbf{m}_k^* \end{bmatrix}. \tag{23}$$

As suggested in paper [6], L/U decomposition of (23) is performed by real number operation, to emphasis it, we rewrite (23) by

$$\begin{bmatrix} \mathrm{Re}[\mathbf{M}_k] \\ \mathrm{Im}[\mathbf{M}_k] \end{bmatrix} \begin{bmatrix} \mathbf{b} \\ \mathbf{a} \end{bmatrix} = \begin{bmatrix} \mathrm{Re}[\mathbf{m}_k] \\ \mathrm{Im}[\mathbf{m}_k] \end{bmatrix}, \tag{24}$$

Note that the set of coefficients of numerator and denominator polynomials (17) is real number.

4.2 Finding Residues

In the Padé approximation (17) at arbitrary point, all of poles are not always found in left-half complex plane, thus, unstable poles must be eliminated. Assuming that $q'+1$ stable poles $p_m (m = 0, 1, \ldots, q')$ are found, the residue k_m to each pole p_m is obtained by

$$k_m = \left.\frac{b_0 + b_1 s + \ldots + b_{q-1}s^{q-1}}{a_1 + a_2 + \ldots + a_q s^{q-1}}\right|_{s=p_m}. \quad (m = 0, 1, \ldots, q') \tag{25}$$

4.3 Recursive Convolution with Piecewise Linear Assumption

Let the impulse response $H(s)$ be given by CFH:

$$H(s) = \sum_{i=0}^{N} \frac{1}{s - p_i} k_i. \tag{26}$$

The time-domain response of a specified output is obtained by the convolution integral:

$$v(t) = \sum_{i=0}^{N} k_i \int_0^t e^{p_i(t-\tau)} v_{in}(\tau) d\tau \tag{27}$$

where $v_{in}(t)$ is the input waveform. The recursive convolution [10] is very efficient in calculating (27). Then, we apply this method to getting the time-domain response as follows:

$$\begin{aligned} v(t_{n+1}) &= \sum_{i=0}^{N} k_i \int_0^{t_{n+1}} e^{p_i(t_{n+1}-\tau)} v_{in}(\tau) d\tau \\ &= \sum_{i=0}^{N} k_i \left\{ e^{p_i h} \int_0^{t_n} e^{p_i(t_n-\tau)} v_{in}(\tau) d\tau \right. \\ &\quad \left. + \int_{t_n}^{t_{n+1}} e^{p_i(t_{n+1}-\tau)} v_{in}(\tau) d\tau \right\} \end{aligned} \tag{28}$$

where $h = t_{n+1} - t_n$. In Ref. [10], the second term of (28) is calculated by the trapezoidal rule. However, if the time interval h is not small, the convolution integral becomes inaccurate [11]. So, the input waveform $v_{in}(t)$ in $[t_n, t_{n+1}]$ is assumed as the piecewise linear function:

$$\begin{aligned} v_{in}(t) &= \frac{v_{in}(t_{n+1}) - v_{in}(t_n)}{t_{n+1} - t_n}(t - t_n) + v_{in}(t_n) \\ &= \frac{v_{in}(t_{n+1}) - v_{in}(t_n)}{h} t \\ &\quad + \frac{v_{in}(t_n)t_{n+1} - v_{in}(t_{n+1})t_n}{h}. \end{aligned} \tag{29}$$

From this assumption, the integral value of the second term (28) is written in the closed form

$$\begin{aligned} &\int_{t_n}^{t_{n+1}} e^{p_i(t_{n+1}-\tau)} v_{in}(\tau) d\tau \\ &= -\frac{1}{hp_i^2} \left(1 - e^{p_i h} + p_i h\right) v_{in}(t_{n+1}) \\ &\quad - \frac{1}{hp_i^2} \left\{e^{p_i h}(1 - p_i h) - 1\right\} v_{in}(t_n). \end{aligned} \tag{30}$$

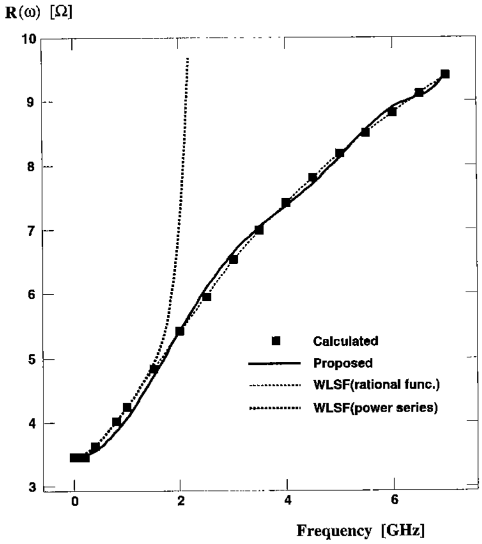

Fig. 1 Frequency-dependent resistance curves of (2,2) element of the series impedance matrix of the transmission lines provided by M. Celik [3].

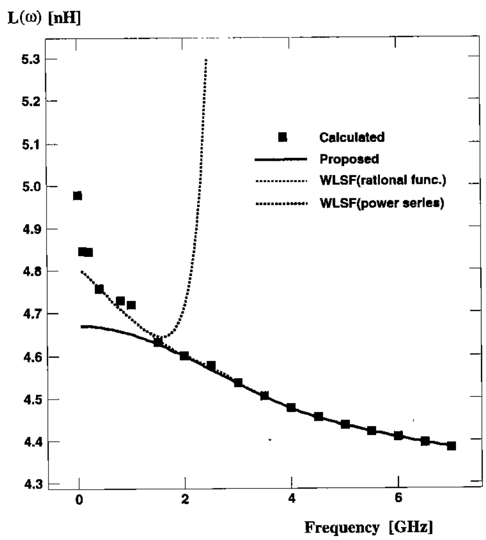

Fig. 2 Frequency-dependent inductance of (2,2) element of the series impedance matrix of the transmission lines provided by M. Celik [3].

5. Numerical Examples

To show the accuracy of our method, the 3-conductor transmission lines provided by M. Celik are considered. The frequency-dependent parameters are listed as the tables II, III in Ref. [7], where the frequency range is from 0 to 7 [GHz]. The series impedance matrix $\mathbf{Z}(s) \in C^{3\times3}$ in (1) is approximated in 10-degree matrix polynomial by using the curve fitting technique in Sect. 3. The parameters at $j\omega_{max}\cos\theta_n$, $\theta_n = \frac{2n+1}{22}\pi$ $(n = 0, 1, \ldots, 4)$, are needed to determine the coefficients a_k in (10a). However, because the tabulated data in reference [7] is not obtained at $j\omega_{max}\cos\theta_n$, the cubic spline in-

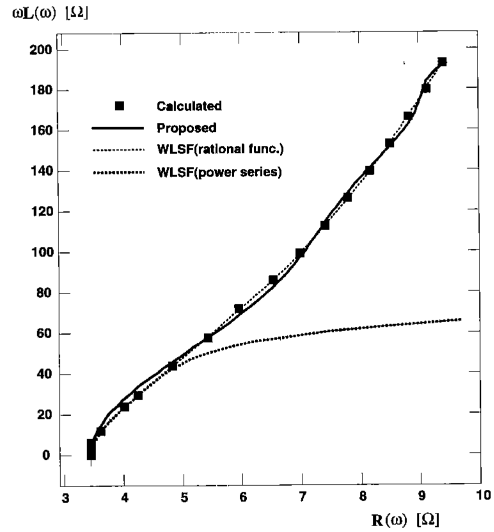

Fig. 3 (2,2) element of the series impedance matrix of the transmission lines provided by M. Celik [3].

Table 1 Coefficients of power series given by the proposed method.

	value		value
c_0	3.448	c_6	-3.292×10^{-8}
c_1	4.670	c_7	1.956×10^{-10}
c_2	-1.684×10^{-2}	c_8	-1.601×10^{-11}
c_3	5.196×10^{-4}	c_9	3.295×10^{-14}
c_4	-3.196×10^{-7}	c_{10}	-2.917×10^{-15}
c_5	4.447×10^{-7}		

terpolation is used to get the parameters at the frequency points. Figs. 1, 2 and 3 show the frequency-dependent resistance, inductance and impedance curves of (2,2) element $z_{22}(s)$ of the series impedance matrix $\mathbf{Z}(s)$, respectively. The coefficients c_i $(i = 0, 1, \ldots, 10)$ in $z_{22}(s) = \sum_{i=0}^{10} c_i s^i$ are listed in Table 1.

For comparison, the WLSF method [7] is applied to the same example. The WLSF method approximates $z(j\omega)$ with rational function of $j\omega$ so that it satisfy

$$\frac{d_0 + d_1 j\omega_i + \ldots + d_m (j\omega_i)^m}{1 + e_1 j\omega_i + \ldots + e_n (j\omega_i)^n} = z(j\omega_i).$$
$$(i = 0, 1, \ldots, m+n)$$

Since the rational function can not be used for the matrix exponential method, the rational function must be converted into a power series:

$$\frac{d_0 + d_1 s + \ldots + d_m s^m}{1 + e_1 s + \ldots + e_n s^n} = c_0 + c_1 s + \ldots + c_{m+n} s^{m+n}.$$

The results by using the WLSF method are also shown in Figs. 1, 2 and 3, where WLSF (rational fuc.) and WLSF (power series) in these figures mean 5-degree/5-

degree rational function and 10-degree power series, respectively.

In Figs. 1, 2 and 3, the WLSF method itself gives good approximation results, but the power series converted from the rational function is not accurate apparently. Since the moment matching technique is essentially Padé approximation, the transfer function of network must be expressed by a power series of complex s. Thus, the accuracy of power series rather than rational function is important for the moment matching technique. This means that the WLSF method is not suitable for the moment matching technique. In Ref. [8], the parameters are expressed by piecewise polynomial. This means many applications of the matrix exponential method for getting the input-output relation of transmission lines, because piecewise polynomial consists of some polynomials. Therefore, the proposed method is more effective for the moment matching technique than the WLSF method [7] or the piecewise polynomial approximation [8].

The moment matching technique extracts a set of the dominant poles and residues in the Laplace-domain, but the proposed curve fitting technique is only verified at $s = j\omega$, not $s = a + j\omega$. Hence, it is not clear whether the proposed curve fitting technique is suitable for the moment matching technique or not. So, the time and frequency responses of the interconnect network provided by M. Celik [7] as shown in Fig. 4 are calculated, in order to illustrate that the moment matching technique incorporating the proposed curve fitting technique gives reliable numerical result. In the example circuit, the 3-conductor transmission lines have the same frequency-dependent parameters approximated by 10-degree matrix polynomial in the previous discussion. Figures 5 and 6 show the transient response at the node V_{out} to a voltage pulse input (0.8 [ns] pulse width, 0.1 [ns] rise and fall time) and the frequency response at the same node to a voltage impulse input, respectively. Here, the impulse response (26) (25-degree/26-degree rational function) is estimated by using the multi point Padé approximation in Sect. 4, where the maximum frequency is selected by 5 [GHz], and 9 expansion points are considered. The transient response shown in Fig. 5 is calculated by using the recursive convolution in Sect. 4.3. The frequency response shown in Fig. 6 is absolute value of the impulse response (26) at $s = j\omega$. These results are compared with the frequency-domain method (FFT) [3] as shown in Figs. 5 and 6. To calculate the transient response shown in Fig. 5, the frequency response is estimated at 128 frequency points, and the waveform in the time-domain is calculated by using inverse fast Fourier transform. Also, the frequency response to a voltage impulse input shown in Fig. 6 is calculated at 128 frequency points. In the frequency-domain method, each frequency response is calculated by directly solving the circuit equation at the frequency, whereas the moment matching technique generates the rational function of complex s by few estimations of the circuit equation. Thus, the frequency-domain method is very accurate. In Figs. 5 and 6, the results by using the proposed curve fitting and the moment matching techniques almost agree with the frequency-domain method. It means that although the curve provided by the proposed method fits the tabulating frequency-dependent parameters on the imaginary axis only, this technique is suitable for the moment matching technique to the analysis of frequency- dependent lossy transmission lines.

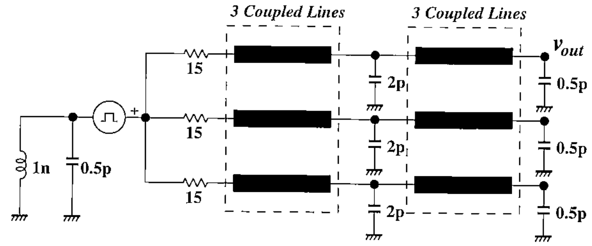

Fig. 4 The circuit including frequency-dependent transmission lines.

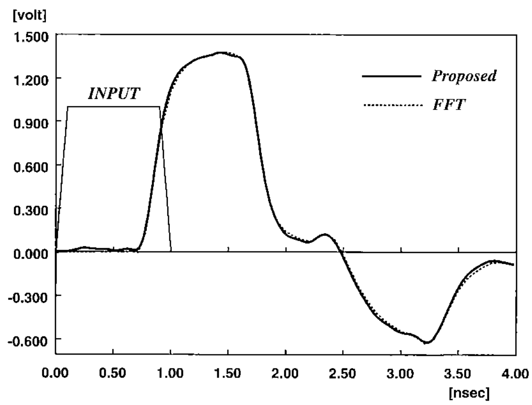

Fig. 5 Transient response to a pulse input.

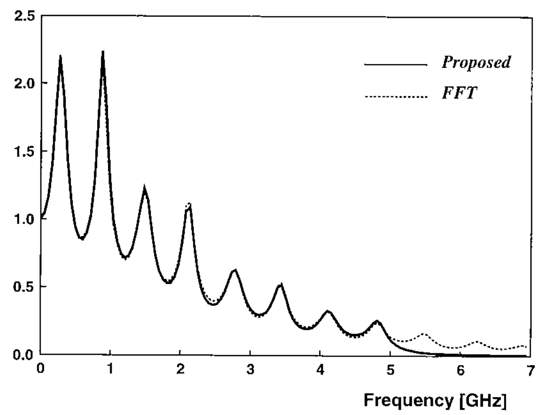

Fig. 6 Frequency response to an impulse input.

6. Conclusions

A new curve fitting technique for the analysis of frequency-dependent lossy transmission lines with tabulated data has been presented. This method is efficiently incorporated with the moment matching technique [5], [6]. Although the object of this paper is turned to the moment mathing technique, this method is easily applied to the method of characteristics by means of a technique in Ref. [12]. This is our future work.

Acknowledgment

The authors are grateful to Prof. Y. Shinohara of Dept. of Mathematics at Tokushima University for his encouragement and comments and would like to thank the anonymous reviewers for their invaluable suggestions in improving the paper.

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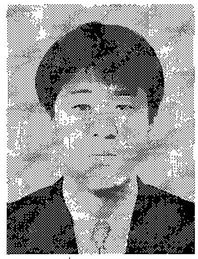
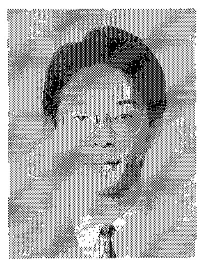


Yuichi Tanji received the B.E., M.E., Ph.D. degrees from Tokushima University, Tokushima, Japan, in 1993, 1995, 1998, respectively. In 1998, he joined the Department of Electrical and Electronic Engineering at Sophia University, Tokyo, Japan, where he is a Research Associate. His research interest is in circuit simulation. Dr. Tanji is a member of the IEEE.

Yoshifumi Nishio received the B.E. and M. E. and Ph.D. degrees in Electrical Engineering from Keio University, Yokohama, Japan, in 1988, 1990 and 1993, respectively. In 1993, he joined the Department of Electrical and Electronic Engineering at Tokushima University, Tokushima Japan, where he is currently an Associate Professor. His research interests are in chaos and synchronization phenomena in nonlinear circuits. Dr. Nishio is a member of the IEEE.



Takashi Shimamoto was born in Tokushima, Japan, on November 22, 1959. He received the B.E. and M.E. degrees in electrical engineering from Tokushima University, and the Dr.E. degree from Osaka University, in 1982, 1984, and 1992, respectively. He joined the Department of Electronic Engineering, College of Industrial Technology, Tokushima University, in 1984 as an Assistant Professor. Presently he is an Associate Professor of the Department of Electrical and Electronic Engineering, Faculty of Engineering, Tokushima University. His interests of research include heuristic algorithms for VLSI CAD.

Akio Ushida received the B.E. and M.E. degrees in electrical engineering from Tokushima University in 1961 and 1966, respectively, and the Ph.D. degree in electrical engineering from University of Osaka Prefecture in 1974. He was an associate professor from 1973 to 1980 at Tokushima University. Since 1980 he has been a Professor in the Department of Electrical Engineering at the university. From 1974 to 1975 he spent one year as a visiting scholar at the Department of Electrical Engineering and Computer Sciences at the University of California, Berkeley. His current research interests include numerical methods and computer-aided analysis of nonlinear systems. Dr. Ushida is a member of the IEEE.