PAPER Special Section on Nonlinear Theory and Its Applications

Circuit Realization of a Coupled Chaotic Circuits Network and Irregular Pattern Switching Phenomenon

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SUMMARY In this study, a coupled chaotic circuits network is realized by real circuit elements. By using a simple circuit converting generating spatial patterns to digital signal, irregular self-switching phenomenon of the appearing patterns can be observed as real physical phenomenon.

key words: *coupled chaotic network, pattern switching, chaotic wandering, chaotic quasi-synchronization, spatio-temporal chaos*

1. Introduction

Recently, spatio-temporal phenomena observed from coupled chaotic networks, namely coupled systems of many chaotic cells, attract many researchers' attentions. The studies on coupled chaotic networks are classified into two categories; namely discrete time systems and continuous time systems. For discrete time systems Kaneko's Coupled Map Lattice seems to be the most interesting and well-studied system [1]. He discovered various nonlinear spatio-temporal chaotic phenomena such as clustering, Brownian motion of defect and so on. Tsuda has mentioned that chaotic wandering observed from coupled chaotic networks is important as a basic mechanism of hermeneutics in brain and mind [2]. Also Aihara's chaos neural network is the most important chaotic network from an engineering point of view [3]. His study indicated new possibility of engineering applications of chaos, namely dynamical search of patterns embedded in neural networks utilizing chaotic wandering. Further, application of chaos neural network to optimization problems is widely studied. Nagashima et al. have investigated the details of pattern switching phenomenon caused by chaotic wandering observed from the chaos neural network [4]. On the other hand, for continuous time systems several studies on one-dimensional or two-dimensional arrays of coupled Chua's circuits have been reported (e.g. some papers in [5]). However, almost studies except some Ogorzałek's studies ([6] and therein) treated only parameter values for which an isolated Chua's circuit does not generate chaotic attractor; namely only two stable sinks or two stable limit cycles. Hence, main subject of such studies was wave propagation phenomena observed for a given set of the initial patterns and there are very few studies on spatial patterns observed after vanishing the effect of initial patterns. Namely, pattern switching phenomenon caused by chaotic wandering as observed in discrete time coupled chaotic networks has not yet observed in analog chaotic circuits network models as far as we know. Further, there seems to be no reports on such spatio-temporal chaotic phenomena observed in laboratory experiments using simple real analog electrical circuits. This is because such spatio-temporal chaotic phenomena are usually observed only from a large number of chaotic cells that is spatially extensive.

The authors have proposed a continuous time coupled chaotic circuits network [7], [8] and have investigated generating spatial patterns and irregular self-switching phenomenon of the patterns only by computer simulation. The important feature of our chaotic network is its coupling structure. Namely adjacent four chaotic circuits are coupled by one resistor. Because such a coupling exhibited quasi-synchronization with phase difference [9], various spatial patterns could be generated even in the network with small size. Therefore, the network was considered to be easily realized by real circuit elements and to be the first real physical electrical circuit exhibiting irregular pattern switching phenomenon. However, because it was difficult to continuously observe how spatial patterns are changing in real circuits, we could not carry out circuits experiments.

In this study, we construct the coupled chaotic circuits network by real circuit elements and try to observe the irregular self-switching phenomenon of appearing spatial patterns as real physical phenomenon. In order to check the appearing spatial patterns, we design a simple circuit converting the generating patterns to digital signal in a simple way. By checking the digital signal, we can observe that the appearing spatial patterns are changing in an irregular way. Such digital informations would be useful to investigate statistical property of the irregular pattern switching phenomenon in future. Further, this study based on real physical system will contribute to elucidation of complex various phenomena in natural fields and fabrication of new parallel information processing mechanism exploiting chaos on analog circuits.

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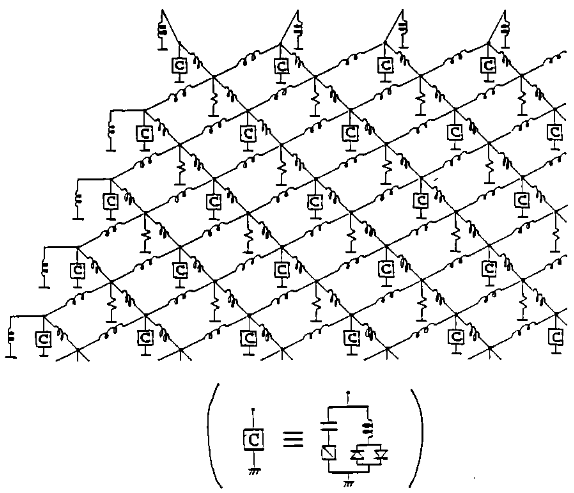
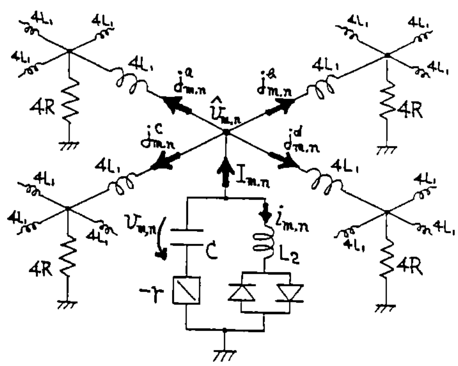
2. Circuit Model

The coupled chaotic circuits network is shown in Fig. 1. This network has local connection structure such that four subcircuits are coupled around one coupling resistor R. The subcircuit is composed of a capacitor, two inductors, a negative resistor and some diodes. When the coupling resistance is zero, each chaotic subcircuit



(a)



(b)

Fig. 1 (a) Coupled chaotic circuits network. (b) Magnification around a cell at position (m,n).

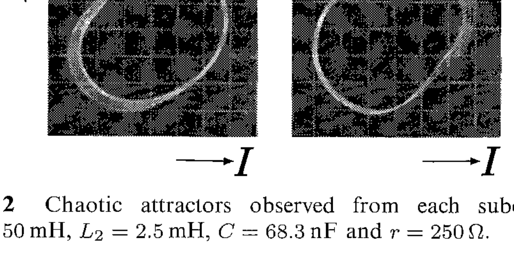


Fig. 2 Chaotic attractors observed from each subcircuit. $L_1 = 50$ mH, $L_2 = 2.5$ mH, $C = 68.3$ nF and $r = 250\,\Omega$.

can produce chaotic attractors as shown in Fig. 2 [10]. At first, we approximate the $v-i$ characteristics of the diodes as follows.

$$v_d(i_{m,n}) = \sqrt[9]{r_d\, i_{m,n}}. \tag{1}$$

By changing the variables and parameters,

$$\begin{aligned}
&I_{m,n} = a\sqrt{\frac{C}{L_1}}\,x_{m,n}, \quad i_{m,n} = a\sqrt{\frac{C}{L_1}}\,y_{m,n},\\
&v_{m,n} = a\,z_{m,n},\\
&j^a_{m,n} = a\sqrt{\frac{C}{L_1}}\,w^a_{m,n}, \quad j^b_{m,n} = a\sqrt{\frac{C}{L_1}}\,w^b_{m,n},\\
&j^c_{m,n} = a\sqrt{\frac{C}{L_1}}\,w^c_{m,n}, \quad j^d_{m,n} = a\sqrt{\frac{C}{L_1}}\,w^d_{m,n},\\
&t = \sqrt{L_1 C}\,\tau, \quad \alpha = \frac{L_1}{L_2}, \quad \beta = r\sqrt{\frac{C}{L_1}},\\
&\gamma = R\sqrt{\frac{C}{L_1}},\\
&\left(\text{where } a = \sqrt[8]{r_d\sqrt{\frac{C}{L_1}}}\right).
\end{aligned} \tag{2}$$

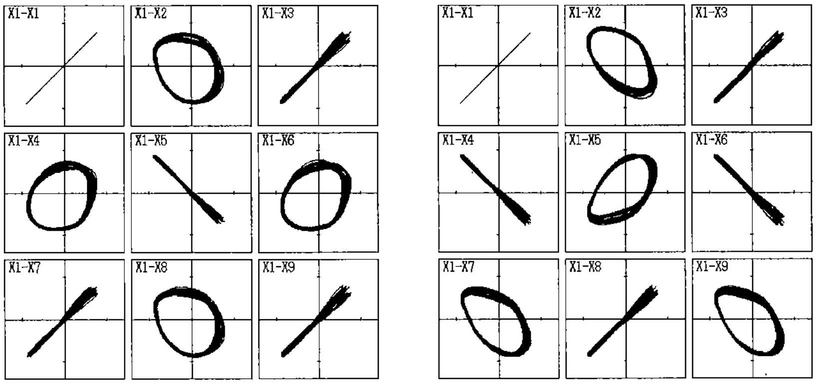
The circuit equations of the network with $N_1 \times N_2$ size are described as

$$\begin{aligned}
x_{m,n} &= w^a_{m,n} + w^b_{m,n} + w^c_{m,n} + w^d_{m,n}\\
\frac{dy_{m,n}}{d\tau} &= \alpha\{\beta(x_{m,n}+y_{m,n}) - z_{m,n} - \sqrt[9]{y_{m,n}}\}\\
\frac{dz_{m,n}}{d\tau} &= x_{m,n} + y_{m,n}\\
\frac{dw^a_{m,n}}{d\tau} &= \beta(x_{m,n}+y_{m,n}) - z_{m,n}\\
&\quad -\gamma(w^a_{m,n} + w^b_{m,n-1} + w^c_{m-1,n} + w^d_{m-1,n-1})\\
\frac{dw^b_{m,n}}{d\tau} &= \beta(x_{m,n}+y_{m,n}) - z_{m,n}\\
&\quad -\gamma(w^a_{m,n+1} + w^b_{m,n} + w^c_{m-1,n+1} + w^d_{m-1,n})\\
\frac{dw^c_{m,n}}{d\tau} &= \beta(x_{m,n}+y_{m,n}) - z_{m,n}\\
&\quad -\gamma(w^a_{m+1,n} + w^b_{m+1,n-1} + w^c_{m,n} + w^d_{m,n-1})\\
\frac{dw^d_{m,n}}{d\tau} &= \beta(x_{m,n}+y_{m,n}) - z_{m,n}\\
&\quad -\gamma(w^a_{m+1,n+1} + w^b_{m+1,n} + w^c_{m,n+1} + w^d_{m,n})\\
&(m = 1,2,3\cdots N_1 \text{ and } n = 1,2,3,\cdots N_2)
\end{aligned} \tag{3}$$

where $w^c_{0,n} = w^d_{0,n} = w^b_{m,0} = w^d_{m,0} = w^a_{N_1+1,n} = w^b_{N_1+1,n} = w^a_{m,N_2+1} = w^c_{m,N_2+1} = 0$ and the value of γ must be neglected for the calculation of the inductors on the edge.

3. Phase Pattern

Because the coupled chaotic circuits network in Fig. 1



(a) (b)

Fig. 3 Snap shots of appearing phase patterns. (a) Type X_1. (b) Type V_1.

tends to minimize the currents through the coupling resistors, it can generate various phase patterns even if the network size is small.

For the case of 3×3 size, 11 kinds of phase patterns appear as follows.

$$\text{Type } \Xi : \ \Xi_1 \begin{pmatrix} A & \bar{A} & A \\ B & \bar{B} & B \\ C & \bar{C} & C \end{pmatrix}, \ \Xi_2 \begin{pmatrix} A & B & C \\ \bar{A} & \bar{B} & \bar{C} \\ A & B & C \end{pmatrix}.$$

$$\text{Type } X : \ X_1 \begin{pmatrix} A & B & A \\ \bar{B} & \bar{A} & \bar{B} \\ A & B & A \end{pmatrix}.$$

$$\text{Type } V : \ V_1 \begin{pmatrix} A & B & A \\ \bar{A} & \bar{B} & \bar{A} \\ B & A & B \end{pmatrix}, \ V_2 \begin{pmatrix} B & A & B \\ \bar{A} & \bar{B} & \bar{A} \\ A & B & A \end{pmatrix},$$

$$V_3 \begin{pmatrix} B & A & \bar{A} \\ \bar{A} & \bar{B} & B \\ B & A & \bar{A} \end{pmatrix}, \ V_4 \begin{pmatrix} A & \bar{A} & B \\ B & \bar{B} & A \\ A & \bar{A} & B \end{pmatrix}.$$

$$\text{Type } L : \ L_1 \begin{pmatrix} A & B & \bar{B} \\ \bar{A} & \bar{B} & B \\ B & A & \bar{A} \end{pmatrix}, \ L_2 \begin{pmatrix} B & \bar{B} & A \\ \bar{B} & B & \bar{A} \\ \bar{A} & A & \bar{B} \end{pmatrix},$$

$$L_3 \begin{pmatrix} B & \bar{A} & A \\ A & \bar{B} & B \\ \bar{A} & B & \bar{B} \end{pmatrix}, \ L_4 \begin{pmatrix} \bar{A} & A & B \\ B & \bar{B} & \bar{A} \\ \bar{B} & B & A \end{pmatrix}.$$

where A and $\bar{A}$ means the pair of the opposite phase quasi-synchronization [6]. Similarly the pair of B and $\bar{B}$ or C and $\bar{C}$ is also quasi-synchronized at the opposite phase. While the different characters A, B and C represent independent group of quasi-synchronization. Please note that the above 11 patterns satisfy the condition that the sum of four subcircuits around any coupling resistors is almost zero (e.g. $A + B + \bar{A} + \bar{B}$). Some snap shots corresponding to the above patterns are shown in Fig. 3. Figure 3 (a) shows an example of the type X_1. While Fig. 3 (b) shows an example of the type V_1.

In our previous study we have carried out only computer calculation for the case of 3×3 size [6]. One example of the time evolution of the Poincaré map is shown in Fig. 4 for the case that asymmetric attractors in Fig. 2 are generated in subcircuits. Although we can observe interesting irregular self-switching phenomenon of phase patterns, it was very difficult to decide appearing phase patterns at certain times. This is due to the fact that the appearing phase patterns in the network contain non-synchronized independent phase states, i.e. we had to continuously observe how phase differences are changing not just checking phase states occasionally.

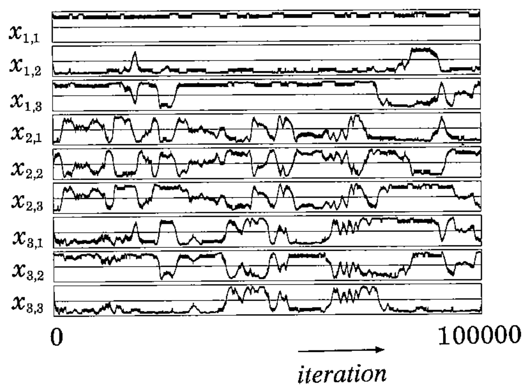


Fig. 4 Time evolution of the Poincaré map (Computer simulated results). $\alpha = 24.0$, $\beta = 0.28$ and $\gamma = 0.30$.

In this study, we construct the coupled chaotic circuits network by real circuit elements and try to observe the irregular self-switching phenomenon of the phase pattern as real physical phenomenon. In order to decide the appearing phase patterns, the task was very difficult even in computer calculation, we designed a simple circuit deciding the patterns in a simple way.

4. Decision Circuit

The appearing phase patterns are decided by checking sums of several voltages obtained from subcircuits. We add some of generated voltages $\hat{v}_{m,n}$ from the subcircuits as (I)–(IV) in Fig. 5. For example, we add $\hat{v}_{1,1}$, $\hat{v}_{2,1}$, $\hat{v}_{2,2}$, $\hat{v}_{2,3}$, $\hat{v}_{3,2}$ and $\hat{v}_{3,3}$ together by an adder as (I) in Fig. 5. If phase pattern V_1 appears at this moment, $\hat{v}_{1,1} + \hat{v}_{2,1} + \hat{v}_{2,2} + \hat{v}_{2,3} + \hat{v}_{3,2} + \hat{v}_{3,3}$ is $A + \bar{A} + \bar{B} + \bar{A} + A + B$ and the sum should be nearly zero. However, if phase pattern X_1 or L_3 appears, the sum leaves $A + \bar{B}$ or $A + B$ and should be non-zero. Namely, we can decide the pattern by judging whether the sum of suitable voltage combinations is zero or not.

Figure 6 shows the block diagram coding the phase patterns to 4-bit digital signal. At first, we take some suitable several voltages from the subcircuits and we add them together using adders. Next, we pass the sum through the absolute value circuits and the comparators in order to judge whether the result is zero or not. In the case that the sum is not zero, the output of the adder crosses the threshold value repeatedly even if pattern does not change. It causes that the output of the comparator oscillates violently between zeros and ones. Then, we pass the output of the comparator through the RC filter. Finally, the output of the RC filter is coded to a digital signal by using the Schmitt trigger.

The decision circuit coding phase patterns to digital signal is shown in Fig. 7. The results of circuit experiments are shown in Fig. 8. In Fig. 8 signals (I)–(IV) correspond to the sum of the voltages (I)–(IV) in Fig. 5. Figure 8 (a) shows the output of the adders. Although

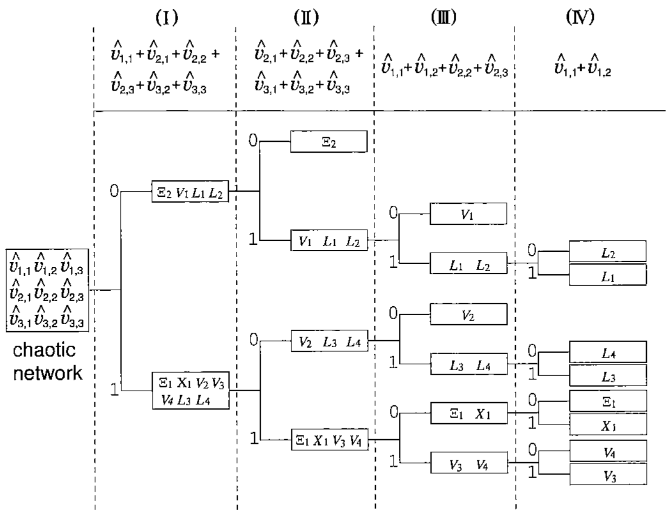


Fig. 5 Decision of appearing phase patterns.

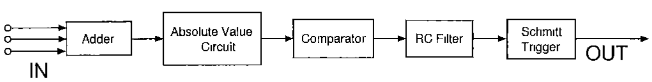


Fig. 6 Block diagram coding phase patterns to digital signal.

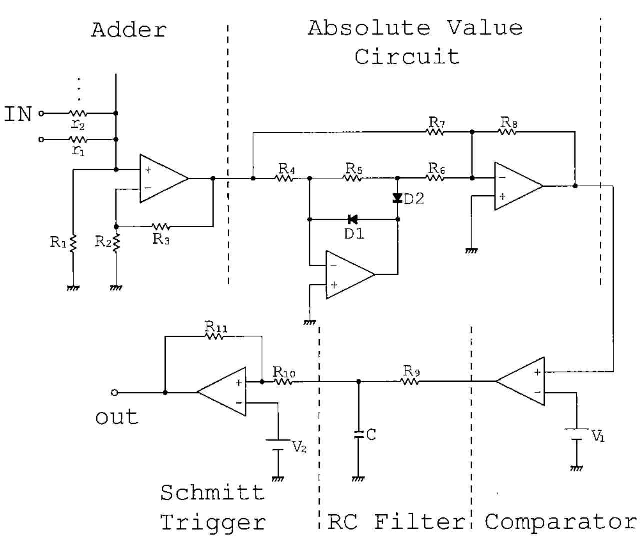


Fig. 7 Decision circuit coding phase patterns to digital signal.

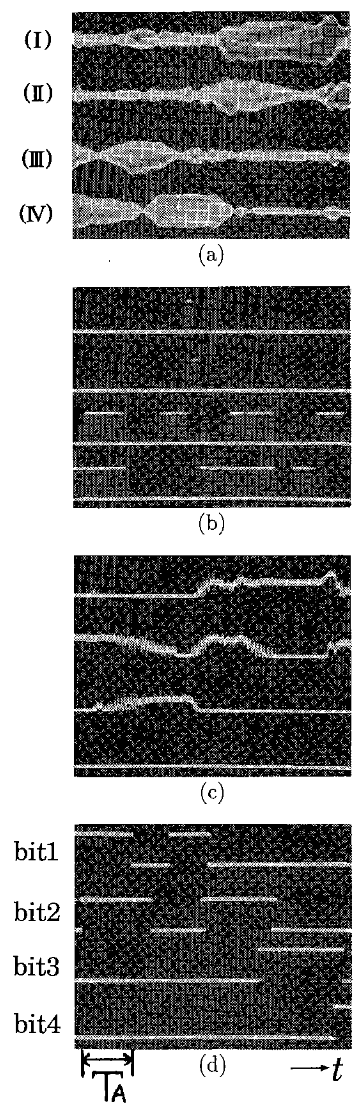


Fig. 8 Experimental results. $r = 250\,\Omega$, $R = 1120\,\Omega$, Vertical: 0.5 V and Horizontal: 20 ms/div. (a) Output of adders. (b) Output of comparators. (c) Output of RC filters. (d) Output of Schmitt triggers.

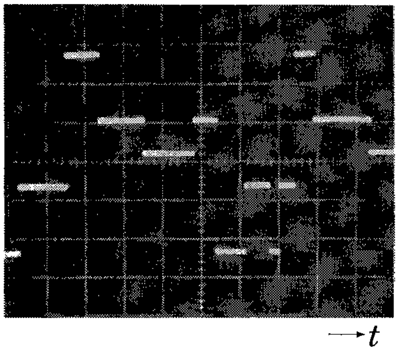


Fig. 9 D/A converted result of the digital signal in Fig. 8 (d).

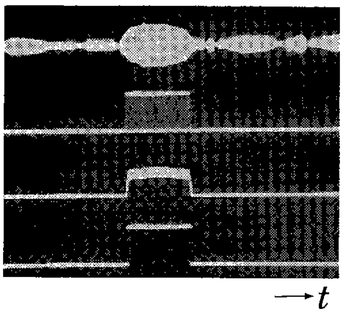


Fig. 10 Decision process (experimental results).

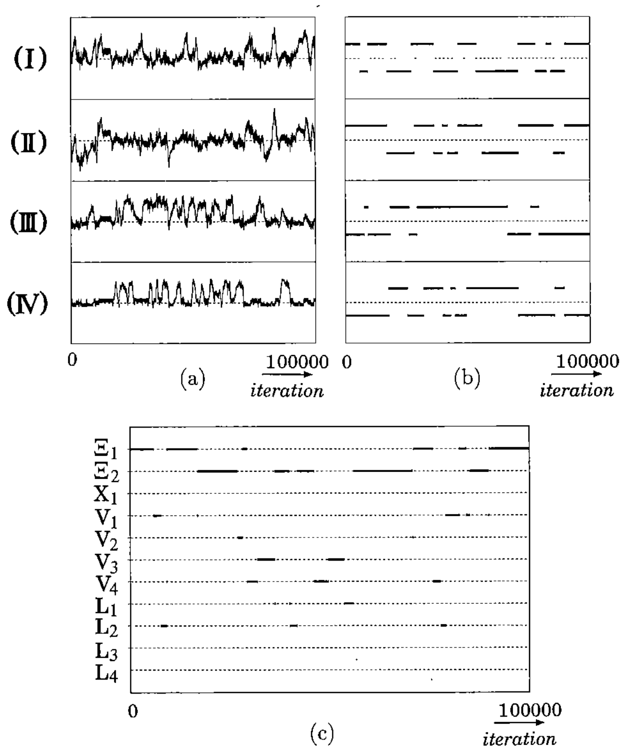


Fig. 11 Computer simulated results. $\alpha = 24.0$, $\beta = 0.2805$ and $\gamma = 0.50$. (a) Sum of the voltages. (b) Coded 4-bit digital signal. (c) Converted 16-level analog signal.

it is difficult to see the presence of oscillation because of the reduced horizontal axis, the sums oscillate violently during it is not zero. Because the quasi-synchronization of chaos is not complete, the sums of the voltages are not completely zero during it should be judged as zero. However, we can judge from the signal whether the sum should be zero or not. Figure 8 (b) shows the output of the comparators. We can see that the output oscillates violently between zeros and ones during it is not zero. Figure 8 (c) shows the output of the RC filters. Finally, Fig. 8 (d) shows the output of the Schmitt triggers and it is the coded 4-bit digital signal. We can confirm that several spatial patterns appear in an irregular manner. For example, for the interval T_A, the 4-bit signal is as (1,1,0,0). Hence, the appearing pattern is decided as Ξ_1 according to Fig. 5. Figure 9 shows the D/A converted result, i.e. 16-level analog signal converted from the 4-bit digital signal. We would like to emphasize again that this is the first result on irregular self-switching phenomenon observed in laboratory experiments using real analog chaotic circuits.

Further, in order to show the decision process more clearly, the outputs of the adder, the comparator, the RC filter and the Schmitt trigger are shown in Fig. 10 at the same time.

The result by computer simulation is shown in Fig. 11. Figure 11 (a) shows the added signals according to Fig. 5. Figure 11 (b) shows the coded 4-bit digital signal and Fig. 11 (c) shows the converted 16-level analog signal.

As a result, we could code spatial patterns observed from the network of the 3×3 size to 4-bit digital signal and we could confirm the generation of irregular self-switching phenomenon of 11 kinds of phase patterns from real analog circuit.

5. Conclusions

In this study, we have realized a coupled chaotic circuits network with 3×3 size and have carried out circuit experiments. By designing a simple circuit converting generating spatial patterns to 4-bit digital signal, we confirmed irregular self-switching phenomenon of the appearing spatial patterns.

As for our future research, we will try statistical analysis of the switching phenomenon by processing the obtained digital information.

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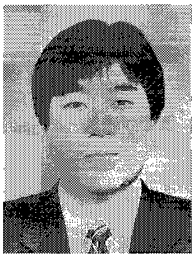

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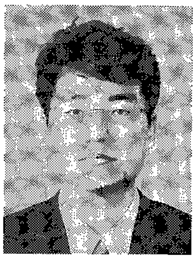

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