PAPER *Special Section on VLSI Design and CAD Algorithms*

Analysis of Nonuniform Transmission Lines Using Chebyshev Expansion Method and Moment Techniques

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SUMMARY *Nonuniform transmission lines* are crucial in integrated circuits and printed circuit boards, because these circuits have complex geometries and layout between the multi layers, and most of the transmission lines possess nonuniform characteristics. In this article, an efficient numerical method for analyzing nonuniform transmission lines has been presented by using the Chebyshev expansion method and moment techniques. Efficiency on computational cost is demonstrated by numerical example.
key words: *nonuniform transmission lines, asymptotic waveform evaluation (AWE) method, moment techniques, Chebyshev expansion method*

1. Introduction

The high-speed performance of microwave or digital circuit systems is limited by the interconnect effects rather than the switching speed of semiconductor devices. To achieve high performance and high packing density, the interconnect scheme for multiple metal layers becomes complicated. The transmission lines are considered as having nonuniform configuration. Therefore, the analysis of nonuniform transmission lines system is important for designing the high performance systems on VLSI circuits, printed circuit boards and multi-chip modules.

Since systems such as VLSI circuits contain very large number of lumped elements and interconnects (uniform/nonuniform transmission lines), a computationally effective method is required for their analysis. The asymptotic waveform evaluation (AWE) method satisfies with such requirements, because the computation speed is two to three orders of magnitude faster than conventional circuit simulators such as SPICE. This method is originally proposed for timing analysis of linear lumped circuits [1]. Due to its efficiency, many researchers pay much attention on the AWE method or moment techniques [2]–[6]. However, for the analysis of distributed networks, the configuration of transmission lines is simplified to be uniform.

In this article, we extend the AWE method to the analysis of nonuniform transmission lines system and discuss fast estimation of transient behavior in the system. In the previous work [7], the calculation procedure is based on the Chebyshev expansion method. In this case, the responses on the spatial space are approximated in the time-domain by using Chebyshev polynomial, and the Telegrapher's equation of nonuniform transmission lines are replaced with a set of ordinary differential equations of which variables are coefficients of Chebyshev polynomials. As a result, the responses are calculated by numerical integration of the ordinary differential equations. In this work, the responses are approximated in the Laplace-domain different from original Palusinski's work [7]. It makes the moment generation [3] possible for nonuniform transmission lines system, and the responses in the Laplace-domain are transformed into the time-domain by the moment matching technique [1]–[3] in the AWE method. Our method is effective in achieving high computational efficiency within acceptable accuracy.

In Sect. 2, we will review briefly the moment generation for the Modified Nodal Analysis (MNA) equation in order to generalize the transmission lines system. In Sect. 3, the calculation procedure for the moments of nonuniform transmission lines is provided by means of the Chebyshev expansion method. In Sect. 4, the efficiency of the proposed method is shown by a numerical example, and we will conclude in Sect. 5.

2. Moment Generation

Details of moment techniques in the AWE method are described in [1] and [2]. In this article, our aim is mainly how to obtain the moments of nonuniform transmission lines. Then we will review briefly moment generation for MNA matrix equations of transmission line systems.

Consider a linear circuit containing lumped components and some uniform/nonuniform multiconductor transmission line systems. Without loss of generality, the frequency-domain MNA matrix Eq. [8] for the circuit can be written as [2]

$$\mathbf{Y}(s)\mathbf{X}(s) = \mathbf{E} \tag{1}$$

where $\mathbf{Y}(s)$ is a MNA matrix, $\mathbf{X}(s)$, $\mathbf{E}$ are an unknown vector and input excitations, respectively. The ports relation of the kth transmission line system in $\mathbf{Y}(s)$ is described by

$$\mathbf{A}_k(s)\mathbf{V}_k(s) + \mathbf{B}_k(s)\mathbf{I}_k(s) = \mathbf{0} \tag{2}$$

where $\mathbf{V}_k(s)$ and $\mathbf{I}_k(s)$ are the Laplace-domain terminal voltage and currents of the kth transmission line system.

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To evaluate the moments of (1), we assume that the coefficient matrices $\mathbf{A}_k(s)$, $\mathbf{B}_k(s)$ of (2) are described by Taylor's series at $s = 0$,

$$\mathbf{A}_k(s) = \sum_{i=0}^{\infty} \mathbf{A}_{k,i} s^i, \tag{3.a}$$

$$\mathbf{B}_k(s) = \sum_{i=0}^{\infty} \mathbf{B}_{k,i} s^i. \tag{3.b}$$

From (3.a), (3.b), the MNA matrix $\mathbf{Y}(s)$ is described as follows:

$$\mathbf{Y}(s) = \sum_{i=0}^{\infty} \mathbf{Y}_i s^i. \tag{4}$$

In the case of lumped components only, $\mathbf{Y}(s)$ becomes a first degree polynomial matrix of s. The solutions $\mathbf{X}(s)$ of (1) are also approximated by the Taylor's series:

$$\mathbf{X}(s) = \sum_{i=0}^{\infty} \mathbf{M}_i s^i \tag{5}$$

where $\mathbf{M}_i$ is the moment vector of $\mathbf{X}(s)$ [1]. Substituting (4) and (5) into (1)

$$\begin{aligned}&\left(\mathbf{Y}_0 + \mathbf{Y}_1 s + \mathbf{Y}_2 s^2 + \cdots\right)\\ &\quad \cdot \left(\mathbf{M}_0 + \mathbf{M}_1 s + \mathbf{M}_2 s^2 + \cdots\right) = \mathbf{E}.\end{aligned} \tag{6}$$

By matching the corresponding powers of s, the recursive relationship for the moments can be derived in the form

$$\mathbf{Y}_0 \mathbf{M}_0 = \mathbf{E}, \tag{7.a}$$

$$\mathbf{Y}_0 \mathbf{M}_i = -\sum_{r=1}^{i} \mathbf{Y}_r \mathbf{M}_{i-r} \quad (i \geqq 1). \tag{7.b}$$

The moment generation on the MNA matrix Eq. (1) is reduced to obtain the ports relation of the transmission line system (2) and its Taylor's series (3.a), (3.b). In the case of uniform transmission lines, an eigenvalue method [2] and matrix exponential method [3] are reported to be efficient. However, if the circuit contains the nonuniform transmission lines, the relation cannot be obtained by only these methods. Thus a new method for nonuniform transmission lines is proposed in the next section.

3. Nonuniform Transmission Lines

3.1 Chebyshev Expansion Method

Consider a system of N transmission lines, where the parameters per unit length are given by $\mathbf{R}(z)$, $\mathbf{L}(z)$, $\mathbf{C}(z)$ and $\mathbf{G}(z)$ which have nonuniform and frequency-independent characteristics and are expressed as functions of a distance z from the input terminals. To analyze the system, a method using Chebyshev polynomials interpolation is reported in [7]. Due to tedious matrix operation, this method seems to be complicated. Thus, we reformulate it appropriately without loss of simplicity and generality. Our derivation is essentially the same as that proposed by Palusinski et al. [7] where the responses in the time-domain are assumed by Chebyshev polynomials. However we assume the responses in the Laplace-domain, making moment generation possible for nonuniform transmission lines.

The Telegrapher's equation in the Laplace-domain is described in the form

$$-\frac{d}{dz}\begin{bmatrix}\mathbf{V}(s,z)\\ \mathbf{I}(s,z)\end{bmatrix} = [\mathbf{f}(z) + s\mathbf{h}(z)]\begin{bmatrix}\mathbf{V}(s,z)\\ \mathbf{I}(s,z)\end{bmatrix} \tag{8}$$

where

$$\mathbf{f}(z) = \begin{bmatrix}\mathbf{0} & \mathbf{R}(z)\\ \mathbf{G}(z) & \mathbf{0}\end{bmatrix},$$

$$\mathbf{h}(z) = \begin{bmatrix}\mathbf{0} & \mathbf{L}(z)\\ \mathbf{C}(z) & \mathbf{0}\end{bmatrix}.$$

First, $z \in [0, l]$ (l is a length of the lines) is converted into $x \in [-1, 1]$, because the Chebyshev polynomials are defined in this interval. Using the transform

$$x = \frac{2}{l} z - 1, \tag{9}$$

the Telegrapher's Eq. (8) is rewritten as

$$-\frac{d}{dx}\begin{bmatrix}\mathbf{V}(s,x)\\ \mathbf{I}(s,x)\end{bmatrix} = \frac{l}{2}[\mathbf{f}(x) + s\mathbf{h}(x)]\begin{bmatrix}\mathbf{V}(s,x)\\ \mathbf{I}(s,x)\end{bmatrix}. \tag{10}$$

Below, the coefficient $l/2$ in the above equation has been dropped for simplicity.

The voltage and current waveforms $[\mathbf{V}(s,x), \mathbf{I}(s,x)]^T$ are modeled by N-1 degree Chebyshev polynomials

$$\begin{bmatrix}\mathbf{V}(s,x)\\ \mathbf{I}(s,x)\end{bmatrix} = \sum_{m=0}^{N-1}{}' \, \mathbf{p}_m(s) \cos m\theta \tag{11}$$

where the symbol $\sum'$ denotes the summation with the first component divided by 2, according to the notation in [7]. Similarly, the derivatives with respect to spatial variable x are described by

$$\frac{d}{dx}\begin{bmatrix}\mathbf{V}(s,x)\\ \mathbf{I}(s,x)\end{bmatrix} = \sum_{m=0}^{N-2}{}' \, \mathbf{p}^{\star}_m(s) \cos m\theta, \tag{12}$$

where

$$\mathbf{p}^{\star}_m(s) = \sum_{k=m+1}^{N-1} k u_{k-m} \mathbf{p}_k \tag{13}$$

$$u_{k-m} = \begin{cases} 2 & \text{(k-m: odd)}\\ 0 & \text{(k-m: even).}\end{cases}$$

Here the derivatives are N-2 degree Chebyshev polynomials.

By substituting (11) and (12) into (10), and introducing an operator $\mathcal{P}_{N-2}$

$$
\begin{aligned}
-\sum_{m=0}^{N-2}{}' \mathbf{p}_m^{\star}(s)\cos m\theta &= \mathcal{P}_{N-2}\left\{[\mathbf{f}(x)+s\mathbf{h}(x)]\sum_{k=0}^{N-1}{}' \mathbf{p}_k(s)\cos k\theta\right\} \\
&= \sum_{m=0}^{N-2}{}' \varphi_m(s)\cos m\theta \qquad (14)
\end{aligned}
$$

where $\mathcal{P}_{N-2}$ is an operator which expresses the Chebyshev polynomials discarding the terms of a higher order than N-2. The coefficients $\varphi_m(s)$ of (14) can be evaluated by using the zeros of N degree Chebyshev polynomial $\cos N\theta$,

$$
\theta_i = \frac{2i+1}{2N}\pi \qquad (i = 0, 1, \ldots, N-1), \qquad (15)
$$

namely,

$$
\begin{aligned}
\varphi_m(s) &= \frac{2}{N}\sum_{i=0}^{N-1}[\mathbf{f}(\cos\theta_i)+s\mathbf{h}(\cos\theta_i)] \\
&\quad \cdot\sum_{k=0}^{N-1}{}' \mathbf{p}_k(s)\cos k\theta_i\cos m\theta_i \\
&= \sum_{k=0}^{N-1}{}' \{(\mathbf{f}_{k+m}+\mathbf{f}_{|k-m|}) \\
&\quad +s(\mathbf{h}_{k+m}+\mathbf{h}_{|k-m|})\}\,\mathbf{p}_k(s) \qquad (16)
\end{aligned}
$$

where

$$
\mathbf{f}_j = \begin{cases} \dfrac{1}{N}\displaystyle\sum_{i=0}^{N-1}\mathbf{f}(\cos\theta_i)\cos j\theta_i & (0 \leqq j < \mathrm{N}) \\ \mathbf{0} \quad (j = N) \\ -\mathbf{f}_{2N-j} & (\mathrm{N}+1 \leqq j \leqq 2\mathrm{N}\text{-}3) \end{cases}
$$

$$
\mathbf{h}_j = \begin{cases} \dfrac{1}{N}\displaystyle\sum_{i=0}^{N-1}\mathbf{h}(\cos\theta_i)\cos j\theta_i & (0 \leqq j < \mathrm{N}) \\ \mathbf{0} \quad (j = \mathrm{N}) \\ -\mathbf{h}_{2N-j} & (\mathrm{N}+1 \leqq j \leqq 2\mathrm{N}\text{-}3). \end{cases}
$$

The fast Fourier transform (FFT) or discrete Fourier transform (DFT) are available for the computation of $\mathbf{f}_j$ and $\mathbf{h}_j$. As a boundary condition, we have

$$
\begin{bmatrix} \mathbf{V}(s,l) \\ \mathbf{I}(s,l) \end{bmatrix} = \sum_{k=0}^{N-1}{}' \mathbf{p}_k(s). \qquad (17)
$$

By matching the corresponding $\cos m\theta$ ($m = 0, 1, \ldots, N-2$) of (14) and combining them with (17), the nonuniform transmission lines are described in terms of the coefficients of the Chebyshev polynomial. It is written in the matrix form

$$
\{(\mathbf{D}+\mathbf{F})+s\mathbf{H}\}\,\mathbf{P}(s) = \mathbf{T}_1\begin{bmatrix} \mathbf{V}(s,l) \\ -\mathbf{I}(s,l) \end{bmatrix} \qquad (18)
$$

where

$$
\mathbf{P}(s) = (\ \mathbf{p}_0\ \ \mathbf{p}_1\ \ \cdots\ \ \mathbf{p}_{N-1}\ )^T
$$

$$
\mathbf{T}_1 = (\ \mathbf{0}\ \ \cdots\ \ \mathbf{0}\ \ \mathbf{U}\ )^T
$$

$$
\mathbf{U} = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & -\mathbf{I} \end{bmatrix}
$$

and $\mathbf{T}_1 \in R^{2N^2\times 2N}$, and $\mathbf{I} \in R^{N\times N}$ is the identity matrix. To simplify the notations of $\mathbf{D}$, $\mathbf{F}$, and $\mathbf{H}$, the following block matrix form of $\mathbf{D}$ is introduced:

$$
\mathbf{D} = \begin{bmatrix} \mathbf{D}_{0,0} & \mathbf{D}_{0,1} & \cdots & \mathbf{D}_{0,N-1} \\ \mathbf{D}_{1,0} & \mathbf{D}_{1,1} & \cdots & \mathbf{D}_{1,N-1} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{D}_{N-1,0} & \mathbf{D}_{N-1,1} & \cdots & \mathbf{D}_{N-1,N-1} \end{bmatrix}.
$$

$\mathbf{F}$, $\mathbf{H}$ are described in the same manner. From (13), (16) and (17), we have

$$
\mathbf{D}_{m,k} = \begin{cases} \mathbf{0} & (m \neq \mathrm{N}\text{-}1,\ 0 \leqq k \leqq m) \\ 2k\mathbf{I}' & (m \neq \mathrm{N}\text{-}1,\ \text{k-m: odd}) \\ \mathbf{0} & (m \neq \mathrm{N}\text{-}1,\ \text{k-m: even}) \\ \mathbf{0} & (m = \mathrm{N}\text{-}1) \end{cases} \qquad (19)
$$

$$
\mathbf{F}_{m,k} = \begin{cases} \mathbf{f}_m & (m \neq \mathrm{N}\text{-}1,\ k = 0) \\ \mathbf{f}_{k+m}+\mathbf{f}_{|k-m|} & (m \neq \mathrm{N}\text{-}1,\ k \neq 0) \\ \mathbf{I}'/2 & (m = \mathrm{N}\text{-}1,\ k = 0) \\ \mathbf{I}' & (m = \mathrm{N}\text{-}1,\ k \neq 0) \end{cases} \qquad (20)
$$

$$
\mathbf{H}_{m,k} = \begin{cases} \mathbf{h}_m & (m \neq \mathrm{N}\text{-}1,\ k = 0) \\ \mathbf{h}_{k+m}+\mathbf{h}_{|k-m|} & (m \neq \mathrm{N}\text{-}1,\ k \neq 0) \\ \mathbf{0} & (m = \mathrm{N}\text{-}1) \end{cases} \qquad (21)
$$

where $\mathbf{I}' \in R^{2N\times 2N}$ is the identity matrix.

The voltage and current waveforms at the input terminal (z=0) are given by

$$
\begin{aligned}
\begin{bmatrix} \mathbf{V}(s,0) \\ \mathbf{I}(s,0) \end{bmatrix} &= \sum_{k=0}^{N-1}{}' (-1)^k\mathbf{p}_k(s) \\
&= \mathbf{T}_2\mathbf{P}(s) \qquad (22)
\end{aligned}
$$

where

$$
\mathbf{T}_2 = \left(\ \frac{1}{2}\mathbf{I}'\ \ -\mathbf{I}'\ \ \mathbf{I}'\ \ \cdots\ \ (-1)^{N-1}\,\mathbf{I}'\ \right).
$$

Combining (18) and (22), the nonuniform transmission lines are expressed in the chain matrix form:

$$\begin{bmatrix} \mathbf{V}(s,0) \\ \mathbf{I}(s,0) \end{bmatrix} = \mathbf{\Phi}(s) \begin{bmatrix} \mathbf{V}(s,l) \\ -\mathbf{I}(s,l) \end{bmatrix} \tag{23}$$

where

$$\begin{aligned} \mathbf{\Phi}(s) &= \begin{bmatrix} \mathbf{\Phi}_{11} & \mathbf{\Phi}_{12} \\ \mathbf{\Phi}_{21} & \mathbf{\Phi}_{22} \end{bmatrix} \\ &= \mathbf{T}_2 \left\{ (\mathbf{D}+\mathbf{F}) + s\mathbf{H} \right\}^{-1} \mathbf{T}_1. \end{aligned} \tag{24}$$

Finally, by rearranging (23), the relationship (2) for the nonuniform transmission lines is obtained by

$$\begin{aligned} &\begin{bmatrix} -\mathbf{I} & \mathbf{\Phi}_{11} \\ \mathbf{0} & \mathbf{\Phi}_{21} \end{bmatrix} \begin{bmatrix} \mathbf{V}(s,0) \\ \mathbf{V}(s,l) \end{bmatrix} \\ &\quad + \begin{bmatrix} \mathbf{0} & \mathbf{\Phi}_{12} \\ -\mathbf{I} & \mathbf{\Phi}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{I}(s,0) \\ -\mathbf{I}(s,l) \end{bmatrix} = \mathbf{0}. \end{aligned} \tag{25}$$

3.2 Moment Computation

To evaluate the moments (7.a), (7.b), the coefficient matrices of (25) must be expanded at $s=0$, such as (3.a), (3.b). Form (24), (25), it is attained by expanding $\{(\mathbf{D}+\mathbf{F})+s\mathbf{H}\}^{-1}\mathbf{T}_1$ in (24) at $s=0$:

$$\left\{ (\mathbf{D}+\mathbf{F}) + s\mathbf{H} \right\}^{-1} \mathbf{T}_1 = \sum_{i=0}^{\infty} \mathbf{C}_i \mathbf{T}_1 s^i. \tag{26}$$

Put

$$\left\{ (\mathbf{D}+\mathbf{F})+s\mathbf{H} \right\} \left(\mathbf{C}_0+\mathbf{C}_1 s+\mathbf{C}_2 s^2+\cdots \right) \mathbf{T}_1 = \mathbf{T}_1. \tag{27}$$

By matching the corresponding powers of s, the recursive relationship can be derived in the form

$$\mathbf{C}_0\mathbf{T}_1 = (\mathbf{D}+\mathbf{F})^{-1}\mathbf{T}_1, \tag{28.a}$$

$$\mathbf{C}_i\mathbf{T}_1 = -(\mathbf{D}+\mathbf{F})^{-1}\mathbf{H}\mathbf{C}_{i-1}\mathbf{T}_1 \quad (i \geqq 1). \tag{28.b}$$

Then, the chain matrix $\mathbf{\Phi}(s)$ is described as

$$\mathbf{\Phi}(s) = \sum_{i=0}^{\infty} \mathbf{T}_2 \mathbf{C}_i \mathbf{T}_1 s^i. \tag{29}$$

4. Numerical Example

To show the efficiency of our algorithm, we consider a double line prototype chip interconnect shown in Fig. 1 (a). The parameter matrices are listed in Table 2 of Ref. [7], and we assume that these matrices are piecewise linear in a distance x. The transient responses to the input voltage shown in Fig. 1 (b) are computed by using the proposed method, introducing the moment matching technique [1]–[3]. For comparison with the proposed method, these responses are also computed by using the FFT method for the nonuniform transmission lines [9] and SPICE3 with the lumped model. In the FFT method, calculating the time-domain response requires convolution integral except for analyzing pulse response [9]. The time step size of convolution integral is selected as (analysis time)/(2 * frequency components). Waveforms in Figs. 2 and 3 are respectively active and quiescent lines responses at the far-end of the transmission lines. The proposed method gets almost the same results as those of SPICE3 and the FFT method (oscillations around 0.17 nano seconds in the FFT's results of Fig. 3 are Gibs phenomena). The CPU time of each method on a Sun SPARK station 5 are listed in Table 1. It is confirmed that the proposed method provides high computational efficiency when compared with the FFT method and SPICE3.

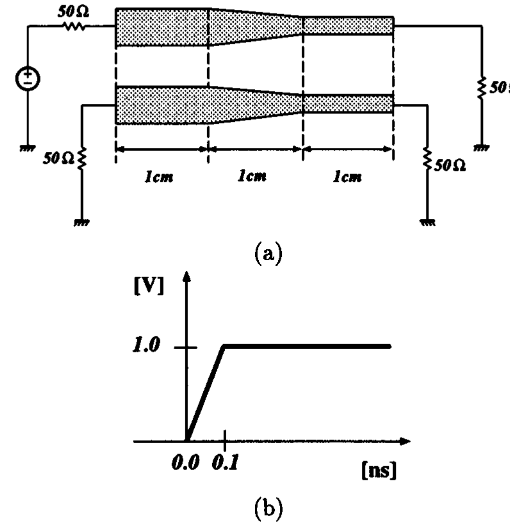

Fig. 1 (a)Nonuniform transmission lines.(b)Input waveform.

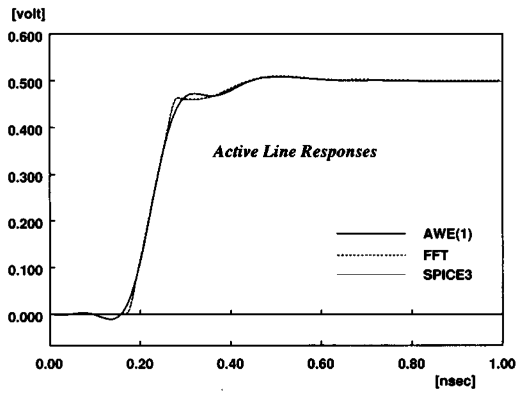

Fig. 2 Transient responses at the active line.

In spite of the great computational efficiency of the proposed method, the waveform in Fig. 3 does not completely coincide with the results of the FFT method and SPICE3. To consider how such inaccuracy of the proposed method generates, the results are also compared with the Chebyshev expansion method in the time-

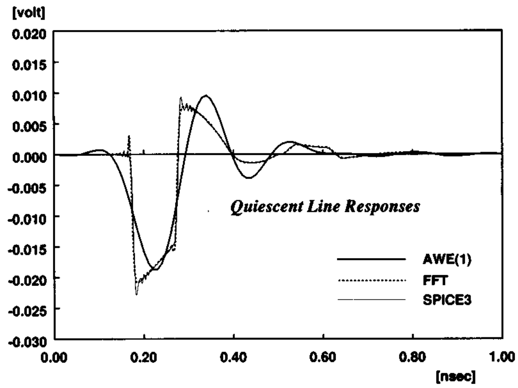

Fig. 3 Transient responses at the quiescent line.

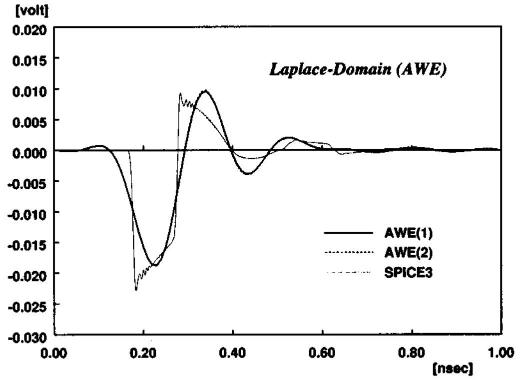

Fig. 4 Transient responses at the quiescent line computed by the Chebyshev expansion in Laplace-domain (AWE).

Table 1 CPU time of numerical example.

Mehtod	Condition	CPU time [sec.]
AWE(1)	AWE order: 10 Chebyshev degree: 8	0.18
FFT	frequency components: 128 line division number: 200	12.12
SPICE3	time step number: 2000 line division number: 200	82.34
AWE(2)	AWE order: 10 Chebyshev degree: 64	6.89
BDF(1)	time step number: 2000 Chebyshev degree: 8	2.42
BDF(2)	time step number: 2000 Chebyshev degree: 64	199.99

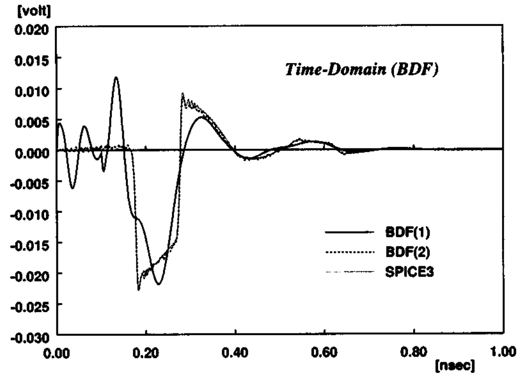

Fig. 5 Transient responses at the quiescent line computed by the Chebyshev expansion method in time-domain (BDF).

domain, Palusinski's work [7]. Waveforms in Fig. 4 are the responses computed by the proposed method with different degree of the Chebyshev polynomials. Waveforms in Fig. 5 are the results in the time-domain, where the Backward Differential Formula (BDF) [10] is used as the solution of the ordinary differential equations of which variables are coefficients of the Chebyshev polynomials [7]. From Figs. 4 and 5, we can see that the results by the proposed method with 8-degree Chebyshev polynomials are the same with 64-degree, whereas the results in time-domain with 64-degree are coincide with SPICE3. The reason is that the AWE method extracts only low frequency portions of a waveform, precisely, the neighboring poles and residues of the function from the origin $s = 0$, because the kernel of Taylor's expansion (3.a), (3.b) is the origin $s = 0$. The order of the AWE method in the results of Fig. 4 is fixed, and the same poles and residues are found by the AWE method, even if the degree of Chebyshev polynomial is different. In other words, the inaccuracy shown in Fig. 3 is due to lack of the higher frequency components or the moments. However, we cannot take more moments, because moment techniques are very sensitive to round-off error. Such disadvantage of the AWE method has been also suggested by other researchers [3]–[5], and they improved it. In [3]–[5], the transfer function is expanded at any point $s = s_0$, and a heuristic procedure so-called complex frequency hopping in [3], a padé approximation via the Lanczos process in [4], a multipoint padé approximation in [5] were introduced to take more moments. Fortunately, the method proposed in Sect. 3 can be incorporated with these methods. Then, (24) is replaced with

$$\mathbf{T}_2 \left\{(\mathbf{D} + \mathbf{F} + s_0\mathbf{H}) + \sigma\mathbf{H}\right\}^{-1} \mathbf{T}_1$$

where $\sigma = s - s_0$. $\mathbf{\Phi}(s)$ of (24) is expanded at $s = s_0$ and described by a power series of σ.

In general, the transfer function of the transmission lines becomes complicated, increasing the frequency. It implies that on the spatial space a few degree of Chebyshev polynomials are enough to approximate a Laplace function in the region covered by the AWE method. This is because the AWE method estimates only the low frequency portions of the waveform. In the time-domain method [7], the accuracy significantly depends on the degree of Chebyshev polynomials, and the analysis with a few degree is no longer meaningful as shown in Fig. 5.

Therefore, extracting low frequency portions only in the AWE method has advantage on the computational point of view, while the accuracy is degraded, because small degree Chebyshev polynomials imply small order matrices **D**, **F**, **H** of (18)

5. Conclusions and Remarks

We have presented an algorithm for calculating the responses of nonuniform transmission lines with the Chebyshev expansion method and moment techniques. Our method is effective in achieving high speed computation.

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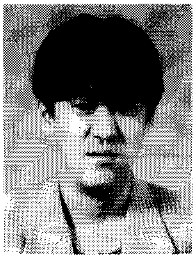

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