PAPER

Analysis of a Coupled Chaotic System Containing Circuits with Different Oscillation Frequencies

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SUMMARY In this study, we show how changing a frequency in one of N chaotic circuits coupled by a resistor effects our system by means of both circuit experiment and computer calculation. In these N chaotic circuits, $N-1$ circuits are completely identical, and the remaining one has altered the value of the oscillation frequency. It is found that for the case of $N=3$ when a value of a coupling resistor is gradually increased, only one circuit with different frequency exhibits bifurcation phenomena including inverse period-doubling bifurcation, and for larger value of coupling resistor, the chaotic circuit with different frequency suddenly stops oscillating and the remaining two chaotic circuits exhibit completely anti-phase synchronization.
key words: *chaos, synchronization of chaos, coupled oscillators, bifurcation phenomena, oscillation death*

1. Introduction

Recently, studies concerned with synchronization of chaos appearing in coupled chaotic circuits have been drawing many researchers' attentions. Also, studies concerned with the collapse of synchronization of chaos have been carried out [1]–[3]. However, not many studies concerned with coupled chaotic circuits with different frequencies have been done. Coupled chaotic systems containing circuits with different frequencies can be models of various phenomena existing in natural field. So, it seems to be important to study how changing oscillation frequencies affects synchronization phenomena of chaos in order to understand the phenomena appearing in natural and real physical systems.

On the other hand, we have studied synchronization phenomena appearing in a coupled system of van der Pol oscillators with different frequencies, and have reported that oscillation of an oscillator stops [4].

In this study, we investigate how changing a frequency in one of N chaotic circuits coupled by a resistor affects our system, by means of both circuit experiment and computer calculation. By the way, for the case that two chaotic circuits with different frequencies are coupled, these two circuits will fight each other. As the result, it will be seen either they do not synchronize or they exhibit drawing phenomenon. So we do not refer to this case. It was found that for the case of $N=3$ when a value of a coupling resistor is gradually increased, only one circuit with different frequency exhibits bifurcation phenomena including inverse period-doubling bifurcation, and for larger value of coupling resistor, the chaotic circuit with different frequency suddenly stops oscillating and the remaining two chaotic circuits exhibit completely anti-phase synchronization. Further we found that by adjusting initial conditions all circuits exhibit limit-cycle and two circuits exhibit in-phase synchronization. Further, we also investigate the case of $N=4$.

2. Circuit Model

The circuit model is shown in Fig. 1. In our system, N chaotic circuits are coupled by a resistor. Each chaotic subcircuit is a symmetric version of the circuit model proposed by Inaba [5] and consists of two inductors, a capacitor, a negative linear resistor, and a nonlinear resistor. Attractors observed from each circuit are shown in Fig. 2. In these N chaotic circuits, $N-1$ circuits are completely identical, and the remaining one has altered values of capacitor and negative resistor, being $\hat{C}$ and $-\hat{r}$ respectively. We approximate the $i-v$ characteristics of the nonlinear resistor as

$$v_d(i_k) = \sqrt[3]{r_d i_k}.$$

The equation governing the circuit in Fig. 1 is represented as follows,

$$L_1 \frac{dI_k}{dt} = r(I_k + i_k) - v_k - R\sum_{j=1}^{N} I_j$$

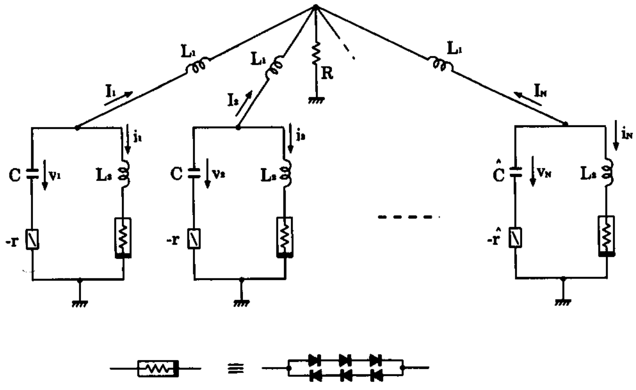

Fig. 1 Circuit model.

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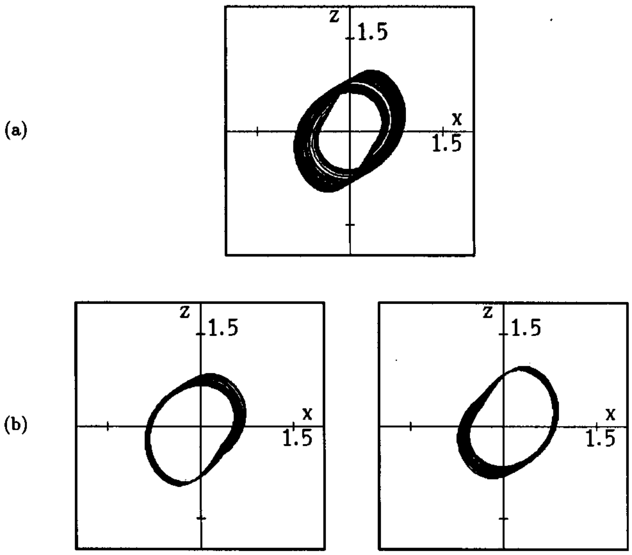

Fig. 2 Example of attractors. (a) symmetric chaos ($\alpha = 20$, $\beta = 0.29$, $\gamma = 0.0$), (b) asymmetric chaos ($\alpha = 20$, $\beta = 0.29$, $\gamma = 0.001$).

$$
\begin{aligned}
L_2\frac{di_k}{dt} &= r(I_k + i_k) - v_k - v_d(i_k)\\
C\frac{dv_k}{dt} &= I_k + i_k \qquad (k = 1, 2, \ldots, N-1)\\
L_1\frac{dI_N}{dt} &= \hat{r}(I_N + i_N) - v_N - R\sum_{j=1}^{N} I_j\\
L_2\frac{di_N}{dt} &= \hat{r}(I_N + i_N) - v_N - v_d(i_N)\\
\hat{C}\frac{dv_N}{dt} &= I_N + i_N. \qquad (1)
\end{aligned}
$$

By changing the variables,

$$
\begin{aligned}
&t = \sqrt{L_1 C}\tau, \quad I_k = V\sqrt{\frac{C}{L_1}}x_k,\\
&i_k = V\sqrt{\frac{C}{L_1}}y_k, \quad v_k = Vz_k,\\
&\alpha = \frac{L_1}{L_2}, \quad \beta = r\sqrt{\frac{C}{L_1}},\\
&\gamma = R\sqrt{\frac{C}{L_1}}, \quad \text{“}\cdot\text{”} = \frac{d}{d\tau},
\end{aligned}
$$

(1) is normalized as

$$
\begin{aligned}
\dot{x}_k &= \beta(x_k + y_k) - z_k - \gamma\sum_{j=1}^{N} x_j\\
\dot{y}_k &= \alpha\{\beta(x_k + y_k) - z_k - f(y_k)\}\\
\dot{z}_k &= x_k + y_k \qquad (k = 1, 2, \ldots, N-1)\\
\dot{x}_N &= \hat{\beta}(x_N + y_N) - z_N - \gamma\sum_{j=1}^{N} x_j\\
\dot{y}_N &= \alpha\{\hat{\beta}(x_N + y_N) - z_N - f(y_N)\}\\
\dot{z}_N &= \delta(x_N + y_N) \qquad (2)
\end{aligned}
$$

where

$$
f(y_k) = \sqrt[3]{y_k}, \quad \hat{\beta} = \hat{r}\sqrt{\frac{C}{L_1}}, \quad \delta = \frac{C}{\hat{C}}.
$$

At first, we consider the case of $C = 0.034\,[\mu\text{F}]$ and $\hat{C} = 0.068\,[\mu\text{F}]$, that is to say the capacitor ratio is 2.0 ($\delta = 0.5$). In this study, we concentrate on the effect on the system due to the difference in the oscillation frequencies, so we need to set the parameter $\hat{\beta}$ so that the same attractors are produced as with β. That is, the following relationship needs to be realized;

$$
r\sqrt{\frac{C}{L_1}} = \hat{r}\sqrt{\frac{2C}{L_1}}.
$$

Then, parameter $\hat{\beta}$ is set as follows

$$
\hat{\beta} = \hat{r}\sqrt{\frac{C}{L_1}} = \frac{r}{\sqrt{2}}\sqrt{\frac{C}{L_1}} = \frac{\beta}{\sqrt{2}}. \qquad (3)
$$

From (2) and (3), we obtain;

$$
\begin{aligned}
\dot{x}_k &= \beta(x_k + y_k) - z_k - \gamma\sum_{j=1}^{N} x_j\\
\dot{y}_k &= \alpha\{\beta(x_k + y_k) - z_k - f(y_k)\}\\
\dot{z}_k &= x_k + y_k \qquad (k = 1, 2, \ldots, N-1)\\
\dot{x}_N &= \frac{\beta}{\sqrt{2}}(x_N + y_N) - z_N - \gamma\sum_{j=1}^{N} x_j\\
\dot{y}_N &= \alpha\left\{\frac{\beta}{\sqrt{2}}(x_N + y_N) - z_N - f(y_N)\right\}\\
\dot{z}_N &= \delta(x_N + y_N). \qquad (4)
\end{aligned}
$$

Strictly speaking, the function corresponding to the diode characteristics $f(y_N)$ also needs to be modified. But it is difficult to modify it in the circuit experiment, and it does not affect the characteristics of attractors largely in comparison with the negative resistor because of its switching operation. Therefore, we ignore the influence in this study.

3. Bifurcation Phenomena

At first, we consider the case of $N = 3$. By setting the parameter $\beta = 0.29$, and changing the parameter γ corresponding to the coupling resistor, attractors in Fig. 3 were obtained by computer calculation. When a value of a coupling resistor is gradually increased, attractors change from Fig. 3 (a) to 3(f). At $\gamma = 0.0$ each circuit exhibits symmetric chaos as in Fig. 2 (a) independently. At $\gamma = 0.01$ (Fig. 3(b3)) the attractor observed from the third circuit changes to asymmetric chaos as in Fig. 2 (b) from symmetric chaos. At $\gamma = 0.02$ asymmetric chaos changes to be almost one periodic Fig. 3 (c3).

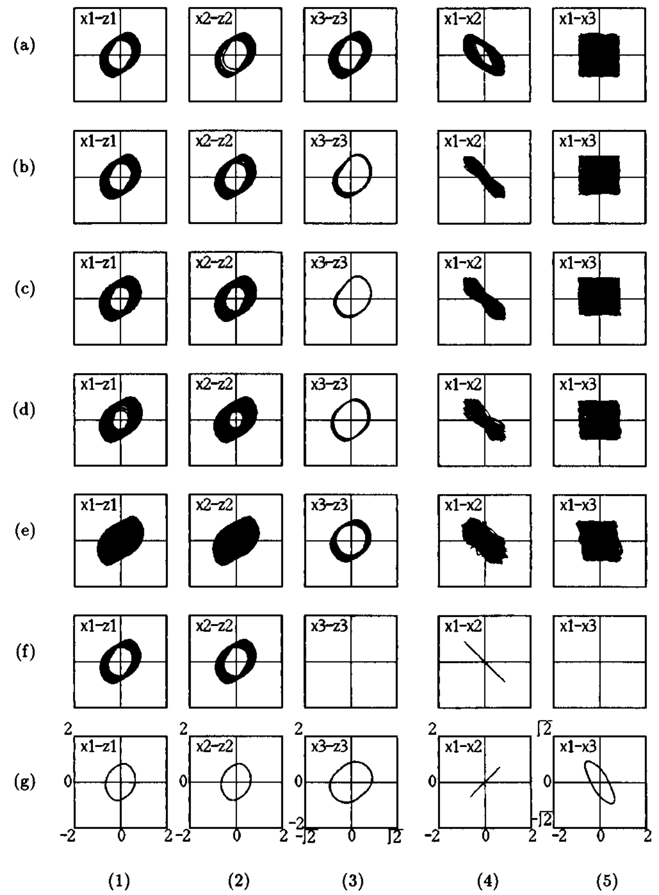

Fig. 3 Result obtained from computer calculation (N = 3). α = 20, β = 0.29, δ = 0.5, (a) γ = 0.0, (b) γ = 0.01, (c) γ = 0.02, (d) γ = 0.065, (e) γ = 0.15, (f) γ = 0.23, (g) γ = 0.3, (1) x_1 vs. z_1, (2) x_2 vs. z_2, (3) x_3 vs. z_3, (4) x_1 vs. x_2, (5) x_1 vs. x_3.

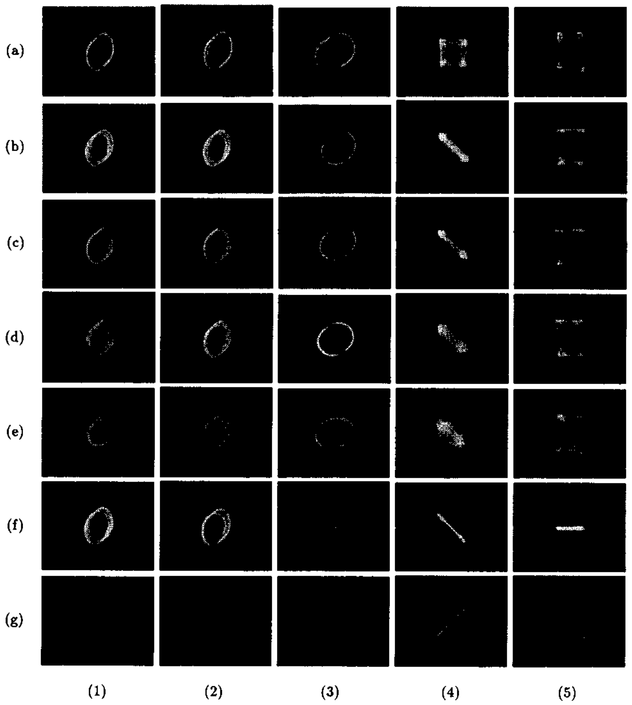

Fig. 4 Attractors obtained from circuit experiment. L_1 = 204 mH, L_2 = 10 mH, C = 0.034 μF, $\hat{C}$ = 0.068 μF, r = 761 Ω, $\hat{r}$ = 556 Ω, (a) R = 0 Ω, (b) R = 30 Ω, (c) R = 53 Ω, (d) R = 161 Ω, (e) R = 300 Ω, (f) R = 500 Ω, (g) R = 7.68 kΩ, (1) I_1 vs. v_1, (2) I_2 vs. v_2, (3) I_3 vs. v_3, (4) I_1 vs. I_2, (5) I_1 vs. I_3.

At $\gamma = 0.065$ the attractor becomes symmetric again. At $\gamma = 0.15$ the attractor grows thicker due to the strong effect of chaotic oscillations of x_1 and x_2. Finally, at $\gamma = 0.23$ x_3 stops oscillating. From Fig. 3 (5) we can see that x_3 is not synchronized to the others. From Fig. 3 (4) we can see that x_1 and x_2 exhibit almost anti-phase synchronization and that they are completely synchronized after x_3 stops oscillating. The one-periodic attractor in Fig. 3 (g) will be explained later. We can also observe these bifurcation phenomena from circuit experiment as shown in Fig. 4.

We drew up one-parameter bifurcation diagram in order to investigate three matters selectively; the first is the behavior of x_1 on changing the coupling resistor value, the second is the anti-phase synchronization of x_1 and x_2, and the third is the behavior of x_3. The bifurcation diagrams are shown in Fig. 5. At first, let us note the behavior of x_3. Figs. 5 (c) and (d) show bifurcation of x_3 obtained by giving different initial conditions. For $0.0 < \gamma < 0.0015$, the effect of the coupling resistor is very small and we can observe symmetric attractor. At $\gamma = 0.0015$, x_3 bifurcates to asymmetric chaos. As γ increases, the asymmetric chaos bifurcates to almost one-periodic (around $\gamma = 0.025$). For $\gamma > 0.0375$, symmetric attractor appears again. This scenario of bifurcation seems to be similar to the bifurcation route observed typical circuits with symmetric structure (see Fig. 11 in [5]). Because of the effect of chaotic oscillations of x_1 and x_2, x_3 could not show periodic behavior clearly. But, we can conclude that the coupling resistor plays a role to produce bifurcation phenomena of the circuit with different oscillation frequency such as one symmetric chaotic attractor → two asymmetric chaotic attractors → (inverse period-doubling bifurcation) → two asymmetric 1-periodic attractors → one symmetric 1-periodic attractor.

As γ increases further ($\gamma > 0.10$), the coupling effect of x_1 and x_2 becomes larger and x_3 vibrates strongly. Finally, for $\gamma > 0.21$, x_3 stops oscillating suddenly.

Next, note the synchronization of x_1 and x_2. From Fig. 5 (b) we can see that anti-phase synchronization of x_1 and x_2 becomes weak as γ increases. After x_3 stops oscillating, x_1 and x_2 exhibit complete anti-phase synchronization.

4. Effect due to the Capacitor Ratio

In this section, we consider how changing the capacitor ratio affects our system for the case of $N = 3$. The Bifurcation diagrams for the capacitor ratios 1.7 ($\delta = 0.59$)

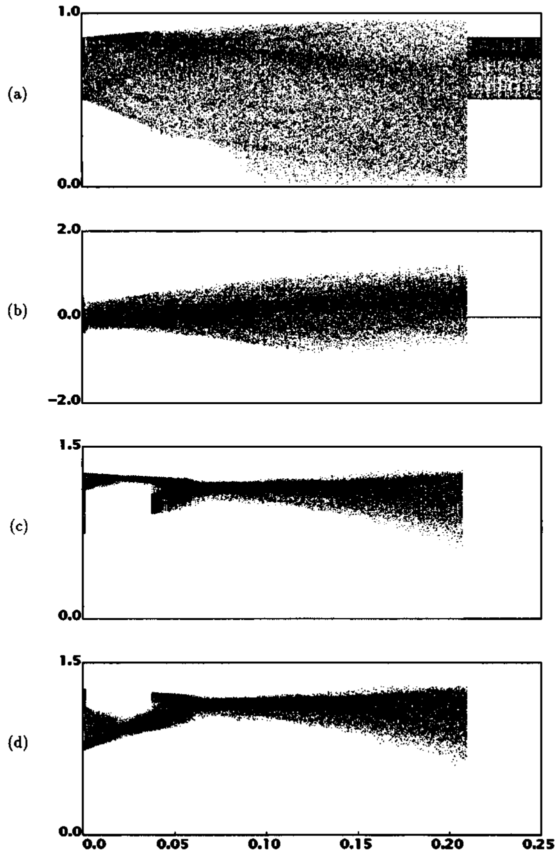

Fig. 5 Bifurcation diagram for $N = 3$. $\alpha = 20, \beta = 0.29, \delta = 0.5$.

and 2.5 ($\delta = 0.4$) are shown in Figs. 6 and 7, respectively.

Figs. 5, 6 and 7 show the change of γ at which oscillating in the circuit with different oscillation frequency stops. It was found that oscillation tends to stop if the capacitor ratio is large. Moreover, as the capacitor ratio increases (δ decreases), the vibration of $x_1 + x_2$ seems to become larger.

5. Coexistence of One-Periodic Attractor

The bifurcation diagrams Figs. 5, 6 and 7 are obtained by giving small initial conditions $(x_3(0), y_3(0), z_3(0)) = (0.001, 0.001, 0.001)$. The bifurcation diagram of x_1 obtained by giving relatively large values of initial conditions $(x_3(0), y_3(0), z_3(0)) = (1.0, 1.0, 1.0)$ is shown in Fig. 8. Up to $\gamma = 0.218$ we can observe the same bifurcation phenomena for both of large and small initial conditions. However, when the value of γ is gradually increased past 0.218, all circuits exhibit limit-cycle for the case of relatively large initial conditions. Moreover, two circuits exhibit in-phase synchronization. For this case, the attractors obtained from computer calculation are shown in Fig. 3 (g), and the attractors obtained from circuit experiment are shown in Fig. 4 (g). We cannot

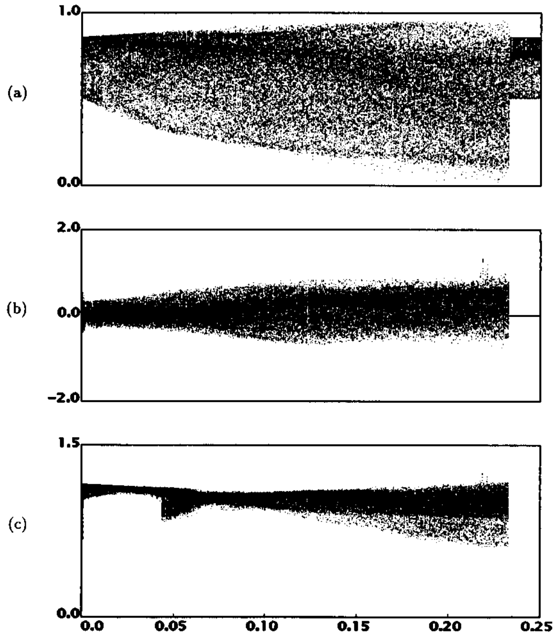

Fig. 6 Bifurcation diagram for different capacitor ratio. $\alpha = 20, \beta = 0.29, \delta = 0.59$.

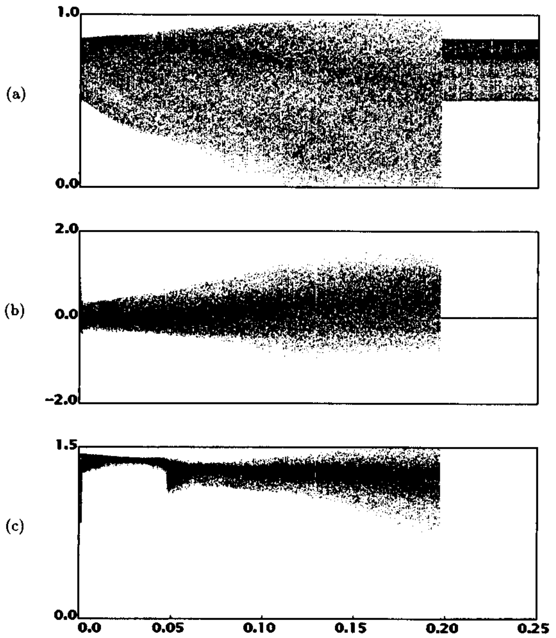

Fig. 7 Bifurcation diagram for different capacitor ratio. $\alpha = 20, \beta = 0.29, \delta = 0.4$.

explain physically why such a synchronization appears. However, we consider that this result would indicates systems including oscillators with different oscillation frequencies can produce various unknown synchronization phenomena.

6. Bifurcation Phenomena for $N = 4$

In this section, we consider the case of $N = 4$.

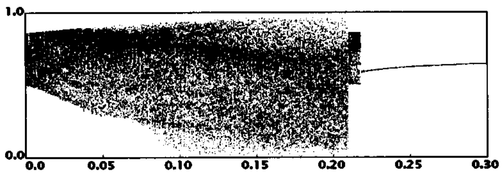

Fig. 8 Bifurcation diagram with relatively large initial conditions. $\alpha = 20, \beta = 0.29, \delta = 0.5$.

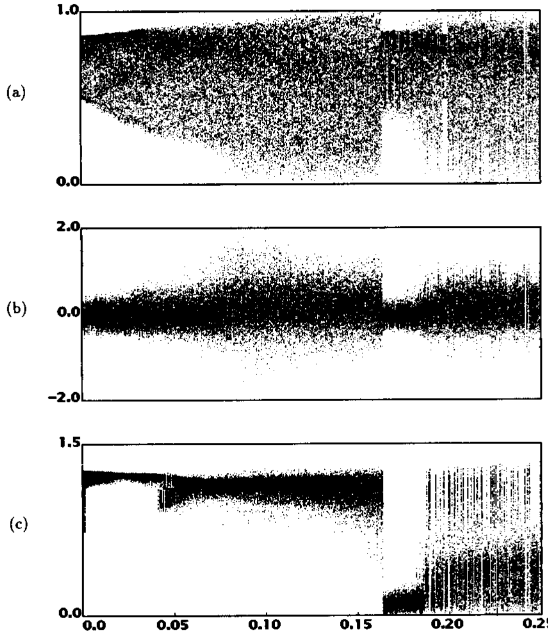

Fig. 9 Bifurcation diagram for $N = 4$. $\alpha = 20, \beta = 0.29, \delta = 0.5$.

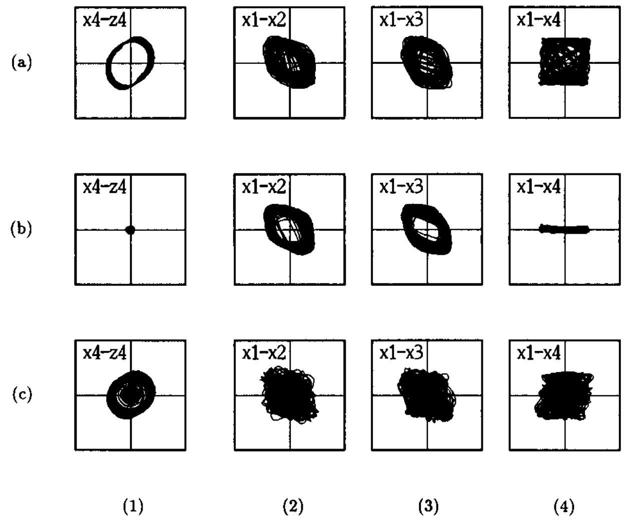

Fig. 10 Result obtained from computer calculation ($N = 4$). $\alpha = 20, \beta = 0.29, \delta = 0.5$, (a) $\gamma = 0.05$, (b) $\gamma = 0.18$, (c) $\gamma = 0.23$, (1) x_4 vs. z_1, (2) x_1 vs. x_2, (3) x_1 vs. x_3, (4) x_1 vs. x_4.

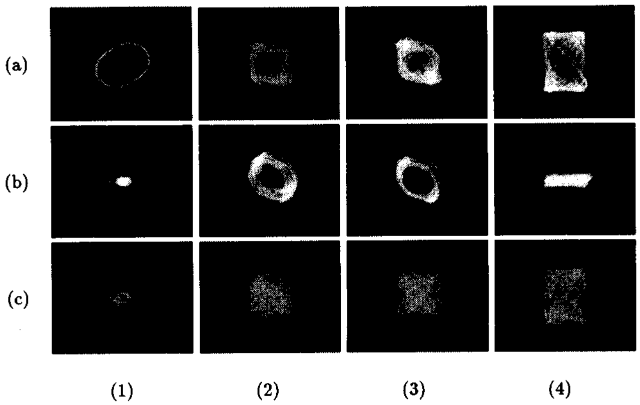

Fig. 11 Attractors obtained from circuit experiment for $N = 4$. $L_1 = 204\,\text{mH}$, $L_2 = 10\,\text{mH}$, $C = 0.034\,\mu\text{F}$, $\hat{C} = 0.068\,\mu\text{F}$, $r = 761\,\Omega$, $\hat{r} = 556\,\Omega$, (a) $R = 120\,\Omega$, (b) $R = 282\,\Omega$, (c) $R = 341\,\Omega$, (1) I_4 vs. v_4, (2) I_1 vs. I_2, (3) I_1 vs. I_3, (4) I_1 vs. I_4.

Similarly to the case of $N = 3$, setting the parameter $\beta = 0.29$, we analyzed our system. One parameter bifurcation diagrams are shown in Fig. 9. When a coupling resistor value is small, inverse period-doubling bifurcation of the circuit with different oscillation frequency can be seen. Also from Figs. 10 (a), 11 (a) and 12 (a), we can see that the remaining three circuits exhibit almost three-phase synchronization. If the coupling resistor value is increased further, the coupling effect of x_1, x_2 and x_3 becomes larger and x_4 vibrates strongly. As γ increases further ($0.164 < \gamma < 0.189$), phenomena of stopping oscillating which can be seen for the case of $N = 3$ can not be seen for this case, but the amplitude of the attractor with different frequency becomes very small. From Figs. 10 (b), 11 (b) and 12 (b), we can see that the vibration of three-phase synchronization also becomes small in comparison with Fig. (a). Beyond $\gamma = 0.189$, irregular self-switching phenomenon of the attractor with small amplitude and one with large amplitude is observed as shown in Figs. 10 (c) and 11 (c). In this case, remaining three circuits cannot be synchronized.

7. Conclusion

In this study, we investigated how changing a frequency in one of N chaotic circuits coupled by a resistor affects our system, by means of both circuit experiment and computer calculation. It was found that for the case of $N = 3$ when a value of a coupling resistor is gradually increased, only one circuit with different frequency exhibits bifurcation phenomena including inverse period-doubling bifurcation, and for larger value of coupling resistor, the chaotic circuit with different frequency suddenly stops oscillating and the remaining two chaotic circuits exhibit completely anti-phase synchronization. Further, we also investigated the case of $N = 4$.

In our previous study, we have studied a coupled system with van der Pol oscillators with different fre-

(a)
(b)

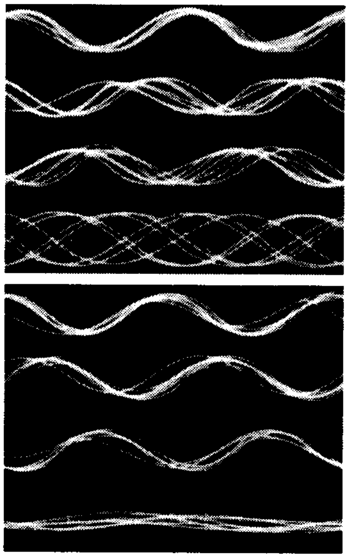

Fig. 12 Time waveform for $N = 4$. $L_1 = 204\,\text{mH}$, $L_2 = 10\,\text{mH}$, $C = 0.034\,\mu\text{F}$, $\hat{C} = 0.068\,\mu\text{F}$, $r = 761\,\Omega$, $\hat{r} = 556\,\Omega$, (a) $R = 120\,\Omega$, (b) $R = 282\,\Omega$.

quencies, and have reported that oscillation of only the oscillator with different frequency stops [4]. From these studies, we can conclude that strength of the coupling resistor will mostly affect the circuit with different oscillation frequency.

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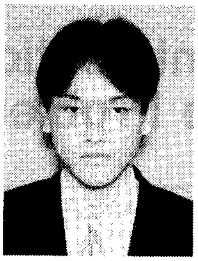

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