PAPER

On Coupled Oscillators Networks for Cellular Neural Networks

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SUMMARY When N oscillators are coupled by one resistor, we can see N-phase oscillation, because the system tends to minimize the current through the coupling resistor. Moreover, when the hard oscillators are coupled, we can see N, $N-1, \cdots$, 3, 2-phase oscillation and get much more phase states. In this study, the two types of coupled oscillators networks with third and fifth-power nonlinear characteristics are proposed. One network has two-dimensional hexagonal structure and the other has two-dimensional lattice structure. In the hexagonal circuit, adjacent three oscillators are coupled by one coupling resistor. On the other hand, in the lattice circuit, four oscillators are coupled by one coupling resistor. In this paper we confirm the phenomena seen in the proposed networks by circuit experiments and numerical calculations. In the system with third-power nonlinear characteristics, we can see the phase patterns based on 3-phase oscillation in the hexagonal circuit, and based on anti-phase oscillation in lattice circuit. In the system with fifth-power nonlinear characteristics, we can see the phase patterns based on 3-phase and anti-phase oscillation in both hexagonal and lattice circuits. In particular, in these networks, we can see not only the synchronization based on 3-phase and anti-phase oscillation but the synchronization which is not based on 3-phase and anti-phase oscillation. As a result, these networks are expected to generate various synchronization patterns. In these networks, each oscillator is connected to only its adjacent oscillators and various patterns are generated according to the initial condition. Therefore, we can consider that we can use these networks as a kind of cellular neural networks.

key words: *coupled oscillators, hexagonal circuit, lattice circuit, cellular neural network*

1. Introduction

There have been many investigations of mutual synchronization and multimode oscillation of coupled oscillators ([1]–[9] and therein). In Refs. [1]–[4], Endo et al. have reported the multimode oscillations in the various types of the coupled oscillators networks. In Refs. [5]–[9] the researches about the synchronization in resistively coupled oscillators have been reported. In particular, we have reported synchronization phenomena observed from N oscillators with the same natural frequency mutually coupled by one resistor as shown in Fig. 1 [7]. In the system, various synchronization phenomena can be observed because the system tends to minimize the current through the coupling resistor. Especially, we have confirmed experimentally that N-phase oscillation can be stably excited when each oscillator has strong nonlinearity. In this case, there exist $(N-1)!$ phase states. Because of the coupling structure and extremely large number of steady states of the system in Fig. 1, this system would be a structural element of the cellular neural network [8].

Moreover, we have investigated the synchronization phenomena in N oscillators with fifth-power nonlinear characteristics coupled by one resistor [8], [9]. Because an oscillator with fifth-power nonlinear characteristics exhibits hard oscillation, we can keep arbitrary number of oscillations to be stationary. Therefore, we can get not only N-phase oscillation but also $N-1, N-2, \cdots, 2$-phase oscillation. As a result, we can get much more phase states than that of the system with third-power nonlinear characteristics.

On the other hand, recently many researches of the networks of the nonlinear elements have been reported [10]–[12], because they are important not only as a model for nonlinear systems but from the viewpoint of biological information processing and possible engineering applications. In Ref. [10] Chua et al. have proposed the network of the sparsely interconnected nonlinear elements called cellular neural network (CNN). CNN has two major features; continuous time feature allowing real-time signal processing and local intercon-

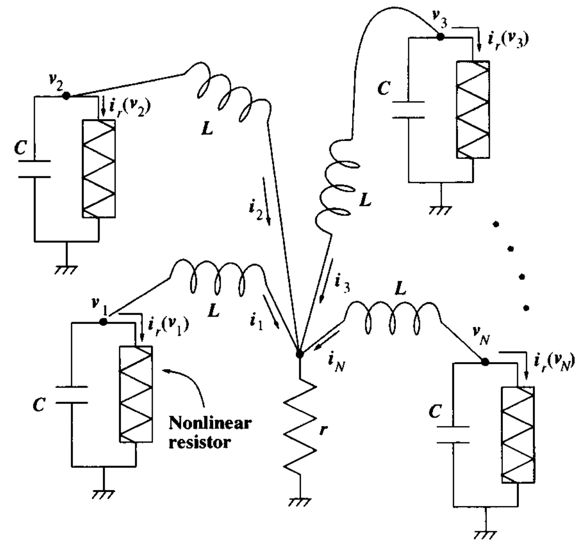

Fig. 1 Oscillators coupled by one resistor.

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nection feature making it tailor made for VLSI implementation. And various applications for image processing and pattern recognition of cellular neural networks have been reported. And in Ref. [11], neural network using a van der Pol oscillator as a neuron has been proposed by Endo et al. This system has the advantage that no simulated annealing is needed to achieve good convergence to an optimum solution, because the static characteristics of the van der Pol oscillators are completely binary and the dynamic characteristics are continuous and smooth. Moreover, it achieves fast convergence. In Ref. [12], Kaneko has suggested the possibility of the information processing using the network of the nonlinear elements by investigating the globally coupled chaotic elements from various aspects.

In this study, we propose two types of networks of coupled oscillators with third and fifth-power nonlinear characteristics. One network has two-dimensional hexagonal structure and the other has two-dimensional lattice structure. In the hexagonal circuits, three oscillators are coupled by one coupling resistor. On the other hand, in the lattice circuits, four oscillators are coupled by one coupling resistor. In the system with third-power nonlinear characteristics, we predict that we can see the phase patterns based on 3-phase oscillation in the hexagonal circuit and based on anti-phase oscillation in lattice circuit. In the system with fifth-power nonlinear characteristics, we predict that we can see the phase patterns based on 3-phase and anti-phase oscillation in both hexagonal and lattice circuits. Therefore, in these networks, we can get many different synchronization patterns by changing the initial states and each oscillator is connected to only its adjacent oscillators. So we can consider that we can use these networks as some kinds of cellular neural networks.

In this paper, we show the proposed networks models in Sect. 2 and we carry out circuit experiments and numerical calculation, and investigate the relation between the initial condition and synchronization patterns in Sect. 3. Section 4 is the conclusions.

2. Networks Models and Circuit Equations

The networks models are shown in Figs. 2 and 3. In the hexagonal circuit shown in Fig. 2, three oscillators are coupled by one coupling resistor. The shape of the network should be an equilateral triangle (see Fig. 4 (a)). When the number of oscillators on one side of the equilateral triangle is N, we call this network size is N. If the network size is N, the number of oscillators in the network is $N(N+1)/2$. Similarly, in the lattice circuit shown in Fig. 3, four oscillators are coupled by one coupling resistor. In this case we can construct the $N \times M$ network, but for simplicity we pay attention to $N \times N$ square networks, so the networks have N^2 oscillators (see Fig. 4 (b)). The v-i characteristics of each nonlinear resistor are approximated by third or fifth-power

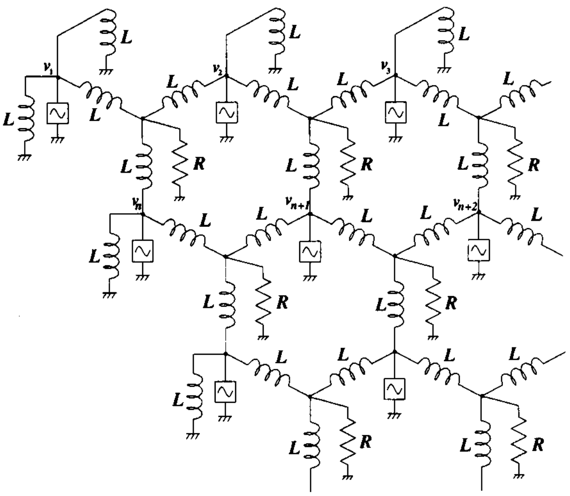

Fig. 2 Coupled oscillators network (hexagonal circuit).

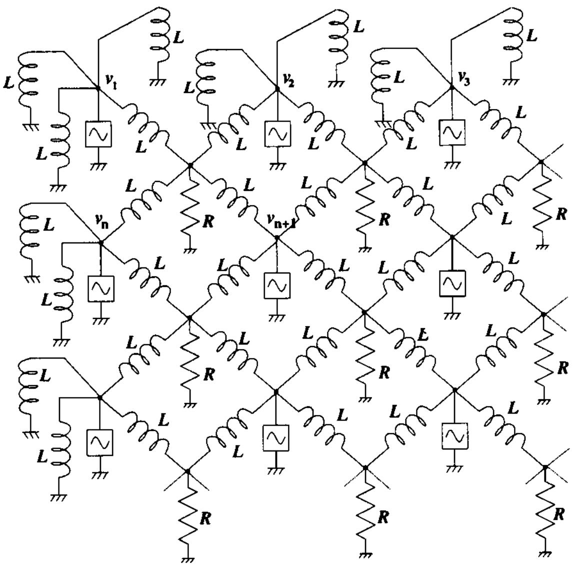

Fig. 3 Coupled oscillators network (lattice circuit).

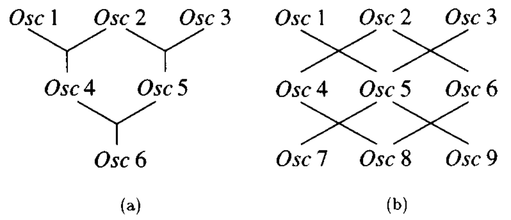

Fig. 4 (a) Size 3 hexagonal circuit. (b) 3×3 lattice circuit.

equation. In these networks, the circuit equations are described as follows,

$$\begin{cases} C_k \dfrac{dv_k}{dt} = - \displaystyle\sum_{L_{ij} \in N_L(k)} i_{ij} - i_r(v_k) \\ L_{kl} \dfrac{di_{kl}}{dt} = v_k - R_m \displaystyle\sum_{L_{ij} \in N_R(kl)} i_{ij} \\ (k = 1, 2, \cdots, K, \quad l = 1, \cdots, 3 \text{ or } 4, \\ \quad m = 1, 2, \cdots, M) \end{cases} \qquad (1)$$

where

$$i_r(v_k) = \begin{cases} -g_1 v_k + g_3 {v_k}^3 \\ \text{(third-power} \\ \quad \text{nonlinear chracteristics)} \\ g_1 v_k - g_3 {v_k}^3 + g_5 {v_k}^5 \\ \text{(fifth-power} \\ \quad \text{nonlinear chracteristics)} \end{cases} \tag{2}$$

and $N_L(k)$ and $N_R(m)$ are defined as

$$N_L(k) = \{L_{ij} | \text{ inductances } L_{ij} \text{ connected to oscillator } k\} \tag{3}$$

$$N_R(m) = \{L_{ij} | \text{ inductances } L_{ij} \text{ connected to resistor } R_m\} \tag{4}$$

and

$$K = \begin{cases} N(N+1)/2 \\ \quad \text{(in the hexagonal circuits.)} \\ N^2 \\ \quad \text{(in the lattice circuits.)} \end{cases} \tag{5}$$

$$M = \begin{cases} N(N-1)/2 \\ \quad \text{(in the hexagonal circuits.)} \\ (N-1)^2 \\ \quad \text{(in the lattice circuits.)} \end{cases} \tag{6}$$

Because each oscillator has the same natural frequency, we take $R_1 = R_2 = \cdots = R_m = \cdots = R_M = R$, $C_1 = C_2 = \cdots = C_N = C$ and $L_{11} = L_{12} = \cdots = L_{ij} = \cdots = L$.

When the nonlinearity is the third-power, by changing variables,

$$\begin{aligned} &t = \sqrt{LC}\tau, \qquad \dot{x} = \frac{dx}{d\tau}, \\ &v_k = \sqrt{\frac{g_1}{3g_3}} x_k, \quad i_{kl} = \sqrt{\frac{Cg_1}{3Lg_3}} y_{kl}, \\ &\varepsilon = g_1 \sqrt{\frac{L}{C}}, \end{aligned} \tag{7}$$

the normalized circuit equations are described as follows

$$\begin{cases} \dot{x}_k = -\sum_{L_{ij} \in N_L(k)} y_{ij} + \varepsilon \left(x_k - \frac{{x_k}^3}{3} \right) \\ \dot{y}_{kl} = x_k - \alpha_m \sum_{L_{ij} \in N_R(kl)} y_{ij} \\ (k = 1, 2, \cdots, K, \quad l = 1, \cdots, 3 \text{ or } 4, \\ \quad m = 1, 2, \cdots, M) \end{cases} \tag{8}$$

and $\alpha_m = R_m \sqrt{C/L}^{\dagger}$. α is coupling factor and ε is the strength of nonlinearity.

When the nonlinearity is the fifth-power and $R_1 = R_2 = \cdots = R_m = \cdots = R_M = R$, $C_1 = C_2 = \cdots = C_N = C$ and $L_{11} = L_{12} = \cdots = L_{ij} = \cdots = L$, by changing variables,

$$\begin{aligned} &t = \sqrt{LC}\tau, \qquad \dot{x} = \frac{dx}{d\tau}, \\ &v_k = \sqrt[4]{\frac{g_1}{5g_5}} x_k, \quad i_{kl} = \sqrt{\frac{C}{L}} \sqrt[4]{\frac{g_1}{5g_5}} y_{kl}, \\ &\beta = \frac{3g_3}{g_1} \sqrt{\frac{g_1}{5g_5}}, \quad \varepsilon = g_1 \sqrt{\frac{L}{C}}, \end{aligned} \tag{9}$$

the normalized circuit equations are described as follows

$$\begin{cases} \dot{x}_k = -\sum_{L_{ij} \in N_L(k)} y_{ij} \\ \qquad -\varepsilon \left(x_k - \frac{1}{3} \beta {x_k}^3 + \frac{1}{5} {x_k}^5 \right) \\ \dot{y}_{kl} = x_k - \alpha_m \sum_{L_{ij} \in N_R(kl)} y_{ij} \\ (k = 1, 2, \cdots, K, \quad l = 1, \cdots, 3 \text{ or } 4, \\ \quad m = 1, 2, \cdots, M) \end{cases} \tag{10}$$

where $\alpha_m = R_m \sqrt{C/L}$ as in the case of the third-power nonlinear characteristics. α is coupling factor and ε is the strength of nonlinearity. The amplitudes of oscillators with hard nonlinear characteristics depend on β.

3. Experimental and Numerical Results

In this section we show experimental and numerical results observed from the proposed oscillators networks and investigate the relation between the initial states and the phase patterns.

3.1 Synchronization in Oscillators Networks

3.1.1 Synchronization in the System with Third-Power Nonlinear Characteristics

In the networks with the third-power nonlinear characteristics each oscillator is excited because of the negative slope of the nonlinear characteristics [8]. The negative resistance with the third-power nonlinear characteristics is realized by the circuit shown in Fig. 5 (a) and its v-i characteristics are shown in Fig. 6 (a). On computer calculation, in order to consider the difference between the natural frequencies of the oscillators, Eq. (8) is rewritten as follows.

$$\begin{cases} \dot{x}_k = -\sum_{L_{ij} \in N_L(k)} y_{ij} + \varepsilon \left(x_k - \frac{{x_k}^3}{3} \right) \\ \dot{y}_{kl} = (1 + \Delta\omega_k) x_k - \alpha_m \sum_{L_{ij} \in N_R(kl)} y_{ij} \\ \Delta\omega_k = (k-1) \times 10^{-3} \\ (k = 1, 2, \cdots, K, \quad l = 1, \cdots, 3 \text{ or } 4, \\ \quad m = 1, 2, \cdots, M) \end{cases} \tag{11}$$

where $\Delta\omega_k$ correspond to the difference between the natural oscillating frequency of the reference oscillator and

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$^{\dagger}\alpha_1 = \alpha_2 = \cdots = \alpha_m = \cdots = \alpha_M = \alpha$

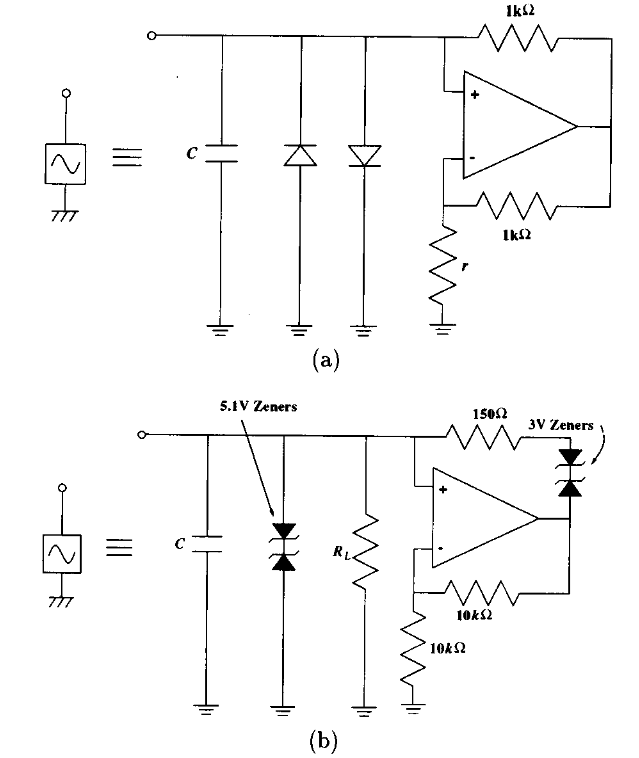


Fig. 5 Construction of oscillator with (a) third-power nonlinear characteristics and (b) fifth-power nonlinear characteristics.

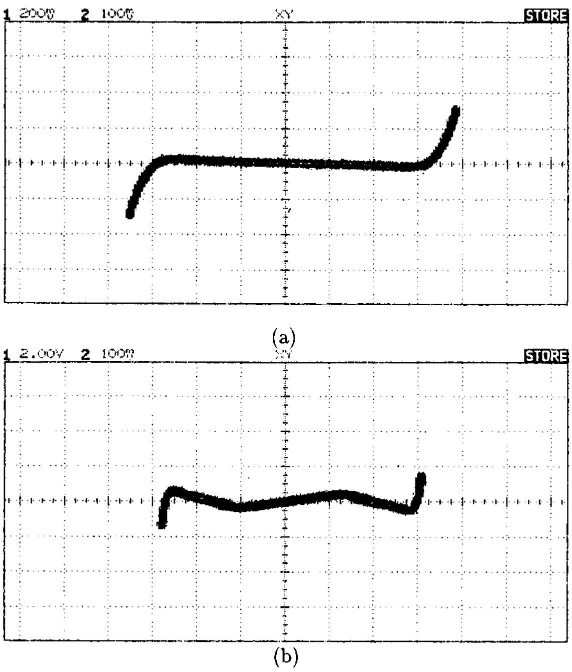


Fig. 6 v-i characteristics of (a) third-power nonlinear characteristics and (b) fifth-power nonlinear characteristics.

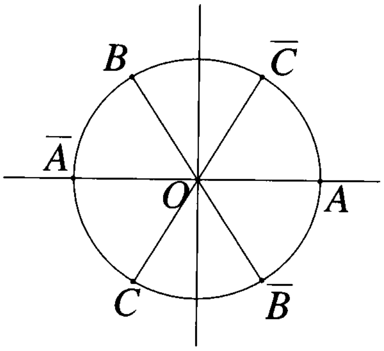


Fig. 7 Notation of the phase states.

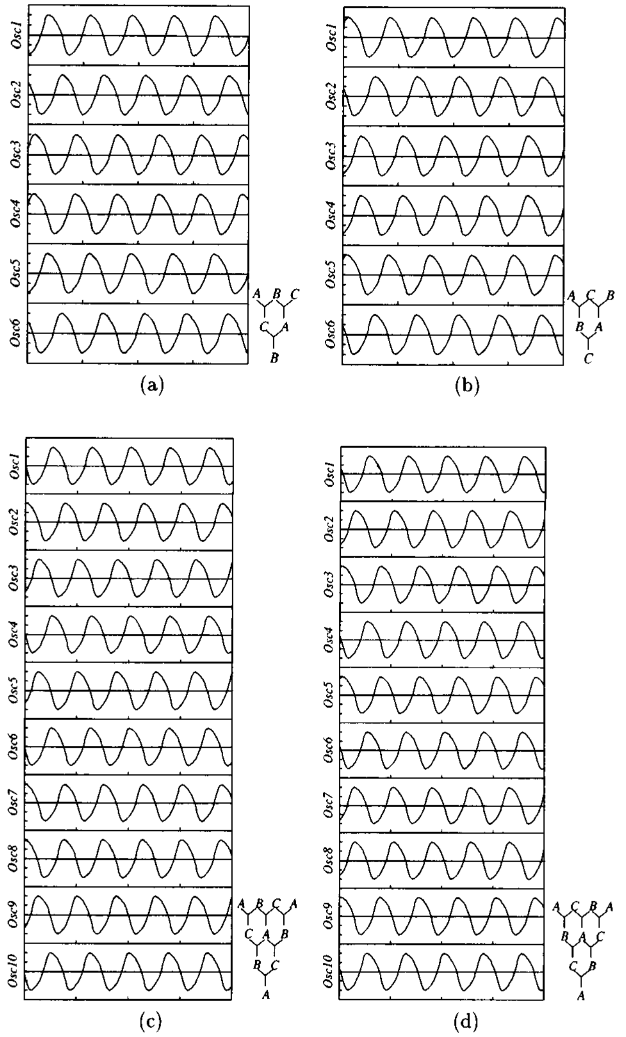
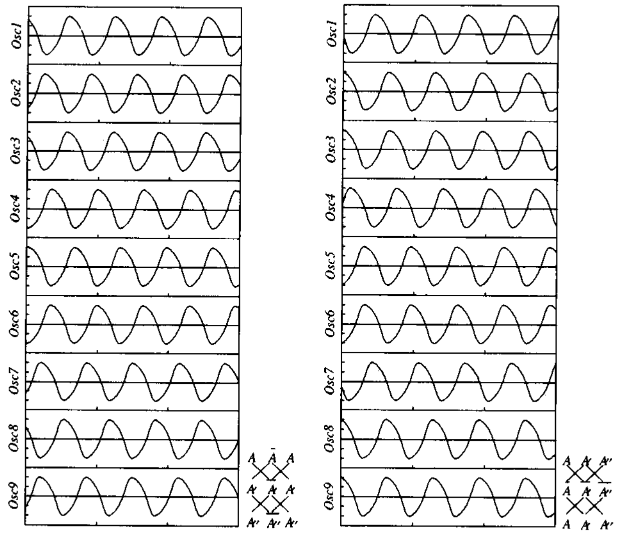


Fig. 8 Numerical results in hexagonal circuits with the third-power nonlinear characteristics when the network size is 3 or 4 ($\alpha = 4.0$, $\varepsilon = 1.5$).

those of the other oscillators and are derived from the conditions on the circuit experiments. In the following results, the notations A–O mean the phase states of the oscillators as shown in Fig. 7.

In the hexagonal circuit with third-power nonlinear characteristics, three oscillators are coupled by one coupling resistor. When 3 oscillators are coupled by one resistor, the system tends to minimize the current through the coupling resistor by minimizing the sum of the voltages of the oscillators and these oscillators exhibit 3-phase oscillation [7], [9]. So in the hexagonal circuit with third-power nonlinear characteristics we can predict that we can see the synchronization based on 3-phase oscillation. To confirm those phenomena we carried out both circuit experiments and numerical calculations. Figure 8 shows the numerical results in the hexagonal circuits with third-power nonlinear characteristics. In these networks we can see the synchronous phase patterns based on 3-phase oscillations because 3 oscillators are coupled by one coupling resistor. In the hexagonal circuits we can see only two synchronous patterns irrespective of the size of the networks, because if the relation of the phases of three oscillators around a coupling resistor is decided, the phase of the next oscillator is decided uniquely. These results are confirmed by circuit experiments, too.

In the lattice circuit with third-power nonlinear characteristics, 4 oscillators are coupled around one coupling resistor. When 4 oscillators are coupled by one resistor, the system tends to minimize the current through the coupling resistor by minimizing the sum of the voltages of the oscillators and we can see 2 independent anti-phase oscillations [7], [9]. So in the lattice circuit with third-power nonlinear characteristics we can predict that we can see the synchronization based on anti-phase oscillation. To confirm those phenomena we carried out both circuit experiments and numerical calculations. Figure 9 shows the numerical results in 3×3 lattice circuits with third-power nonlinear characteristics. In this network we cannot get any synchronous pattern but many asynchronous phase patterns because the phase states based on the independent anti-phase oscillations. These results are confirmed by circuit experiments, too.



(a) (b)

Fig. 9 Numerical results in lattice circuit with the third-power nonlinear characteristics in 3×3 network ($\alpha = 4.0$, $\varepsilon = 1.5$).

3.1.2 Synchronization in the System with Fifth-Power Nonlinear Characteristics

In the networks with the fifth-power nonlinear characteristics we can keep arbitrary number of oscillations to be stationary because of the positive slope near 0V of the nonlinear characteristics [8]. The negative resistance with the fifth-power nonlinear characteristics is realized by the circuit shown in Fig. 5 (b) and its v-i characteristics are shown in Fig. 6 (b). On computer calculation, in order to consider the difference between the natural frequencies of the oscillators, Eq. (10) is rewritten as follows.

$$\begin{cases} \dot{x}_k = -\displaystyle\sum_{L_{ij} \in N_L(k)} y_{ij} \\ \qquad -\varepsilon\left(x_k - \dfrac{1}{3}\beta {x_k}^3 + \dfrac{1}{5}{x_k}^5\right) \\ \dot{y}_{kl} = (1 + \Delta\omega_k)x_k - \alpha_m \displaystyle\sum_{L_{ij} \in N_R(kl)} y_{ij} \\ \Delta\omega_k = (k-1) \times 10^{-3} \\ (k = 1, 2, \cdots, K, \quad l = 1, \cdots, 3 \text{ or } 4, \\ \quad m = 1, 2, \cdots, M) \end{cases} \tag{12}$$

where $\Delta\omega_k$ correspond to the difference between the natural oscillating frequency of the reference oscillator and those of the other oscillators and are derived from the conditions on the circuit experiments.

In the hexagonal circuit with fifth-power nonlinear characteristics, three oscillators are coupled by one coupling resistor. When three oscillators with fifth-power nonlinear characteristics are coupled by one resistor, we can see three cases of the phenomena as follows, because we can choose the number of excited oscillators by changing the initial states [8].

1. 3 oscillators make three-phase synchronization.
2. 2 oscillators make anti-phase oscillation but the other one is not excited.
3. No oscillator is excited.

So in the hexagonal circuit with fifth-power nonlinear characteristics, we can predict that we can see the synchronization based on 3-phase and anti-phase oscillation.

To confirm those phenomena we carried out both circuit experiments and numerical calculations. Figures 10 and 11 show the experimental and numerical results observed from the hexagonal circuits with fifth-power nonlinear characteristics when the network size is 3. From these figures we can see much more synchronous patterns than that of the network with third-power nonlinear characteristics. Actually we can see 15 phase patterns in the Size 3 hexagonal network while

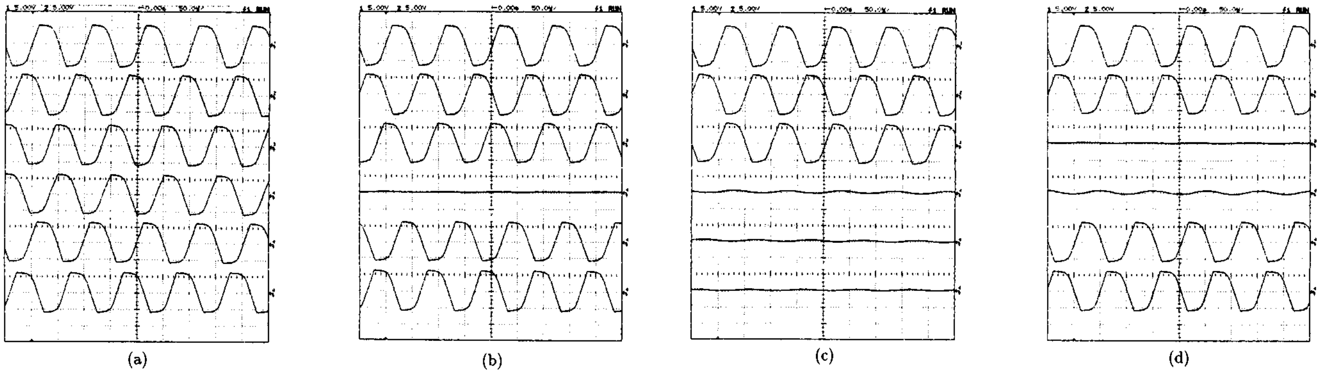


Fig. 10 Experimental results in hexagonal circuit with the fifth-power nonlinear characteristics when the network size is 3 (L = 10 mH, C = 0.068 μF, R = 1.0 kΩ, horizontal scale: 50 μsec/div., vertical scale: 5 V/div.).

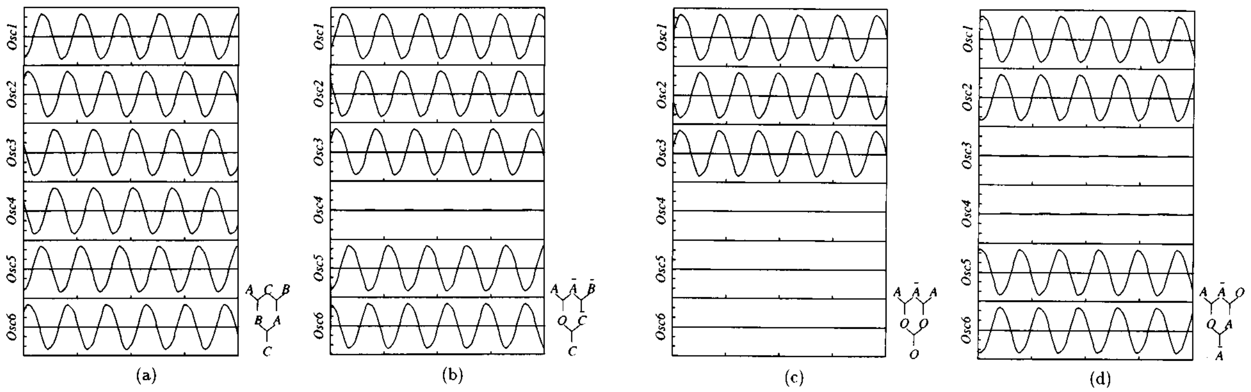


Fig. 11 Numerical results in hexagonal circuit with the fifth-power nonlinear characteristics when the network size is 3 ($\alpha = 4.0$, $\beta = 3.5$, $\varepsilon = 0.5$).

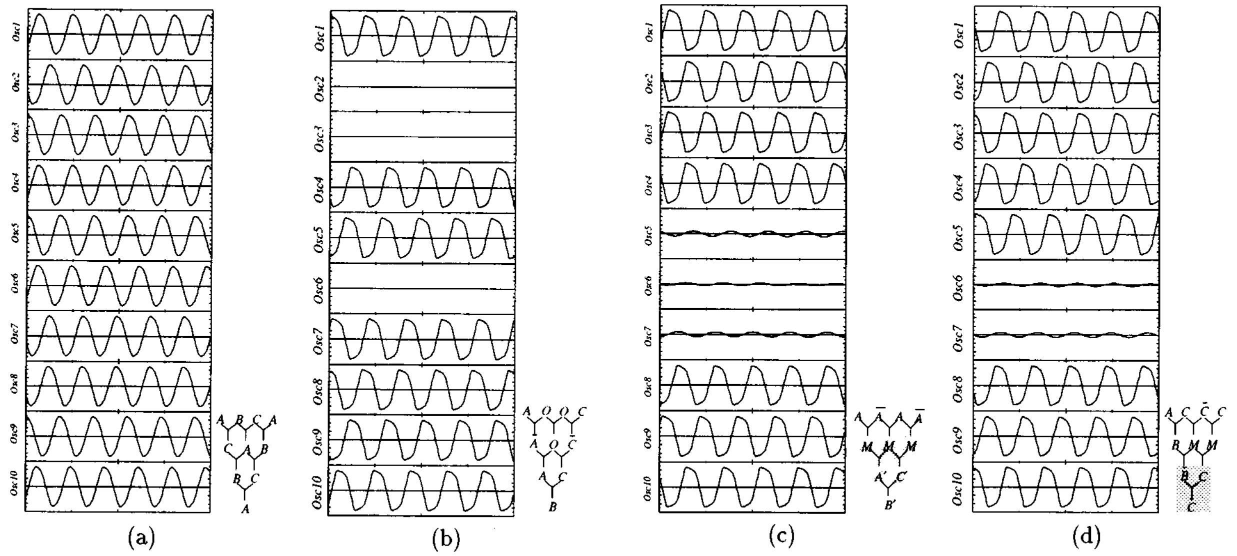


Fig. 12 Numerical results in hexagonal circuit with the fifth-power nonlinear characteristics when the network size is 4 ($\alpha = 1.0$, $\beta = 3.5$, $\varepsilon = 1.5$).

only 2 patterns in the circuit with third-power nonlinear characteristics.

Figure 12 shows the examples of the numerical results observed from the hexagonal circuits with fifth-power nonlinear characteristics when the network size is 4. In this network we can see much more synchronous

Table 1 The number of the phase patterns in proposed networks.

Nonlinearity	Hexagonal		Lattice
	Size 3	Size 4	3×3
Third-power	2	2	12
Fifth-power	15	28	126

phase patterns than that of the system with third power nonlinear characteristics. In Fig. 12 (a) and (b), we can see the phase patterns based on 3-phase and anti-phase oscillations as we predict. We call them coherent patterns. Moreover we can see asynchronous patterns as shown in Fig. 12 (c). In this figure, M means the oscillation with smaller amplitude than those of ordinary excited oscillators. In this case, Oscillators 1–4 make anti-phase oscillation and Oscillators 8–10 make 3-phase oscillation but the phase difference between Oscillator 1 and 8 is independent, because Oscillators 5–7 are almost stopped. In this case, Oscillators 5–7 are not stopped completely because the oscillators around the resistors R_4 and R_5 make neither 3-phase nor anti-phase oscillations. Moreover Fig. 12 (d) shows another pattern which is not based on either 3-phase or anti-phase oscillations. In the shaded part of the phase pattern, 3 oscillators do not make 3-phase oscillation though they are excited, and Oscillators 6 and 7 are excited with the small amplitudes to compensate such incoherence. These incoherent patterns are the phenomena we do not predict. In the Size 4 network we can get 22 coherent synchronous patterns and 6 asynchronous phase patterns as shown in Fig. 12 (c). And we can get also many incoherent patterns.

Moreover in the lattice circuits with fifth-power nonlinear characteristics, four oscillators are coupled by one coupling resistor. When four oscillators with fifth-power nonlinear characteristics are coupled by one resistor, we can see four cases of the phenomena as follows, because we can choose the number of excited oscillators by changing the initial states [8].

1. 2 oscillators make anti-phase synchronization and the other 2 oscillators make anti-phase synchronization independently.
2. 3 oscillators make three-phase synchronization but the other one is not excited.
3. 2 oscillators make anti-phase synchronization but the other 2 oscillators are not excited.
4. No oscillator is excited.

So in the lattice circuit with fifth-power nonlinear characteristics, we can predict that we can see the synchronization based on 3-phase and anti-phase oscillation.

To confirm those phenomena we carried out both circuit experiments and numerical calculations. Figure 13 shows the examples of the numerical results observed from the 3×3 lattice circuit with fifth-power nonlinear characteristics. When all the oscillators are excited, we can see only asynchronous patterns just as shown in the case with third-power nonlinear characteristics (see Fig. 13 (a)). When some oscillators are stopped, we can see both synchronous and asynchronous patterns. In Fig. 13 (b), we can see a coherent synchronous phase pattern, and in Fig. 13 (c) we can see a coherent asynchronous pattern. In this case Oscillators 1–3 and Oscillators 7–9 make the independent anti-phase oscillations because Oscillators 4–6 are completely stopped. In Fig. 13 (d), however, we can see an incoherent pattern as seen in Fig. 12 (d). In the shaded part of the phase pattern, 4 oscillators do not make either 3-phase or anti-phase oscillation though they are excited, and Oscillator 6 is excited with the small amplitude to compensate such incoherence. In the 3×3 lattice circuit we can get 12 coherent asynchronous pat-

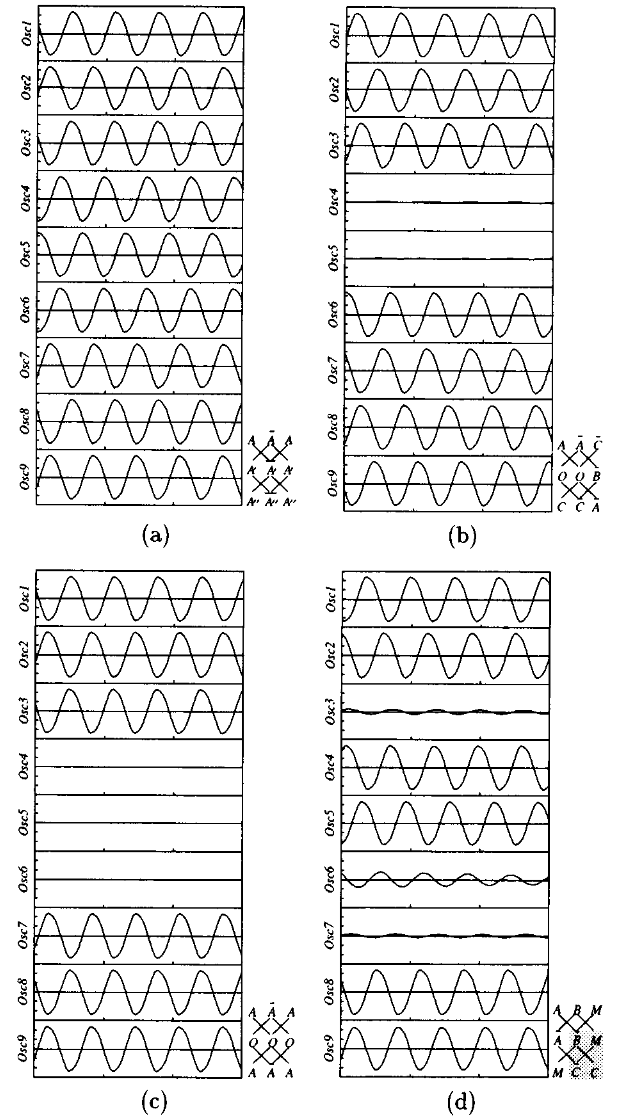


Fig. 13 Numerical results in lattice circuit with the fifth-power nonlinear characteristics when the network size is 3 (α = 4.0, β = 3.5, ε = 0.5).

Table 2 Relation between the initial states and the obtained phase patterns in Size 4 hexagonal circuit ($\beta = 3.5$, $\varepsilon = 1.5$).

initial voltage		phase pattern	
		$\alpha = 1.0$	$\alpha = 3.0$
x_4	2.0	$AOOC\bar{A}O\bar{C}ACB$	$AOOC\bar{A}O\bar{C}ACB$
	1.1	$AOOO\bar{A}MMACB$	$AOOO\bar{A}MMACB$
	−1.3	$AOOC\bar{A}O\bar{C}ACB$	$AOOC\bar{A}O\bar{C}ACB$
	−2.0	$AOOC\bar{A}O\bar{C}ACB$	$AOOC\bar{A}O\bar{C}ACB$
x_5	2.0	$AOD\bar{D}\bar{A}\bar{D}M\bar{C}\bar{A}\bar{B}$	$ABCACABBCA$
	1.5	$AOD\bar{D}\bar{A}\bar{D}M\bar{C}\bar{A}\bar{B}$	$ABCACABBCA$
	1.4	$AOD\bar{D}\bar{A}\bar{D}M\bar{C}\bar{A}\bar{B}$	$AM\bar{A}\bar{B}\bar{A}D\bar{C}\bar{C}\bar{A}\bar{B}$
	−1.0	$AOOC\bar{A}O\bar{C}ACB$	$AM\bar{A}\bar{B}\bar{A}D\bar{C}\bar{C}\bar{A}\bar{B}$
	−2.0	$AOOC\bar{A}O\bar{C}ACB$	$AOOC\bar{A}O\bar{C}ACB$
x_{10}	2.0	$AOOC\bar{A}O\bar{C}ACB$	$AOOC\bar{A}O\bar{C}ACB$
	1.3	$AOOC\bar{A}O\bar{C}ACB$	$AM\bar{A}\bar{B}\bar{A}D\bar{C}\bar{C}\bar{A}\bar{B}$
	1.2	$AOO\bar{A}\bar{A}OAA\bar{A}O$	$AM\bar{A}\bar{B}\bar{A}D\bar{C}\bar{C}\bar{A}\bar{B}$
	1.1	$AOO\bar{A}\bar{A}OAA\bar{A}O$	$AMMCCMMBDM$
	−1.3	$AOOC\bar{A}O\bar{C}ACB$	$AOOC\bar{A}O\bar{C}ACB$
	−2.0	$AOOC\bar{A}O\bar{C}ACB$	$AOOC\bar{A}O\bar{C}ACB$

Table 3 Relation between the initial states and the obtained phase patterns in 3×3 lattice circuit ($\beta = 3.5$, $\varepsilon = 0.5$).

initial voltage		phase pattern	
		$\alpha = 1.0$	$\alpha = 3.0$
x_3	2.0	$A\bar{A}AOOOA'\bar{A}'A'$	$A\bar{A}AOOOA'\bar{A}'A'$
	1.0	$A\bar{A}MMMMA'\bar{A}'A'$	$A\bar{A}AOOOA'\bar{A}'A'$
	−1.4	$A\bar{A}MMMMA'\bar{A}'A'$	$A\bar{A}\bar{B}OO\bar{C}B\bar{B}\bar{A}$
	−1.5	$A\bar{A}\bar{B}OO\bar{C}A\bar{A}\bar{B}$	$A\bar{A}\bar{B}OO\bar{C}B\bar{B}\bar{A}$
	−2.0	$A\bar{A}\bar{B}OO\bar{C}A\bar{A}\bar{B}$	$A\bar{A}\bar{B}OO\bar{C}B\bar{B}\bar{A}$
x_8	2.0	$ACAOBOCAC$	$ACAOBOACA$
	1.6	$ACAOBOCAC$	$ACAOBOACA$
	1.3	$A\bar{A}AOOOA'\bar{A}'A'$	$ACAOBOACA$
	−2.0	$A\bar{A}AOOOA'\bar{A}'A'$	$A\bar{A}AOOOA'\bar{A}'A'$
x_9	2.0	$A\bar{A}AOOOA'\bar{A}'A'$	$A\bar{A}AOOOA'\bar{A}'A'$
	1.0	$A\bar{A}AMMMA'\bar{A}'M$	$A\bar{A}AOOOA'\bar{A}'A'$
	−1.4	$A\bar{A}AMMMA'\bar{A}'M$	$A\bar{A}\bar{C}OO\bar{B}C\bar{C}\bar{A}$
	−1.5	$A\bar{A}\bar{C}OO\bar{B}C\bar{C}\bar{A}$	$A\bar{A}\bar{C}OO\bar{B}C\bar{C}\bar{A}$
	−2.0	$A\bar{A}\bar{C}OO\bar{B}C\bar{C}\bar{A}$	$A\bar{A}\bar{C}OO\bar{B}C\bar{C}\bar{A}$

terns when all the oscillators are excited, and 112 coherent synchronous patterns and 2 coherent asynchronous patterns when some oscillators are stopped. And we can get also many incoherent patterns.

3.2 Relation between Initial States and Phase Patterns

In this section, we investigate the relation between the initial states and the obtained phase patterns by numerical calculations. It may suggest certain ways to apply the proposed systems to cellular neural networks.

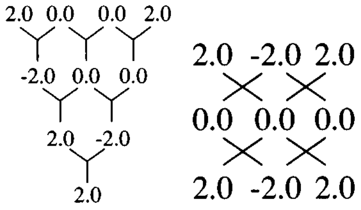
Tables 2 and 3 show the stable phase states obtained when some initial states are changed from the states shown in Fig. 14 in the Size 4 hexagonal circuit and 3×3 lattice circuit, respectively. In these tables the notation A–O mean the phase states as in the previous sections. The notation D means that the wave is synchronized with A but does not exhibits either 3-phase oscillation or anti-phase oscillation and that it has the same amplitude as A. Note that D produces the incoherent phase patterns as M does.

From both results we can see the change of the phase patterns around the initial values ±1.44 that are the threshold voltage of the single oscillator with fifth-power nonlinear characteristics when $\beta = 3.5$ whether it oscillates or not. In each network, we can see 2–4 different phase patterns by changing the initial values from −2.0 to +2.0.

In the networks with different α (i.e. coupling factor), we can see the different phase patterns from the same initial state. This is very important feature because we can decide the character of the networks by choosing α like choosing the cloning templates in CNN.

When one oscillator changes the phase state, the distant oscillators change their phase states, too. In this case the transition of the phase state of one oscillator affects the phase states of the whole network though each oscillator is connected to only its adjacent oscillators.

In these proposed systems, we can consider the initial states as the inputs and the steady phase patterns as the outputs. Because so many inputs are mapped into several phase patterns in these systems, we can use them for pattern recognition and associative memory. In particular, the systems with fifth-power nonlinear characteristics can recognize many different patterns because they have many stable states.



(a) (b)

Fig. 14 (a) The initial state of the Size 4 hexagonal circuit. (b) The initial state of the 3×3 lattice circuits.

These features are quite similar to those of the CNN, because we can get many different stable states by changing the initial states of the networks and each oscillator is connected to only its adjacent oscillators [10]. When we consider one oscillator as a cell, we expect that we can use these networks as some kinds of CNN. The practical ways of application to CNN, however, are our future problems.

4. Conclusions

In this paper we have proposed the coupled oscillators networks with hexagonal and lattice structure and investigated the phenomena in the proposed circuits by both circuit experiments and numerical calculations. In the hexagonal circuit with third-power nonlinear characteristics, we can see the stable phase patterns based on 3-phase oscillation because all the oscillators are excited. Similarly, in the lattice circuit with third-power nonlinear characteristics, we can see the stable asynchronous phase patterns based on anti-phase oscillations. On the other hand, in the networks with fifth-power nonlinear characteristics, various synchronous and asynchronous phase patterns can be observed because the oscillation of the oscillator depends on the initial states. In these networks, we can see both the coherent phase patterns which are based on 3-phase or anti-phase oscillations and the incoherent phase patterns which are not based on either 3-phase or anti-phase oscillations. These incoherent patterns are interesting phenomena which we have not predicted. And in the networks with fifth-power nonlinear characteristics, we can get much more phase states than that of ones with third-power nonlinear characteristics.

Moreover from the same initial state we can get the different phase patterns by changing the coupling factor α. Therefore we can decide the character of the networks by choosing α like choosing the cloning template in CNN. And the transition of the phase state of just one oscillator affects the phase patterns of whole networks though each oscillator is connected to its adjacent oscillators. These features are quite similar to those of the CNN, because we can get many different stable states by changing the initial states of the networks and each oscillator is connected to only its adjacent oscillators. And because so many initial states are mapped into several phase patterns in these systems, we can use them for pattern recognition and associative memory. When we consider one oscillator as a cell, we expect that we can use these networks as some kinds of CNN. The practical ways of application to CNN are our future problems.

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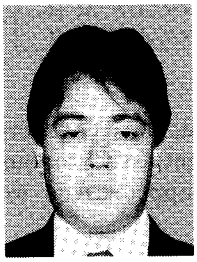

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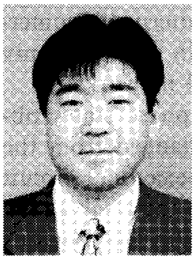

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