PAPER Special Section on Nonlinear Theory and its Applications

SPICE Oriented Steady-State Analysis of Large Scale Circuits

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SUMMARY In this paper, we propose a novel SPICE oriented steady-state analysis of nonlinear circuits based on the circuit partition technique. Namely, a given circuit is partitioned into the linear and nonlinear subnetworks by the application of the substitution theorem. Each subnetwork is solved using SPICE simulator by the different techniques of AC analysis and transient analysis, respectively, whose steady-state response is found by an iteration method. The novel points of our algorithm are as follows: Once the linear subnetworks are solved by AC analysis, each subnetwork is replaced by a simple equivalent RL or RC circuit at each frequency component. On the other hand, the response of nonlinear subnetworks are solved by transient analysis. If we assume that the sensitivity circuit is approximated at the DC operational point, the variational value will be also calculated from a simple RL or RC circuit. Thus, our method is very simple and can be also applied to large scale circuits, efficiently. To improve the convergency, we introduce a compensation technique which is usefully applied to stiff circuits containing components such as diodes and transistors.
key words: *steady-state analysis, SPICE oriented algorithm, circuit partition technique, operational point, AC analysis, transient analysis*

1. Introduction

There are three basic approaches for calculating the steady-state response. The first one is a numerical integration method, which is widely used for a large damping circuits. However, it is inefficient for a small damping circuits, because the transient behavior will continue for a long period. The second one is time-domain shooting methods [1], [2] which are useful for small scale circuits containing strong nonlinear elements. However, it is inefficient for large scale circuits, because the number of the state variables is generally increased for large scale circuits. The third one is the frequency-domain harmonic balance method [4] which is useful for weakly nonlinear circuits having few nonlinear elements. However, it is inefficient for strong nonlinear circuits, because we must consider many frequency components for the approximation of steady-state waveform.

We have proposed a modified hybrid method based on circuit partitioning technique [5]. In Ref. [5], the sensitivity circuit is obtained by replacing each nonlinear element with the time-invariant element, which is evaluated by the average value over one period [0,T]. This sensitivity circuit must be estimated at every iteration. The circuit has the same configuration as that of the original one except that all of the nonlinear elements are replaced by the time-invariant elements. Therefore, to obtain the variational value, we must also solve the large scale sensitivity circuit, which is very time consuming.

In this paper, we propose a SPICE oriented steady-state analysis of large scale circuits, which is based on the circuit partition technique by the application of the substitution theorem. We first partition a circuit into linear and nonlinear subnetworks with substitution theorem. Each subnetwork is solved by the different techniques of AC and transient analysis using SPICE simulator. At the first step of the algorithm, the linear subnetwork is solved by SPICE AC analysis and it is replaced by a simple RL or RC circuit at each frequency component. Similarly, the nonlinear subnetwork is also solved by SPICE AC analysis at the DC operating point, and it is also replaced by a simple RL or RC circuit using the same technique as for the linear subnetwork. These modified sensitivity circuits are used to calculate the variational value. Therefore, we need no longer estimate the sensitivity circuits at each iteration. If the nonlinearity is not very strong, the circuit will be efficiently used as the sensitivity circuit for calculating the variational value $\Delta v(t)$. Thus, the difference between the method [5] and our method is in obtaining the simple sensitivity circuit. If we can partition the circuit such that the nonlinear subnetwork N_2 is small scale compared to the linear subnetwork N_1, our algorithm can be efficiently applied to large scale circuits.

To improve the convergency, we introduce a compensation technique which is usefully applied to stiff circuits containing diodes and transistors.

2. Hybrid Method Using Frequency Responses

To understand the basic ideas of our method, consider a circuit as shown in Fig. 1 (a). We choose a linear subnetwork N_1 such that it includes as many L's and C's as possible, and nonlinear subnetwork N_2 containing as few L's and C's as possible. In this case, the transient behavior of the nonlinear subnetwork will finish after a short period. Thus, we can easily solve the

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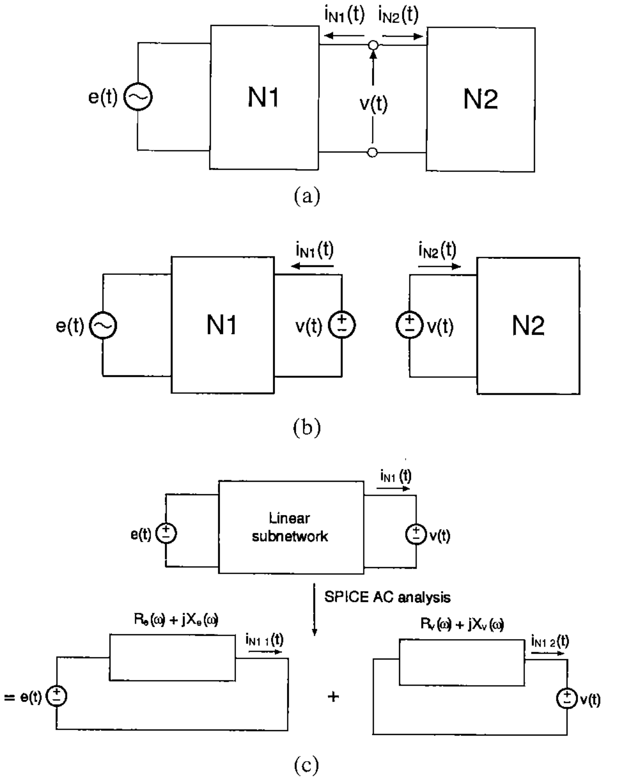


Fig. 1 (a) A schematic diagram of our circuit partition technique. (b) Partitioning the circuit into a linear subnetwork N_1 and a nonlinear subnetwork N_2 using a substitution voltage source $v(t)$. (c) Simplified $R(\omega)+jX(\omega)$ model of linear subnetwork N_1.

steady-state response of N_2 using SPICE simulator. The schematic diagram of our partition technique is shown in Fig. 1 (b). For obtaining steady-state response, we approximate the substitution voltage source as follows:

$$v(t) = V_0 + \sum_{k=1}^{M}(V_{2k-1}\cos k\nu t + V_{2k}\sin k\nu t) \qquad (1)$$

where M is a number of frequency components in our consideration. For multi-frequency components $\omega_1 \cdots \omega_n$, we choose the fundamental frequency ν in the following manner:

$$\nu = \min\{\omega_1 \cdots \omega_n\}/m,$$
(for an arbitrary large integer m)

and approximate the other frequency

$$\omega_{n_i} \cong n_i\nu \quad \text{(for an integer } n_i\text{)}.$$

Note that if we choose large m, we can approximate any frequency ω_i in a given accracy.

Now, we assume that the original circuit in Fig. 1 (a) has a unique steady-state solution described by (1). Then, the substitution theorem says that the solution $v(t)$ satisfies

$$F(v(t)) = i_{N_1}(v(t)) + i_{N_2}(v(t)) = 0 \qquad (2)$$

where $i_{N_1}(\cdot)$ and $i_{N_2}(\cdot)$ are currents through the subnetworks N_1 and N_2 as shown in Fig. 1 (b). We solve the steady-state solution satisfying (2) by an iteration method, and assume the waveform at the jth iteration by

$$v^j(t) = V_0^j + \sum_{k=1}^{M}(V_{2k-1}^j\cos k\nu t + V_{2k}^j\sin k\nu t). \qquad (3)$$

For the linear subnetwork N_1 we can calculate the steady-state response by AC analysis using SPICE simulator. Once the linear subnetwork to the substitution voltage source is solved, it is replaced by a simple equivalent RL or RC circuit at each frequency component, as shown in Fig. 1 (c). Thus, we can easily get the response at each frequency component.

$$i_{N1}^j(t) = \mathcal{Y}_1\{v^j(t)\} + \mathcal{Y}_2\{e(t)\} \qquad (4)$$

where the symbols $\mathcal{Y}_1$, $\mathcal{Y}_2$ denote linear operators. The waveform is described by Fourier expansion into the following form:

$$\begin{aligned} i_{N1}^j(t) &= I_{N1,0}^j + \sum_{k=1}^{M}\{I_{N1,2k-1}^j\cos k\nu t \\ &\quad + I_{N1,2k}^j\sin k\nu t\}. \end{aligned} \qquad (5)$$

For the nonlinear subnetwork N_2, we calculate the steady-state response using SPICE transient simulator. Note that if the damping is very small, we recommend to use the time-domain approaches [1], [2]. We also describe the waveform by Fourier expansion

$$\begin{aligned} i_{N2}^j(t) &= \mathcal{N}\{v(t)\} \\ &= I_{N2,0}^j + \sum_{k=1}^{M}\{I_{N2,2k-1}^j\cos k\nu t \\ &\quad + I_{N2,2k}^j\sin k\nu t\} \end{aligned} \qquad (6)$$

where $\mathcal{N}$ denotes a nonlinear operator for obtaining the state-steady response of nonlinear subnetwork N_2. Thus, we have from (5) and (6) the following system of determining equations for calculating the Fourier coefficients of the state-steady response:

$$F(V) = \begin{bmatrix} I_{N1,0}(V) \\ I_{N1,1}(V) \\ \vdots \\ I_{N1,2M}(V) \end{bmatrix} + \begin{bmatrix} I_{N2,0}(V) \\ I_{N2,1}(V) \\ \vdots \\ I_{N2,2M}(V) \end{bmatrix} = 0 \qquad (7)$$

where

$$V \equiv [V_0, V_1, \cdots, V_{2M}]^T. \qquad (8)$$

To calculate the solution at the $(j+1)$th iteration, assume the solution

$$v^{j+1}(t) = v^j(t) + \Delta v(t) \qquad (9)$$

where $\Delta v(t)$ is a variational waveform, which is described by

$$\Delta v(t) = \Delta V_0 + \sum_{k=1}^{M}\{\Delta V_{2k-1}\cos k\nu t + \Delta V_{2k}\sin k\nu t\}. \quad (10)$$

Substituting $v^{j+1}(t)$ into (2), we obtain

$$\begin{aligned} F(v^j + \Delta v) &= i_{N1}(v^{j+1}) + i_{N2}(v^{j+1}) \\ &= \mathcal{Y}_1\{v^{j+1}(t)\} + \mathcal{Y}_2\{e(t)\} + \mathcal{N}\{v^{j+1}(t)\} \\ &= \mathcal{Y}_1\{v^j(t)\} + \mathcal{Y}_1\{\Delta v(t)\} + \mathcal{Y}_2\{e(t)\} + \mathcal{N}\{v^{j+1}(t)\} \quad (11) \end{aligned}$$

where we approximate $\mathcal{N}\{v^{j+1}(t)\}$ by the Taylor expansion as follows:

$$\mathcal{N}\{v^{j+1}(t)\} = \mathcal{N}\{v^j(t)\} + \left.\frac{\partial \mathcal{N}}{\partial v}\right|_{v=v^j} \Delta v(t). \quad (12)$$

Using the responses of each the subnetworks N_1 and N_2, we define the residual error $\varepsilon^j(t)$

$$\varepsilon^j(t) \equiv i_{N1}(v^j(t)) + i_{N2}(v^j(t)). \quad (13)$$

Substituting (12) into (11), we obtain

$$\begin{aligned} F(v^j + \Delta v) &\cong \mathcal{Y}_1\{v^j(t)\} + \mathcal{Y}_1\{\Delta v(t)\} + \mathcal{Y}_2\{e(t)\} + \mathcal{N}\{v^j(t)\} + \left.\frac{\partial \mathcal{N}}{\partial v}\right|_{v=v^j} \Delta v(t) \\ &= \varepsilon^j(t) + \mathcal{Y}_1\{\Delta v(t)\} + G^j(t)\Delta v(t) \\ &= \mathcal{Y}^j_{N1}(\Delta v) + \mathcal{Y}^j_{N2,t}(\Delta v) + \varepsilon^j(t) \quad (14) \end{aligned}$$

where $G^j(t)$ corresponds to the time-varying element described by

$$G^j(t) = G^j_0 + \sum_{k=1}^{M}\left\{G^j_{2k-1}\cos k\nu t + G^j_{2k}\sin k\nu t\right\}. \quad (15)$$

However, it is not easy to solve the time-varying circuit given by (14). Therefore, we approximate it by the time-invariant circuit as follow:

$$\mathcal{Y}^j_{N_1}(\Delta v) + \mathcal{Y}^j_{N_2,0}(\Delta v) + \varepsilon^j(t) = 0. \quad (16)$$

The symbols $\mathcal{Y}^j_{N_1}(\Delta v), \mathcal{Y}^j_{N_2,t}(\Delta v)$ and $\mathcal{Y}^j_{N_2,0}(\Delta v)$ in (14) and (16) denote linear operators which transform the substitution voltage source $\Delta v(t)$ into the time-domain responses of the associated subnetworks, where the subscript "t" denotes the time-varying operator and "0" the time-invariant operator.

Now, consider the sensitivity of nonlinear elements at jth solution. For the example of a nonlinear resistor $i_R = \hat{i}_R(v_R)$, the time-varying resistor is obtained as follows:

$$R(t) = \left.\frac{d\hat{i}_R}{dv_R}\right|_{v_R=v^j_R}$$

Since, it is not easy to solve the response of the time-varying circuit, we approximate it by time-invariant element

$$R_0 = \left.\frac{d\hat{i}_R}{dv_R}\right|_{v_R=v_{R0}}$$

where v_{R0} is the DC operational point. For the other elements, the time-invariant sensitivity elements are given by Table 1 in the same manner. Note that usual AC analysis of the SPICE simulation is the response of the sensitivity circuits at the DC operational point. The schematic frequency response is shown in Fig. 2 (a), and we can easily replace the sensitivity circuit by a simple RL or RC circuit as shown in Fig. 2 (b), where the impedance is estimated by

$$Z(k\nu) = \frac{V(k\nu)}{I(k\nu)} = R(k\nu) + jX(k\nu) \quad (k = 1, 2, \cdots, M)$$

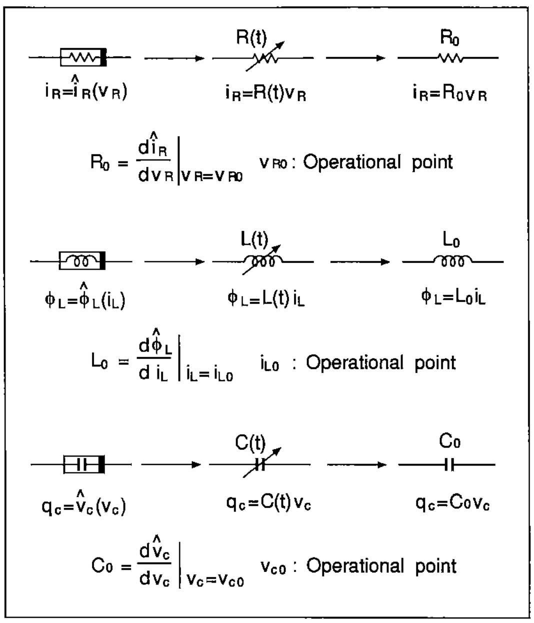
at each frequency component, and $Z(k\nu)$, $V(k\nu)$ and $I(k\nu)$ are complex value. The variational value $\Delta v(t)$ can be independently calculated by using superposition theorem at each frequency component of $\varepsilon^j(t)$. Thus, we can estimate the variational value $\Delta v(t)$ by very simple calculation of Fig. 2 (b),

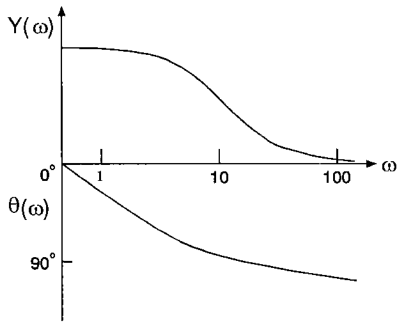
$$\mathcal{Y}_{N_1}(\Delta v) + \mathcal{Y}_{N_2,0}(\Delta v) + \varepsilon^j(t) = 0. \quad (17)$$

Therefore, we need not estimate the sensitivity circuit at each iteration.

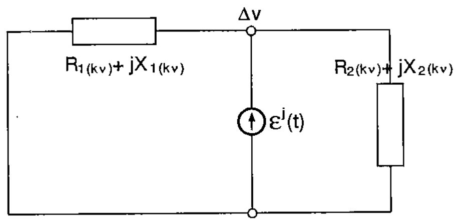
Since the variational value $\Delta v(t)$ obtained by the

Table 1 Sensitivity elements.



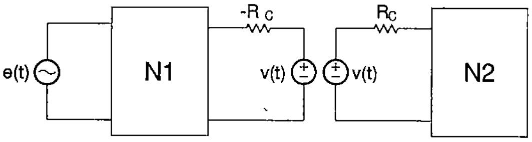


(a)

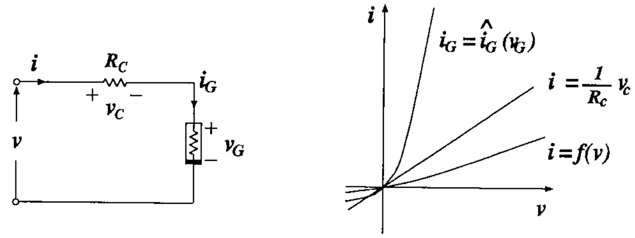


(b)

Fig. 2 (a) Frequency responses of N_1 or N_2. (b) Sensitivity circuit for each frequency component obtained AC analysis.



(a)



(b)

Fig. 3 (a) Compensation technique. (b) A schematic diagram weakening the nonlinearity.

above method is not the solution of the original time-varying sensitivity circuit, the convergence ratio will not be so large as that using the Newton method. However, we can hope for sufficient convergence ratio for the weakly nonlinear circuits.

The iteration is continued until the variation satisfies $||\Delta V|| < \delta$ for a given small δ, where

$$\Delta V = [\Delta V_0, \Delta V_1, \cdots, \Delta V_{2M}]^T. \tag{18}$$

If the residual error defined by

$$\begin{aligned}\varepsilon_H^j &= \frac{1}{T}\int_0^T [i_{N_1}(v^j) + i_{N_2}(v^j)]^2 dt \\ &= \sum_{k=2M+1}^{\infty} |I_{N,k}^j|^2.\end{aligned} \tag{19}$$

is not small enough, we must increase the number of frequency components and repeat algorithms (9) and (16). Thus, we get the steady-state response $v(t)$.

Our algorithm can be efficiently applied to weakly nonlinear circuits. However, for stiff nonlinear circuits, the iteration sometimes becomes unstable. Then, we recommend to introducing compensation resistors R_c and $-R_c$ as shown in Fig. 3 (a), [5]. The resistor R_c in the nonlinear subnetwork plays a very important role in weakening the nonlinearity, as shown in Fig. 3 (b).

Now, we summarize our method in the following algorithm:

Algorithm

Step 0: Partition the circuit into two subnetworks N_1 and N_2 using the substitution sources $v(t)$. Set the frequency component $M^\dagger$. Next, to obtain the sensitivity circuit, apply SPICE AC analysis to both the linear and nonlinear subnetworks at the DC operating point.

Step 1: Set $v^0 = 0$ and $j = 0$.

Step 2: Calculate the response $i_{N1}(t)$ of $v^j(t)$ from the simplified circuit Fig. 1 (c), and transient response $i_{N2}(t)$ of the nonlinear subnetwork using SPICE.

Step 3: Calculate the variational value Δv for the time-invariant sensitivity circuit††.

Step 4: If the variational value satisfies $||\Delta V|| < \delta$ for a given small δ, then go to Step 5. Otherwise, set $j = j+1$, $v^j = v^{j-1} + \Delta v$, and go to Step 2.

Step 5: If the residual error (19) is not small enough, then, we must increase the number of frequency components M, and go to Step 0. Otherwise, stop.

3. Error Estimation

We will estimate an error bound for the solution with the help of a proposition due to Urabe [3], where the following notation is adopted:

$$||v|| \equiv \sqrt{\frac{2}{T}\int_0^T v^2(t)dt}. \tag{20}$$

$^\dagger M$ should be chosen 2^N, because we apply FFT for the frequency domain analysis.

††For a weakly nonlinear circuit, the sensitivity circuits for linear and nonlinear subnetworks can be approximated by the simple RL or RC circuit at each frequency at Step 0 as shown in Fig. 2 (b).

If we define the Euclidean norm for a vector by

$$||V|| \equiv \sqrt{|V_0|^2 + |V_1|^2 + |V_2|^2 + \cdots + |V_{2M}|^2} \quad (21)$$

and if each element of the vector V is composed of the Fourier coefficient of $v(t)$, then both norms from (20) and (21) will coincide. Now, we derive a sufficient convergence condition of our iterational method due to Urabe's theorem [3].

Theorem: Assume the circuit is partioned into a linear and a nonlinear subnetworks. Then, we have the following determining equation:

$$F(v) = \mathcal{Y}_1(v) + \mathcal{Y}_2(e) + i_N(v) = 0 \quad (22)$$

where the symbols $\mathcal{Y}_1(v)$ and $\mathcal{Y}_2(e)$ denote *linear operators*, and the subscript "N" denote the nonlinear subnetworks. And there exist positive constants δ, κ $(0 \leqq \kappa < 1)$ and r, M, for a given approximate solution v^0 as follows:

(i) $\Omega \equiv \{v| \, ||v - v^0|| \leqq \delta\}$

(ii) $||Y_N - J_N(v)|| \leqq \dfrac{\kappa}{M}$, $(0 \leqq \kappa < 1 \ \text{for all} \ v \in \Omega)$

(iii) $||F(v^0)|| \leqq r$

(iv) $\dfrac{Mr}{1-\kappa} \leqq \delta$

(v) $||Y^{-1}|| \leqq M$

where $J_N(v)$ is the Jacobian matrix of the nonlinear subnetwork, and Y is an admittance matrix at the partitioning point, and Y_N is an admittance matrix obtained from the time-invariant equivalent circuit as follows:

$$Y_N = \left[Y_{N,R}(0)\, diag \begin{bmatrix} Y_{N,R}(k\nu) & Y_{N,I}(k\nu) \\ -Y_{N,I}(k\nu) & Y_{N,R}(k\nu) \end{bmatrix} \right] \quad (23)$$

where the admittance at $k\nu$ frequency component is obtained from Fig. 1 (e) as follows:

$$Y_N(k\nu) = Y_{N,R}(k\nu) + jY_{N,I}(k\nu) \quad (24)$$

and the admittance matrix at partitioning point Y is given by

$$Y = \left[Y_R(0)\, diag \begin{bmatrix} Y_R(k\nu) & Y_I(k\nu) \\ -Y_I(k\nu) & Y_R(k\nu) \end{bmatrix} \right] \quad (25)$$

where each component is given by

$$Y_R(k\nu) \equiv Y_{L,R}(k\nu) + Y_{N,R}(k\nu) \quad (26)$$
$$Y_I(k\nu) \equiv Y_{L,I}(k\nu) + Y_{N,I}(k\nu). \quad (27)$$

Then, our algorithm (17) converges to the exact solution $v^*(t)$ and the *truncation error* is given by

$$||v^* - v^0|| \leqq \frac{Mr}{1-\kappa}. \quad (28)$$

Proof: For $j = 0$, we have from (17):

$$\mathcal{Y}_1(v^1 - v^0) + \mathcal{Y}_{N,0}(v^1 - v^0) = -F(v^0). \quad (29)$$

From the definitions (20) and (21) and the assumptions (iii) and (iv), we have from (29),

$$||V^1 - V^0|| \leqq Mr. \quad (30)$$

For jth and $(j+1)$th iterations, we have from (16):

$$\mathcal{Y}_1(v^{j+1} - v^j) + \mathcal{Y}_{N,0}(v^{j+1} - v^j) + i_L(v^j) + i_N^j(v^j) = 0 \quad (31)$$

$$\mathcal{Y}_1(v^j - v^{j-1}) + \mathcal{Y}_{N,0}(v^j - v^{j-1}) + i_L(v^{j-1}) + i_N^{j-1}(v^{j-1}) = 0. \quad (32)$$

Since

$$i_L(v) = \mathcal{Y}_1(v) + \mathcal{Y}_2(e) \quad (33)$$

for the linear subnetwork, we have

$$\mathcal{Y}_1(v^{j+1} - v^j) + \mathcal{Y}_{N,0}(v^{j+1} - v^j) = \mathcal{Y}_{N,0}(v^j - v^{j-1}) - i_N^j(v^j) + i_N^{j-1}(v^{j-1}). \quad (34)$$

Now, describing the relation (34) in the frequency-domain, we obtain from the *mean-value theorem* the following relation:

$$Y[V^{j+1} - V^j] = \int_0^1 \{Y_N - J_N[V^{j-1} + \theta(V^j - V^{j-1})]\}(V^j - V^{j-1})d\theta. \quad (35)$$

By assumptions of V^{j-1}, V^j belonging to Ω, $V^{j-1} + \theta(V^j - V^{j-1})$ also belongs to Ω, where θ is $(0 \leqq \theta \leqq 1)$. Thus, we have from the assumptions of (ii) and (v).

$$||V^{j+1} - V^j|| \leqq M \cdot \frac{\kappa}{M} ||V^j - V^{j-1}|| = \kappa ||V^j - V^{j-1}||. \quad (36)$$

By assumption for mathematical induction,

$$||V^j - V^{j-1}|| \leqq \kappa^{j-1} ||V^1 - V^0||. \quad (37)$$

So, we have the following relation,

$$||V^{j+1} - V^j|| \leqq \kappa^j ||V^1 - V^0||. \quad (38)$$

Hence, we have the following relation from (29),

$$\begin{aligned} ||V^{j+1} - V^0|| &\leqq ||V^{j+1} - V^j|| + ||V^j - V^{j-1}|| + \cdots + ||V^1 - V^0|| \\ &\leqq (\kappa^j + \kappa^{j-1} + \cdots + \kappa + 1)||V^1 - V^0|| \\ &\leqq \frac{Mr}{1-\kappa} \leqq \delta. \end{aligned} \quad (39)$$

Thus, we have $V^{j+1} \in \Omega$. So, we obtain (28) for the error bound. Q.E.D.

If the nonlinearity of the circuit is strong, the assumption (ii) will not be small. However, it will be largely reduced by introducing the compensation element which weakens the nonlinearity of the circuit.

4. Illustrative Examples

4.1 Simple Diode Circuit

To show the efficiency of our method, consider a simple diode circuit as shown in Fig. 4 (a).

We introduce compensation resistors R_c and $-R_c$, where $-R_c$ is contained in the linear circuit and R_c in the diode circuit. The compensation resistors plays an important role in weakening the nonlinearity. We partition the circuit into two subcircuits at $a - a'$. If we choose the fundamental frequency component $\nu = 0.4433$ [rad/sec], then we have

$$\omega_1 = 3\nu, \qquad \omega_2 = 16\nu.$$

Then, we start the iteration with $v^0(t) = 0$ at the initial state, and the steady-state is obtained in 9 iterations. The waveform, frequency spectrum and convergence ratio are shown in Figs. 4 (b)–(d), where we assumed 256 frequency components.

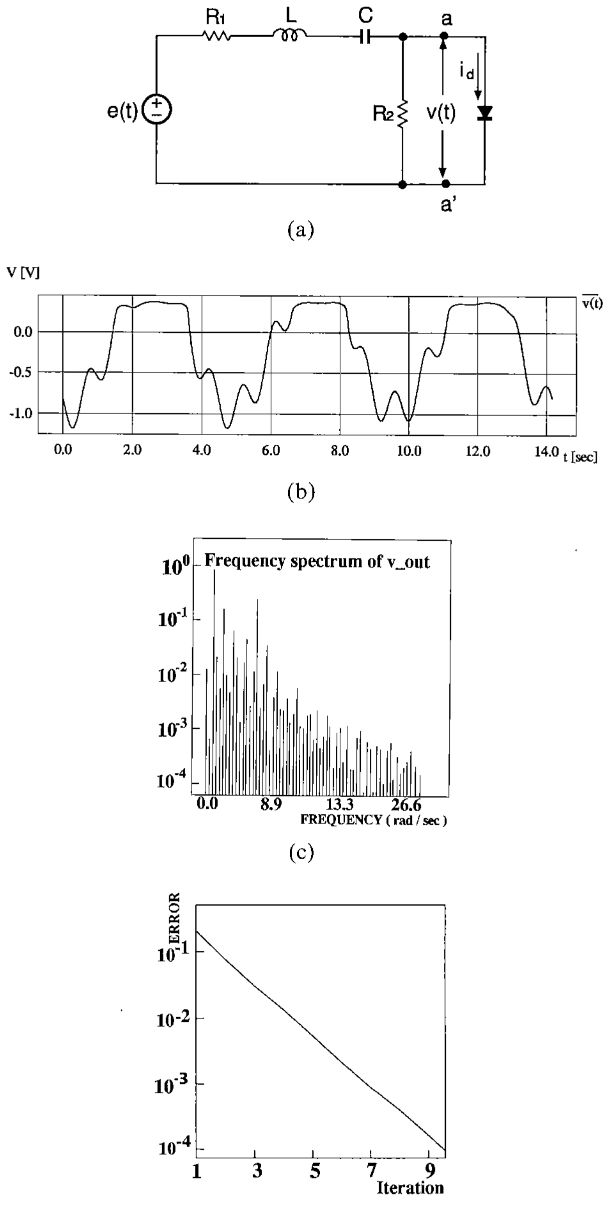


Fig. 4 (a) Simple diode circuit. The circuit parameter is $R_1 = 1\,[\Omega]$, $R_2 = 5[\Omega]$, $L = 1\,[\mathrm{H}]$, $C = 1\,[\mathrm{F}]$, $i_d = 10^8$ (exp $(40v_d) - 1)$, $e(t) = 1.5\cos 3\nu t + 1.0\cos 16\nu t$ [V]. (b) Steady-state waveform of $v_{out}(t)$. (c) Frequency spectrum of v_{out}. (d) Convergence ratio.

4.2 Amplitude Modulation Circuit

Figure 5 (a) is an example of amplitude modulation circuit. Since the circuit has a high Q resonance circuit, if we apply a brute force method to obtain the steady-state

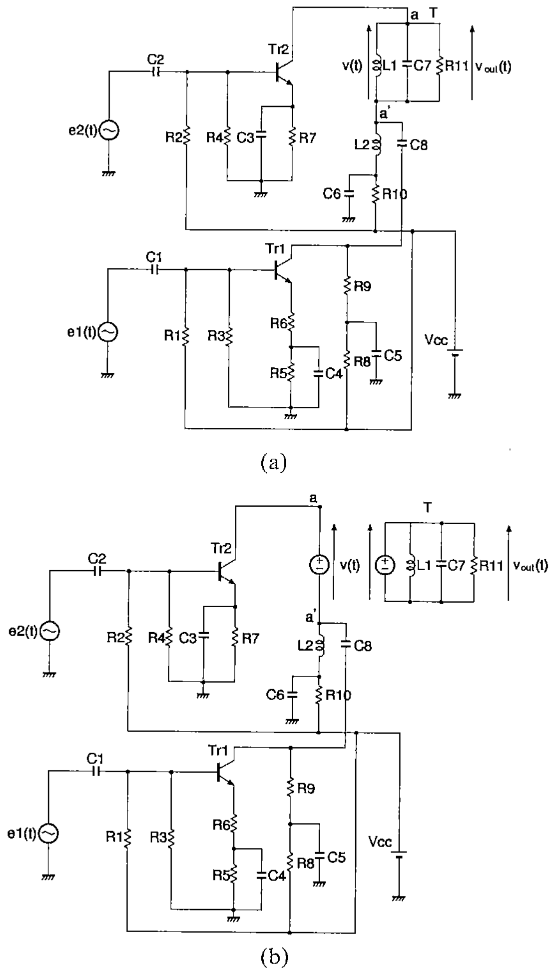


Fig. 5 (a) Amplitude modulation circuit. The circuit parameter is $R_1 = 11\,[\mathrm{k}\Omega]$, $R_2 = 43\,[\mathrm{k}\Omega]$, $R_3 = 1.7\,[\mathrm{k}\Omega]$, $R_4 = 2.7\,[\mathrm{k}\Omega]$, $R_5 = 100\,[\Omega]$, $R_6 = 10\,[\Omega]$, $R_7 = 100\,[\Omega]$, $R_8 = 100\,[\Omega]$, $R_9 = 500\,[\Omega]$, $R_{10} = 10\,[\Omega]$, $R_{11} = 3.3\,[\mathrm{k}\Omega]$, $C_1 = 100\,[\mu\mathrm{F}]$, $C_2 = 20\,[\mu\mathrm{F}]$, $C_3 = 100\,[\mu\mathrm{F}]$, $C_4 = 100\,[\mu\mathrm{F}]$, $C_5 = 10\,[\mu\mathrm{F}]$, $C_6 = 10\,[\mu\mathrm{F}]$, $C_7 = 0.2\,[\mu\mathrm{F}]$, $L_1 = 100\,[\mu\mathrm{H}]$, $L_2 = 100\,[\mathrm{mH}]$, $e_1(t) = 0.3\sin 4\nu t$ [V], $e_2(t) = 1.0\sin 20\nu t$ [mV]. (b) Partitioning amplitude modulation circuit into two subnetworks.

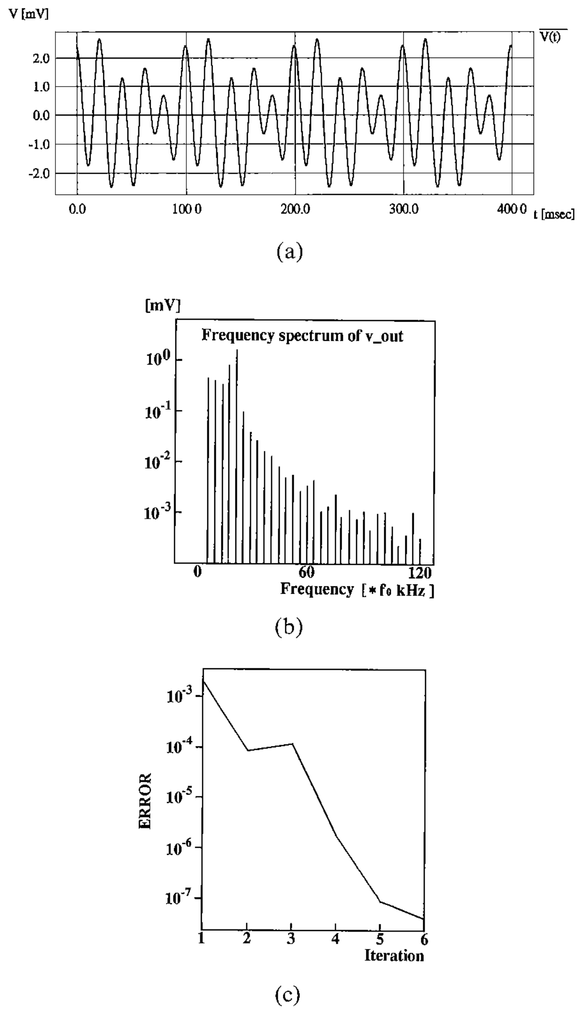


Fig. 6 (a) Steady-state waveform of $v_{out}(t)$. (b) Frequency spectrum of v_{out}. (c) Convergence ratio.

solution, it will take a large computation time. We partition the circuit into two subcircuits at $a - a'$ as shown in Fig. 5 (b). If we choose the fundamental frequency component $\nu = 2\pi \times 2.5 \times 10^3$ [rad/sec], then we have

$$\omega_1 = 4\nu, \qquad \omega_2 = 20\nu.$$

Then, we start the iteration with $v^0(t) = 0$ at the initial state, the and the steady-state is obtained in 6 iterations. The waveform, frequency spectrum and convergence ratio are shown in Figs. 6 (a)–(c), where we assumed 128 frequency components.

5. Conclusions and Remarks

We partition a circuit into linear and nonlinear subnetworks with substitution theorem. Each subnetwork is solved by the different techniques of AC and transient analysis using SPICE simulator. At the first step of the algorithm, the linear subnetwork is solved by SPICE AC analysis, and it is replaced by a simple RL or RC circuit at each frequency component. Similarly, the nonlinear subnetwork is also solved by SPICE AC analysis at the DC operating point, and it is also replaced by the simple RL or RC circuit with the same technique as for the linear subnetwork. If the nonlinearity is not very strong, the circuit will be used as the sensitivity circuit for calculating the variational value $\Delta v(t)$. Thus, our algorithm is very simple. If we can partition the circuit such that the nonlinear subnetwork N_2 is small scale compared to the linear subnetwork N_1, our algorithm will be efficiently applied to large scale circuits.

The convergence ratio in this paper may be small compared with the Newton methods such as Refs. [1]–[4], [6], because our method belongs to the relaxation method in the frequency domain. However, the convergence ratio depends on the nonlinearity. If the nonlinearity is not very strong, we will have sufficient convergence ratio. Note that time-domain shooting methods [1], [2] are efficiently applied to the small scale circuits because they depend on the Newton method. However, they are inefficient for large scale circuits, where the number of state-variables is so great that it takes much computation time to obtain the variational values. The frequency domain methods [3]–[4], [6] can be only applied when the circuit is partitioned into linear subnetwork and nonlinear resistive elements. Therefore, it is not allowed when the nonlinear elements have the dynamics. Many nonlinear elements such as transistor, diode and FET have dynamics, so that the method can not be applied. The ideas of our method are similar to those of the reference [5]. Hence, the convergence ratio is equal to [5], and it can be applied to a large class of nonlinear networks. Furthermore, we have improved the method such that the steady-state response can be obtained by the use of only SPICE simulator.

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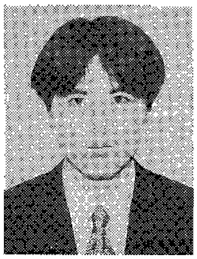

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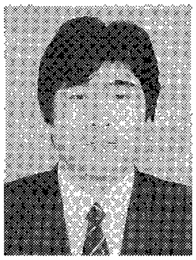

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