Prof. A. Tesi for helpful comments and advise for preparing the manuscript.

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Quasi-Synchronization Phenomena in Chaotic Circuits Coupled by One Resistor

Yoshifumi Nishio and Akio Ushida

Abstract— **In this brief, synchronization phenomena observed from simple chaotic circuits coupled by one resistor are investigated. A simple three-dimensional autonomous circuit is considered as a chaotic subcircuit. By carrying out circuit experiments and computer calculations for two, three or four subcircuits case, various interesting synchronization phenomena of chaos, which are different types from the results reported before, are confirmed to be stably generated. Further, quasi-synchronization of asymmetric chaos are investigated with attention on the number of synchronization states.**

I. INTRODUCTION

Coupled oscillators systems are good model to describe various nonlinear phenomena in the field of natural science and a number of excellent studies on mutual synchronization of oscillators have been carried out ([1]–[3] and therein). In [1], Kimura *et al.* investigated synchronization phenomena observed from two van der Pol oscillators coupled by resistors and confirmed that the two oscillators synchronized at the opposite-phase. They considered that such synchronization occurred to minimize the current through the coupling resistors. Later they investigated three oscillators case and confirmed the generation of the three-phase synchronization [2]. Also, we investigated the synchronization phenomena of many oscillators with strong nonlinearity coupled by one resistor [3].

On the other hand, many nonlinear dynamical systems in the various fields have been clarified to exhibits chaotic oscillations and recently applications of chaos to engineering systems attract many researchers' attentions, for example, chaos noise generator ([4] and therein), control of chaos [5], [6], synchronization of chaos [7]–[10], and so on. Among the studies on such applications, synchronization of chaotic systems or signals is extremely interesting, because the chaotic solution is unstable and small error of initial values must be expanded as time goes. As far as we know, such phenomena have been firstly reported to be generated in simple real circuits by a group of Saito [7]. Since Pecora *et al.* have investigated such phenomena theoretically [8], many papers have been published until now. Further, the technique of synchronization of chaos is also applied to realize secure communication systems using chaos ([11] and therein). However, almost all studies on synchronization of chaos treat only the case that chaotic signals generated from two identical chaotic systems are synchronized at the in-phase. Namely, as far as we know, another types of synchronizations of chaotic systems have not been reported at all. Though secure communication systems do not need another types of synchronizations, the investigation of various synchronizations of chaotic systems will open the way to another applications of chaos.

In this brief, we investigate quasi-synchronization phenomena observed from simple chaotic circuits coupled by one resistor. In this paper we use the term synchronization of chaos as follows. Two chaotic signals (e.g., currents through inductors) which are the function of continuous time $S_1(t)$ and $S_2(t)$ are said to be

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synchronized at the opposite phase, if $S_1(t)+S_2(t) \simeq 0$. For another types of synchronization states rigorous definition of synchronization seems to be extremely difficult. Then, we use the term quasi-synchronization. The term means that a chaotic signal from one subcircuit $S_i(t)$ and the other chaotic signal from the other subcircuit $S_j(t)$ have the relation such as $|S_i(t) \pm S_j(t-T)| < \varepsilon$ where a constant T represents phase shift between two signals and a constant ε is much smaller than the average amplitude of each chaotic signal. This is not mathematical definition and is used only for qualitative explanation of the observed phenomena. We consider a simple three-dimensional autonomous circuit as a chaotic subcircuit. This circuit is a symmetric version of the chaotic circuit proposed by Inaba *et al.* [12] and one of the simplest autonomous chaotic circuits. They used ideal piecewise linear model of diodes in [12], but in this brief the V–I characteristics of the nonlinear resistor consisting of diodes are approximated by a smooth function. This is more real than piecewise linear approximation in the sense that every real elements in the natural field are not piecewise linear. By carrying out circuit experiments and computer calculations for two, three or four subcircuits case, various interesting synchronization phenomena which are different types from the results reported before, namely opposite-phase quasi-synchronization of two chaos, three-phase quasi-synchronization of three chaos and various kinds of quasi-synchronizations of four chaos, are confirmed to be stably generated. Such types of synchronization of chaos have never been reported yet. Moreover, quasi-synchronization of asymmetric chaos are investigated with attention on the number of synchronization states.

II. Circuit Model

The circuit model is shown in Fig. 1(a). In our system N same chaotic circuits are coupled by one resistor. Each chaotic subcircuit is a symmetric version of the circuit model proposed by Inaba *et al.* [12]. It consists of three memory elements, one linear negative resistor and one nonlinear resistor, which is realized by connecting some diodes, and is one of the simplest chaotic circuits. This circuit exhibits bifurcation phenomena which are similar to those reported in [13], namely, bifurcation route from one-periodic attractor to chaotic attractor is explained as follows:

One symmetric one-periodic attractor—(symmetry breaking transition) → Two asymmetric one-periodic attractors—(period-doubling route) → Two asymmetric chaotic attractors—(symmetry recovering crisis) → One symmetric chaotic attractor.

Fig. 1(b) and (c) shows typical examples of chaotic attractors obtained from the chaotic subcircuit. In the following circuit experiments, the values of the inductors and the capacitor in each chaotic subcircuit are fixed and those values are measured as $L_1 = 204.15$ mH $\pm$ 0.073%, $L_2 = 9.933$ mH $\pm$ 0.030% and $C = 0.034\ 25$ μF $\pm$ 0.29%.

At first, we approximate the IV characteristics of the nonlinear resistor consisting of diodes by the following function.

$$v_d(i_k) = \sqrt[9]{r_d i_k}. \tag{1}$$

By changing the variables and parameters

$$\begin{aligned} t &= \sqrt{L_1 C}\tau, \quad \text{"}\cdot\text{"} = \frac{d}{d\tau}, \\ I_k &= a\sqrt{\frac{C}{L_1}}x_k, \quad i_k = a\sqrt{\frac{C}{L_1}}y_k, \quad v_k = az_k \\ \alpha &= \frac{L_1}{L_2}, \quad \beta = r\sqrt{\frac{C}{L_1}}, \quad \gamma = R\sqrt{\frac{C}{L_1}} \\ &\left(\text{where } a = \sqrt[8]{\sqrt{r_d\sqrt{\frac{C}{L_1}}}}\right) \end{aligned} \tag{2}$$

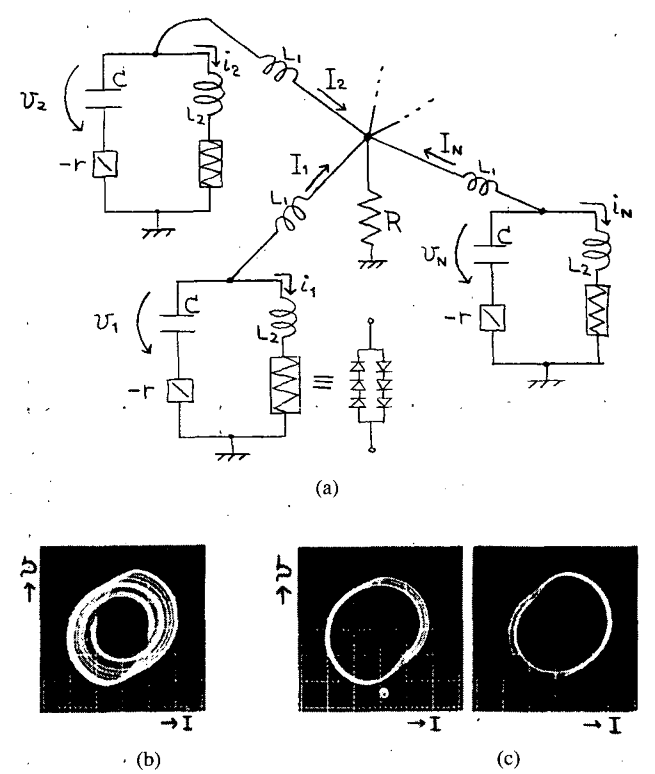

Fig. 1. Circuit model. (a) Chaotic circuits coupled by one resistor. (b) Symmetric chaotic attractor (r =650 Ω). (b) Coexistence of two asymmetric chaotic attractors (r =554 Ω). Horizontal: 0.4 mA/div. Vertical: 1 V/div.

the circuit equation is normalized and described as

$$\begin{aligned} \dot{x}_k &= \beta(x_k + y_k) - z_k - \gamma\sum_{j=1}^{N} x_j \\ \dot{y}_k &= \alpha\{\beta(x_k + y_k) - z_k - f(y_k)\} \\ \dot{z}_k &= x_k + y_k \qquad (k = 1, 2, \cdots, N) \end{aligned} \tag{3}$$

where

$$f(y_k) = \sqrt[9]{y_k}. \tag{4}$$

For computer calculations, in order to consider the difference of real circuit elements, (3) is rewritten as follows:

$$\begin{aligned} \dot{x}_k &= \beta(x_k + y_k) - z_k - \gamma\sum_{j=1}^{N} x_j \\ \dot{y}_k &= \alpha\{\beta(x_k + y_k) - z_k - f(y_k)\} \\ \dot{z}_k &= (1 + \Delta\omega_k)(x_k + y_k) \qquad (k = 1, 2, \cdots, N). \end{aligned} \tag{5}$$

In the following computer calculations, the parameter values corresponding to the inductors and the capacitor are fixed as $\alpha = 20.6$ and $\Delta\omega_k = 0.005(k-1)$ and (6) is calculated by using the Runge–Kutta method with step size $\Delta t = 0.01$.

The circuit in Fig. 1(a) can be regarded as a chaotic circuit version of the coupled van der Pol oscillators in [1]–[3]. Hence, this coupled chaotic circuits are expected to minimize the current through the coupling resistor (R) and to exhibit various types of synchronization phenomena.

III. Two Subcircuits Case

In this section, we consider the case of $N = 2$, namely only two chaotic subcircuits are coupled by one resistor.

At first, fix $\beta = 0.325$(r = 650 Ω) and vary coupling parameter γ (R). Fig. 2 shows computer calculated results and the

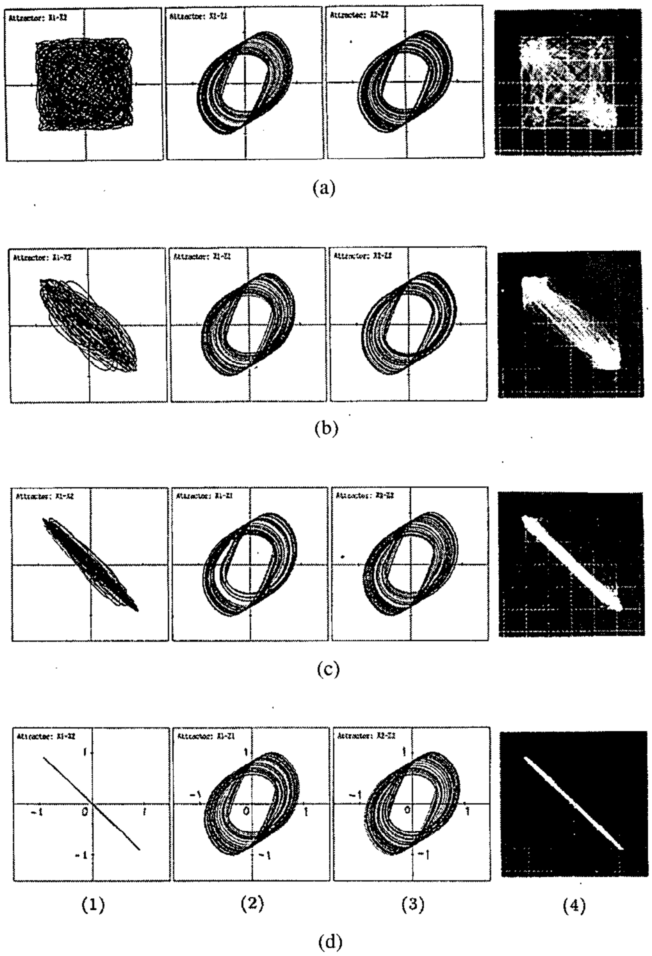

Fig. 2. Synchronization of two symmetric chaos. $\beta = 0.325$ ($r = 650\,\Omega$). (a) $\gamma = 0.0$ ($R = 0\,\Omega$). (b) $\gamma = 0.10$ ($R = 48\,\Omega$). (c) $\gamma = 0.22$ ($R = 292\,\Omega$). (d) $\gamma = 0.35$ ($R = 540\,\Omega$). (1) x_1 versus x_2. (2) x_1 versus z_1. (3) x_2 versus z_2. (4) Circuit experimental results. I_1 versus I_2. 0.4 mA/div.

corresponding circuit experimental results. As shown in the figure two subcircuits generate symmetric chaos for this parameter value. However, as $\gamma(R)$ increases, chaotic signals obtained from two subcircuits become to be synchronized at the opposite-phase. When $\gamma = 0.35$ ($R = 540\ \Omega$), two chaotic signals seem to be completely synchronized at the opposite-phase.

Second, let us investigate the synchronization of asymmetric chaos. In this case there exist two different types of synchronization of chaos as shown in Fig. 3. In the Fig. 3(a) two asymmetric chaos located symmetrically with respect to the origin are completely synchronized. We call this type of synchronization as symmetric synchronization. While, in the Fig. 3(b) two chaos tends to be synchronized at the opposite-phase but they cannot completely, because two asymmetric attractors are not located symmetrically. We call this type of synchronization as asymmetric synchronization. For small γ values (namely when coupling is small), asymmetric synchronization of chaos can coexist with symmetric synchronization of chaos. However, for relatively larger γ values as Fig. 3, the value of β at which asymmetric synchronization stably exist must be larger than the value for symmetric synchronization. Namely, symmetric and asymmetric synchronizations cannot coexist for such γ values. We can explain this reason physically as follows. For symmetric synchronization, two chaotic signals can be synchronized completely. Hence, the current through the coupling resistor is extremely small

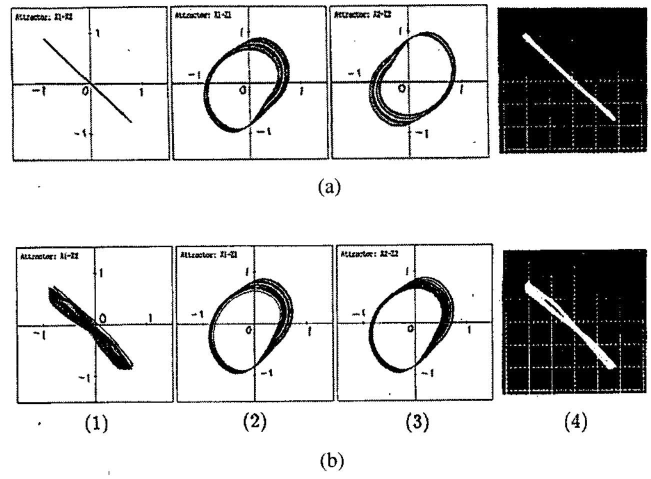

Fig. 3. Synchronization of two asymmetric chaos. $\gamma = 0.10(R = 190\ \Omega)$. (a) Symmetric synchronization for $\beta = 0.287$ ($r = 554\ \Omega$). (b) Asymmetric synchronization for $\beta = 0.308$ ($r = 586\ \Omega$). (1) x_1 versus x_2. (2) x_1 versus z_1. (3) x_2 versus z_2. (4) Circuit experimental results. I_1 versus I_2. 0.4 mA/div.

and the loss consumed by the coupling resistor may be negligible. While for asymmetric synchronization, two chaotic signals cannot be synchronized completely and hence some power must be consumed by the coupling resistor. Therefore, in order to generate chaotic attractors larger power must be supplied to the circuit by the negative resistor.

IV. Three Subcircuits Case

In this section, we consider the case of $N = 3$. In this case, three-phase quasi-synchronization of chaos can be observed.

Fig. 4 shows the computer calculated results and the corresponding circuit experimental results. Each subcircuit exhibits symmetric chaos as shown in Figs. 1(b) and 2(d(2)(3)) for these parameter values. In the two subcircuits case two symmetric chaos can be synchronized completely for larger γ value, while three symmetric chaos cannot be synchronized completely even if γ is large. The reason is explained as follows. The circuit tends to be synchronized at the three-phase in order to minimize the current through the coupling resistor. However, the sum of chaotic signals cannot be zero even if the phase-difference between two signals is equal to $2\pi/3$, because each chaotic signal is not completely sinusoidal. The three-phase quasi-synchronization has two different phase-states, namely the phase of one phase-state is arranged as $\{1, 2, 3\}$, while the phase of the other phase-state is arranged as $\{1, 3, 2\}$. We have also confirmed that one-periodic attractors bifurcate to chaotic attractors by varying the value of β keeping the three-phase quasi-synchronization as well as the two subcircuits case.

Next, let us consider three-phase quasi-synchronization of three asymmetric chaos. Because each subcircuit has two asymmetric chaotic attractors located symmetrically with respect to the origin as shown in Fig. 1(c), there exist $2^3 = 8$ different combinations of attractors coexist for three subcircuits. Further, each synchronization has two different phase-states as well as symmetric attractors case ($\{1, 2, 3\}$ and $\{1, 3, 2\}$). Totally, 16 different synchronization states coexist. Fig. 5 shows two different synchronization states for three-phase quasi-synchronization. Note that when three attractors obtained from three subcircuits have the same shape, attractor is one-periodic as Fig. 5(a). There coexist four such synchronization states. While attractor is chaotic for the other 12 cases as Fig. 5(b). This reason is explained as well as the asymmetric synchronization for two

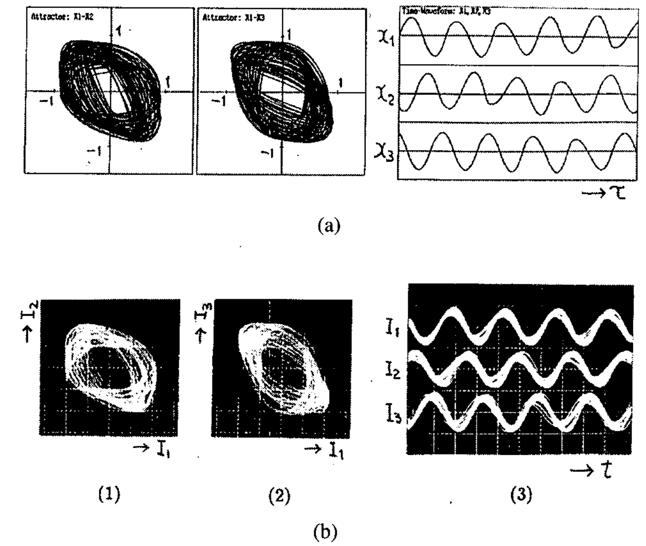

Fig. 4. Three-phase quasi-synchronization of three symmetric chaos. (a) Computer calculated results. $\beta = 0.305$ and $\gamma = 0.05$. (a1) x_1 versus x_2. (a2) x_1 versus x_3. (a3) Time-waveforms. (b) Circuit experimental results. $r = 650\ \Omega$ and $R = 50.8\ \Omega$. (b1)(b2) 0.4 mA/div. (b3) Horizontal: 0.2 ms/div. Vertical: 1 mA/div.

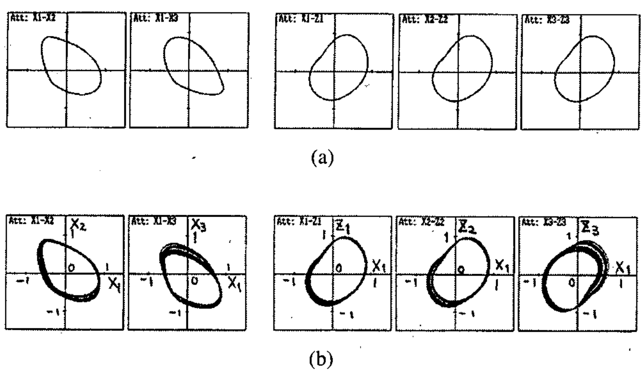

Fig. 5. Three-phase quasi-synchronization of three asymmetric chaos (computer calculated results). $\beta = 0.287$ and $\gamma = 0.05$. (a) One-period. (b) Chaos.

subcircuits case. Namely, because the current through the coupling resistor for Fig. 5(a) is larger than that for Fig. 5(b). As γ increases, the synchronization states shown in Fig. 5(a) also bifurcates to chaos. For such parameter values, the attractor corresponding to Fig. 5(b) becomes to be unstable and there exist only four synchronization states.

V. Four Subcircuits Case

In this section, we consider the case of $N = 4$.

At first, let us investigate the quasi-synchronization phenomena of symmetric chaos. In this case we observed three different types of quasi-synchronization phenomena; in and opposite-phases quasi-synchronization, two-pairs of opposite-phase quasi-synchronizations, and self-switching of three opposite-phase quasi-synchronizations. These three types of quasi-synchronizations were observed for different parameter values and they cannot coexist.

In and opposite-phases quasi-synchronization is observed for relatively larger γ values. This quasi-synchronization is shown in Fig. 6(a). If we take the subcircuit one as reference circuit, one of remaining three subcircuits is almost synchronized at the in-phase and the remaining two subcircuits are almost synchronized at the

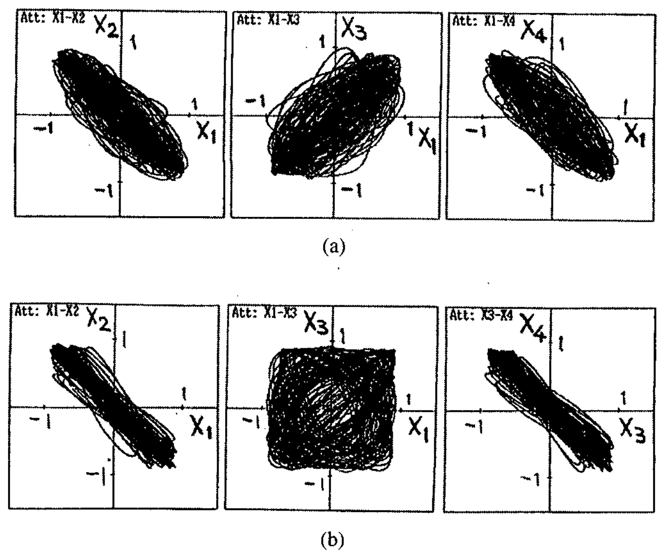

Fig. 6. Quasi-synchronization of four symmetric chaos (computer calculated results). $\beta = 0.30$. (a) In and opposite-phases quasi-synchronization for $\gamma = 0.40$. (b) Two pairs of opposite-phase quasi-synchronizations for and $\gamma = 0.10$.

opposite phase. There coexist three different phase-states for this type of quasi-synchronization.

Two pairs of opposite-phase quasi-synchronizations are shown in Fig. 6(b). Two of four subcircuits are almost synchronized at the opposite-phase. Also the remaining two subcircuits are almost synchronized at the opposite-phase. However, two opposite-phase quasi-synchronizations are independent. Before we carried out experiments, we had considered that the number of the combination of subcircuits must be three because of the symmetry of the coupling, namely {1, 2} and {3, 4}, {1,3 } and {2, 4} and {1, 4} and {2, 3}. However, the combination seems to be decided by the difference of real circuit elements and we observed only one combination for both of computer calculations and circuit experiments. For computer calculations x_1 and x_2 are synchronized at the opposite phase and x_3 and x_4 are also synchronized, but x_1 and x_3 are independent. Other combination states can be observed only in the long transient states and they are eventually attracted to the state in Fig. 6(a). For circuit experiments I_1 and I_3 are synchronized at the opposite phase and I_2 and I_4 are also synchronized, but I_1 and I_2 are independent. Other combination states are not observed.

Self-switching of three opposite-phase quasi-synchronizations is that three phase-states corresponding to the two pairs of opposite-phase quasi-synchronizations are switched alternately. The order of the appearance of three phase-states is truly chaotic. Further switching period is also chaotic, namely a state may be switched to the next state instantly and a state may be switched after about one second. Because of the difference of real circuit elements, stability of three phase-states must be different and the difference must influence the switching period. We also observed such quasi-synchronizations from computer calculations. The detailed results on this phenomenon will be discussed elsewhere.

Next, let us investigate the quasi-synchronization phenomena of asymmetric chaos. In this case we observed only the two-pairs of opposite-phase quasi-synchronization. The in and opposite-phases or the self-switching cannot be observed. As well as symmetric chaos, the combination the synchronized subcircuits seems to be decided by the difference of real circuit elements and we observed only one combination; {1, 2} and {3, 4}. Because two asymmetric attractors coexist in each subcircuit, we had considered that there

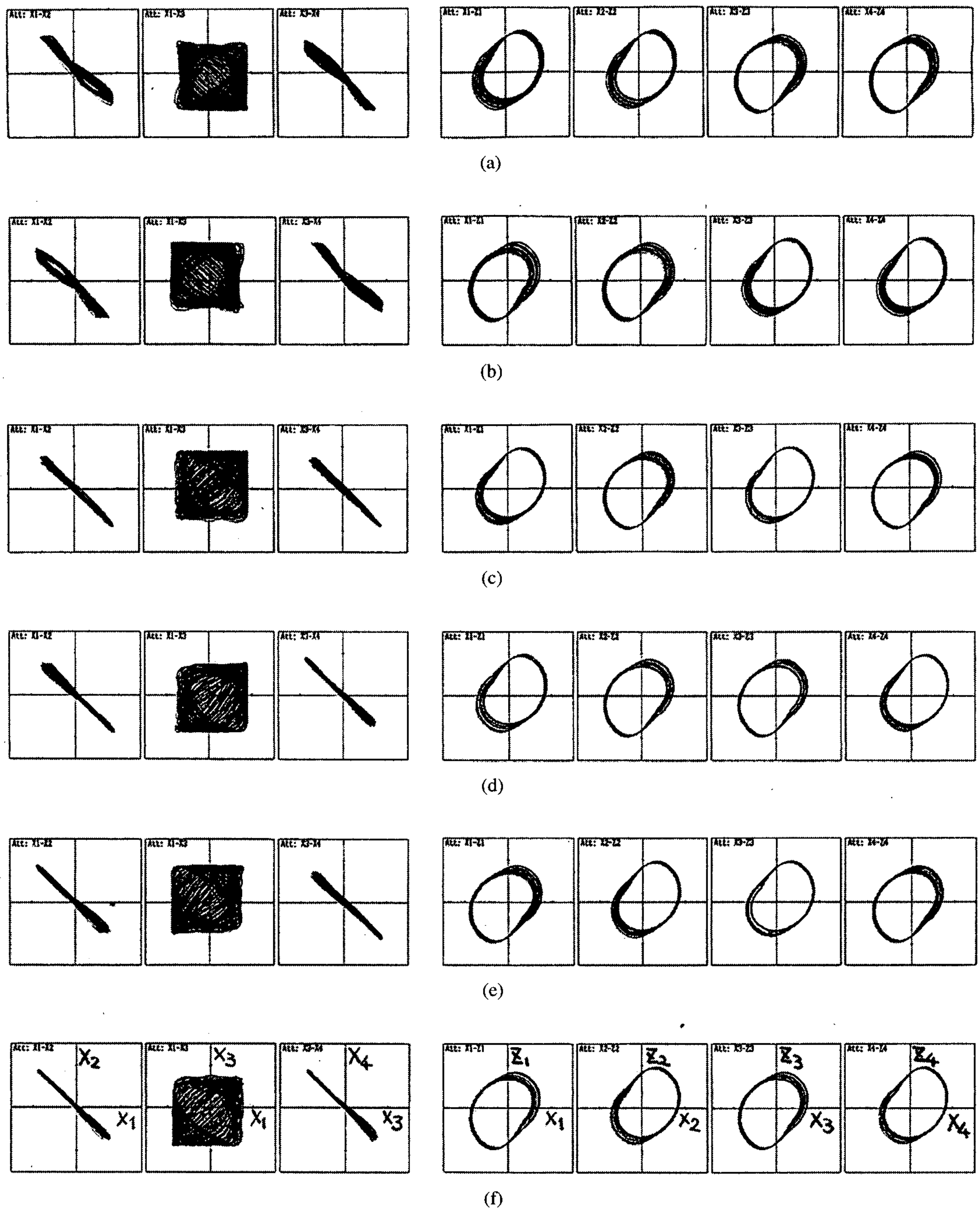

Fig. 7. Two pairs of opposite-phase quasi-synchronizations of four asymmetric chaos (computer calculated results). $\beta = 0.285$ and $\gamma = 0.10$. (a) and (b) Two pairs of asymmetric synchronizations. (c)–(f) Two pairs of symmetric synchronizations.

exist $2^4 = 16$ different synchronization states. However, we observed only six different synchronization states for both of computer calculations and circuit experiments as shown in Fig. 7. The six different synchronization states are divided into two groups, namely two pairs of asymmetric synchronizations [Fig. 7(a) and (b)] and two pairs of symmetric synchronizations [Fig. 7(c)–(f)]. As well as two subcircuits, these two different types of quasi-synchronizations cannot coexist for larger γ values.

VI. Conclusion

In this brief, we investigated quasi-synchronization phenomena observed from simple chaotic circuits coupled by one resistor. By carrying out circuit experiments and computer calculations for two, three or four subcircuits case, we confirmed that various kinds of quasi-synchronization phenomena of chaos were stably observed.

We would like to emphasize that almost synchronization phenomena in this paper had not been reported yet. Moreover such interesting phenomena have been observed from real circuit model made up easily.

We enumerate the synchronization phenomena introduced in this paper. The numbers in ( ) denote the number of the coexisting synchronization states.

- 2 subcircuits case
  - a) Opposite-phase synchronization of symmetric chaos (1)
  - b) Symmetric synchronization of asymmetric chaos (2)
  - c) Asymmetric synchronization of asymmetric chaos (2)

- 3 subcircuits case
  a) Three-phase quasi-synchronization of symmetric chaos (2)
  b) Three-phase quasi-synchronization of asymmetric chaos (same shape 4, not same shape 12)
- 4 subcircuits case
  a) In and opposite-phases quasi-synchronization of symmetric chaos (3)
  b) Two pairs of opposite-phase quasi-synchronizations of symmetric chaos (1)
  c) Self-switching of three opposite-phase quasi-synchronizations of symmetric chaos (1)
  d) Two pairs of asymmetric synchronization of asymmetric chaos (2)
  e) Two pairs of symmetric synchronization of asymmetric chaos (4)

For five or more subcircuits case, we could not observe any synchronization phenomena of chaos. We have confirmed that five or more van der Pol oscillators could not be synchronized when the nonlinearity is weak [3]. Because the chaotic signals obtained from the present subcircuit are similar to sinusoidal wave, five or more chaotic subcircuits could not be synchronized. If we use circuits generating chaotic waveform looks like rectangular wave, they may be synchronized.

Though we omit to introduce some results for another chaotic circuits in this paper, we have carried out circuit experiments for some types of chaotic circuits and have confirmed the generation of similar types of quasi-synchronization of chaos. Some examples of the asymmetric synchronization and the three-phase quasi-synchronization for another chaotic circuit can be seen in [14]. Hence, various interesting quasi-synchronization phenomena introduced in this brief are considered to be generated in various coupled systems. Namely, quasi-synchronization phenomena of chaos are not special phenomena observed from only a few systems, but common phenomena as well as chaos.

We consider that dimension of chaotic attractors must be deeply related with quasi-synchronization of chaos and that it will be very useful to classify quasi-synchronization of chaos. Hence, the analysis of Lyapunov exponents of quasi-synchronization of chaos is extremely important as our future research. Further, we must establish the method for theoretical analysis of the quasi-synchronizations of chaos. We hope that our study would motivate the establishment of analyzing method for quasi-synchronizations of chaos and that the phenomena in this brief would be applied to various engineering systems.

Acknowledgment

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Robustness of Pole-Clustering in a Ring for Structured Perturbation Systems

Chun-Hsiung Fang and Chun-Lin Lu

Abstract—**A simple approach is proposed to ensure that all poles of the uncertain system are clustered in a prescribed ring. The explicit bounds on linear time-invariant structured perturbations are obtained. Under these allowable highly structured perturbations, both stability robustness and certain performance robustness will thus be ensured. In the literature, as far as we are aware, little effort has been devoted to investigating pole-clustering robustness in such region.**

Notations

First of all, we introduce some notations which will be used throughout this brief.

$\mathbb{C}$	field of complex number
$\lambda_i(M)$	the ith eigenvalue of $M \in \mathbb{C}^{n\times n}$
$\rho(M)$	spectral radius of matrix $M \in \mathbb{C}^{n\times n}$
$\lvert M\rvert$	determinant of matrix $M \in \mathbb{C}^{n\times n}$
$\lvert z\rvert$	magnitude of $z \in \mathbb{C}$
$[m_{ij}]$	the (i, j)th element of matrix M

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