LETTER

Synchronization Phenomena in RC Oscillators Coupled by One Resistor

Seiichiro MORO†, *Student Member*, Yoshifumi NISHIO††, *and* Shinsaku MORI†, *Members*

SUMMARY In this study, we propose a system of N Wien-bridge oscillators with the same natural frequency coupled by one resistor, and investigate synchronization phenomena in the proposed system. Because the structure of the system is different from that of LC oscillators systems proposed in our previous works, this system cannot exhibit N-phase oscillations but 3-phase and in-phase oscillations. Also in this system, we can get an extremely large number of steady phase states by changing the initial states. In particular, when N is not so large, we can get more phase states in this system than that of the LC oscillators systems. Because this system does not include any inductors and is strong against phase error this system is much more suitable for applications on VLSI compared with coupled system of van der Pol type LC oscillators.
key words: *coupled oscillators, Wien-bridge oscillator, phase states, VLSI implementation*

1. Introduction

There have been many investigations of the mutual synchronization of oscillators ([1]–[8] and therein). Endo et al. have analyzed the systems of large number of coupled van der Pol oscillators [1]–[3]. Kimura et al. have confirmed that two oscillators coupled by one resistor are synchronized at opposite phase [4] and three oscillators coupled by one resistor are synchronized at 3-phase [5]. Moreover, we have investigated the synchronization phenomena in N oscillators coupled by one resistor [6]–[8]. When the nonlinearity is the third-power characteristic, N-phase oscillations are stably excited and the system has $(N-1)!$ phase states [6]. When the nonlinearity is the fifth-power characteristic, not only N but also $N-1, N-2, \cdots 3, 2$-phase oscillations are stably excited and the system has much more phase states than that of system with the third-power nonlinear characteristics [7]. Moreover we have proposed the coupled oscillators networks [9] based on the system described in Ref. [6] for cellular neural networks (CNN) [10]. Thus the coupled oscillators systems are expected to applications to neural networks and large scale memories.

In this study, we investigate synchronization phenomena in the Wien-bridge oscillators with the same natural frequency coupled by one resistor by both of computer calculations and circuit experiments. In this system the synchronization phenomena which have never been seen can be observed; only in-phase and 3-phase oscillation can be seen in spite of N. This system can generate a large number of steady states and has the similar structure as the systems described in Refs. [6]–[8]. Further, this system does not include any inductors and is strong against phase error. Hence, this system is much more suitable for applications on VLSI compared with coupled system of van der Pol type LC oscillators.

2. Circuit Model

Recently, we have reported synchronization phenomena in N van der Pol LC oscillators with the same natural frequency coupled by one resistor. In the systems, various synchronization phenomena can be stably observed, because they tends to minimize the current through the coupling resistor. When the nonlinear characteristics are the third-power, we can see N-phase oscillations. When we take the waveform observed in one oscillator as a reference signal, the other oscillators can take any phase differences among $\phi_k = 2k\pi/N$ $(k = 1, 2, \cdots, N-1)$. Therefore, this system can take $(N-1)!$ phase states. This means that this system can take 479,001,600 steady states when $N = 13$. On the other hand, when the nonlinear characteristics are the fifth-power, we can see not only N-phase oscillation but also $N-1, N-2, \cdots, 2$-phase oscillations by choosing the initial states of oscillators. In this system, we can take much more steady states than that of the system with the third-power nonlinear characteristics. For example, we can take 792,712,283 steady states for $N = 13$.

In this study we investigate N Wien-bridge oscillators shown in Fig. 1 with the same natural frequency coupled by one resistor. The circuit model is shown in Fig. 2. We consider the negative-feedback amplifiers have nonlinear gain characteristics approximately as follows because of the saturation characteristics of the op amps.

$$v'_k = g_1 v_{k3} - g_3 v_{k3}^3 \qquad (1)$$

So circuit equations are described as follows,

Manuscript received April 3, 1995.
Manuscript revised June 12, 1995.
†The authors are with the Faculty of Science and Technology, Keio University, Yokohama-shi, 223 Japan.
††The author is with the Faculty of Engineering, Tokushima University, Tokushima-shi, 770 Japan.

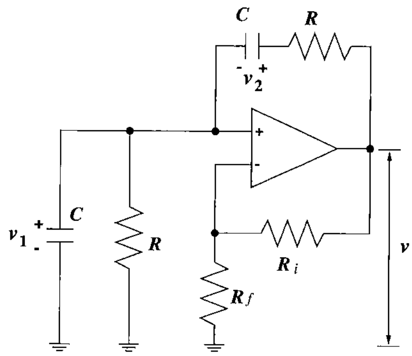


Fig. 1 Wien-bridge oscillator.

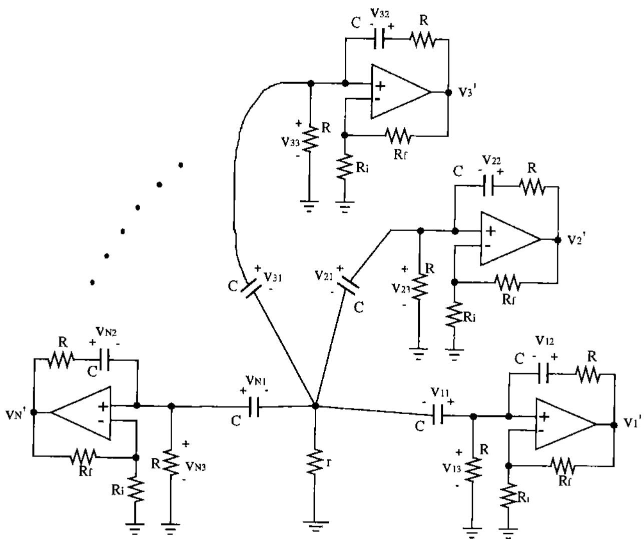


Fig. 2 Circuit model.

$$\begin{cases} C\dfrac{dv_{k1}}{dt} = C\dfrac{dv_{k2}}{dt} - \dfrac{1}{R}v_{k3} \\ C\dfrac{dv_{k2}}{dt} = \dfrac{1}{R}(g_1 v_{k3} - g_3 v_{k3}^3 - v_{k2} - v_{k3}) \\ C\displaystyle\sum_{j=1}^{N}\dfrac{dv_{k1}}{dt} = \dfrac{1}{r}(v_{k3} - v_{k1}) \\ (k = 1, 2, \cdots, N) \end{cases} \tag{2}$$

By changing variables,

$$\begin{aligned} & t = RC\tau, \\ & x_k = \sqrt{\frac{g_1 - 3}{3g_3}}v_{k1}, \quad y_k = \sqrt{\frac{g_1 - 3}{3g_3}}v_{k2}, \\ & z_k = \sqrt{\frac{g_1 - 3}{3g_3}}v_{k3}, \quad \alpha = \frac{r}{R}, \quad \varepsilon = g_1 - 3 \end{aligned} \tag{3}$$

Equation (2) is normalized as

$$\begin{cases} \dot{x}_k = \dot{y}_k - z_k \\ \dot{y}_k = \varepsilon\left(z_k - \dfrac{z_k^3}{3}\right) - y_k + 2z_k \\ z_k = x_k + \alpha\displaystyle\sum_{j=1}^{N}\dot{x}_j \\ (k = 1, 2, \cdots, N) \end{cases} \tag{4}$$

In Eq. (4), α is coupling factor and ε is the strength of nonlinearity. Because the condition for oscillation of the Wien-bridge oscillators is $g_1 > 3$, the strength of nonlinearity ε must be larger than 0. When α and ε are sufficiently small, the waveform of each oscillator can be regarded as almost purely sinusoidal, and we can rewrite Eq. (4) as described below.

$$\ddot{x}_k + x_k = \varepsilon(1 - {x_k}^2)\dot{x}_k - \alpha\sum_{j=1}^{N}(\dot{x}_j - \ddot{x}_j) \tag{5}$$

Comparing with Eq. (5) in Ref. [8], we notice the existence of the term $\sum \ddot{x}_j$. So we can predict that the dynamics of this *RC* oscillators' system should be different from that of the system with *LC* oscillators shown in Ref. [8].

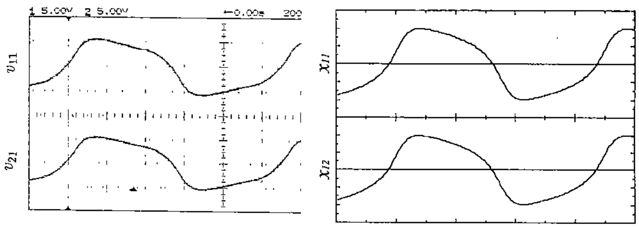
3. Circuit Experiments and Numerical Calculations

Next, in order to confirm the phenomena observed from the proposed circuit, we show examples of the circuit experimental results and the corresponding results of the numerical calculations for the case of $N = 2$–5 (Figs. 3–6). On computer calculations, we have to consider the differences among the natural frequencies of real oscillators, so normalized Eq. (4) is written as follows.

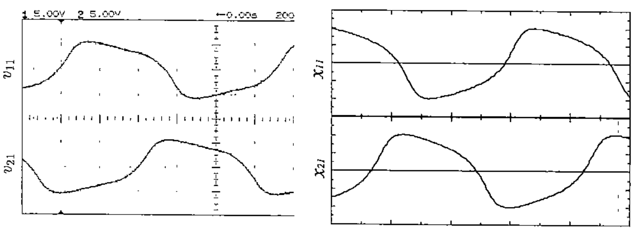
$$\begin{cases} \dot{x}_k = \dot{y}_k - (1 + \Delta\omega_k)z_k \\ \dot{y}_k = \varepsilon\left(z_k - \dfrac{z_k^3}{3}\right) - y_k + 2z_k \\ z_k = x_k + \alpha\displaystyle\sum_{j=1}^{N}\dot{x}_j \\ (k = 1, 2, \cdots, N) \end{cases} \tag{6}$$

On circuit experiments, we take $R_i = 4.7\,\text{k}\Omega$ and $R_f = 14.7\,\text{k}\Omega$.

From these results, we can see the different type of synchronization from N-phase oscillation. When $N = 2$, we can see only the opposite phase oscillation in the system with *LC* oscillators. But in the proposed system, we can see synchronization with in-phase (Figs. 3 (a), (b)) and 120° phase shift (Figs. 3 (c), (d)). When $N = 3$, we can see 3-phase oscillation just like the system with *LC* oscillators shown in Figs. 4 (a), (b) and in-phase and 120° phase shift synchronization shown in Figs. 4 (c), (d). When $N = 4$, we can see the opposite phase oscillations in the system with *LC* oscillators as shown in Ref. [8]. But in the proposed *RC* systems, we can see 3-phase and in-phase synchronization. In Figs. 5 (a), (b), v_{11}, v_{21} and v_{31} make 3-phase oscillation

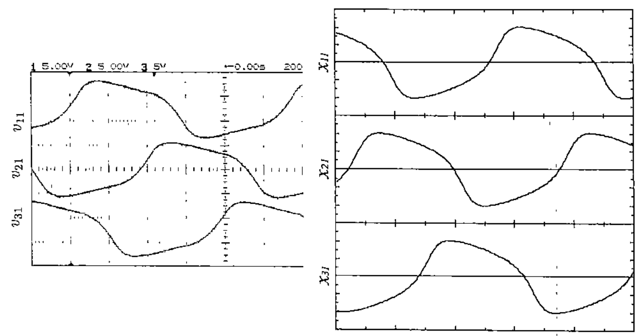


(a) Experimental result. (b) Numerical result.

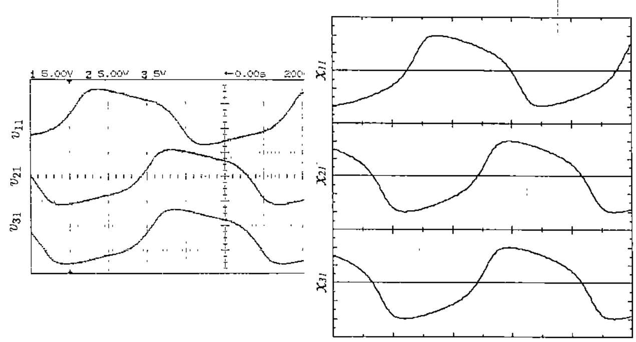


(c) Experimental result. (d) Numerical result.

Fig. 3 Experimental and numerical results for the case of $N = 2$ ($C = 0.015\,\mu$F, $R = 10\,\text{k}\Omega$, $r = 200\,\Omega$, horizontal scale: $200\,\mu$sec/div, vertical scale: 5 V/div, $\alpha = 0.02$, $\varepsilon = 1.5$).



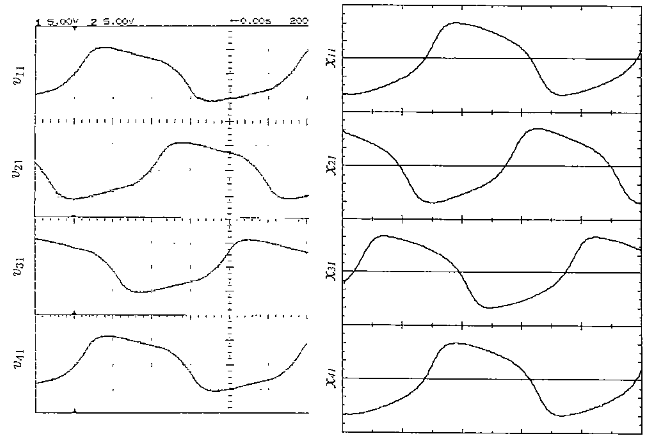
(a) Experimental result. (b) Numerical result.



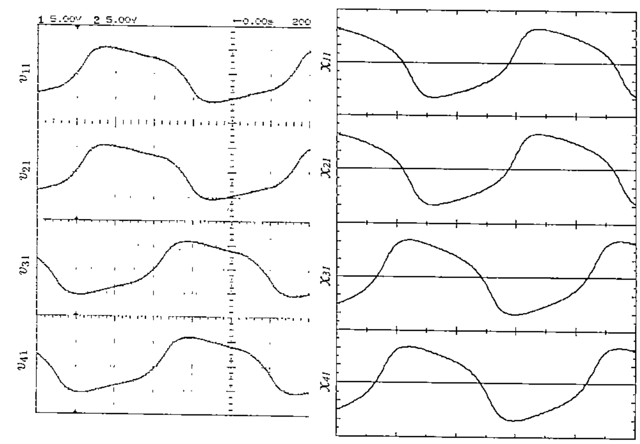
(c) Experimental result. (d) Numerical result.

Fig. 4 Experimental and numerical results for the case of $N = 3$ ($C = 0.015\,\mu$F, $R = 10\,\text{k}\Omega$, $r = 200\,\Omega$, horizontal scale: $200\,\mu$sec/div, vertical scale: 5 V/div, $\alpha = 0.02$, $\varepsilon = 1.5$).

and v_{11} and v_{41} synchronize at in-phase. In Figs. 5 (c), (d), v_{11} and v_{31} synchronize at 120°, and v_{11} and v_{21} synchronize at in-phase. Moreover, we carried out the circuit experiments and numerical calculations for the case of $N = 5, 6$. From Figs. 6 and 7, we can see the



(a) Experimental result. (b) Numerical result.



(c) Experimental result. (d) Numerical result.

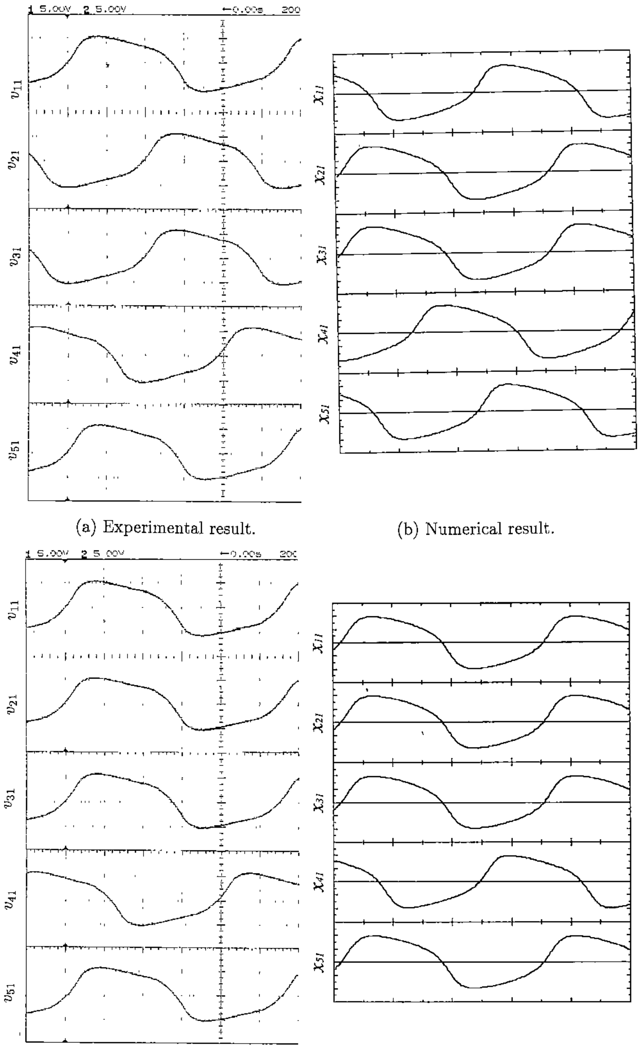
Fig. 5 Experimental and numerical results for the case of $N = 4$ ($C = 0.015\,\mu$F, $R = 10\,\text{k}\Omega$, $r = 200\,\Omega$, horizontal scale: $200\,\mu$sec/div, vertical scale: 5 V/div, $\alpha = 0.02$, $\varepsilon = 1.5$).

similar phenomena with $N = 4$. These synchronization patterns are determined by the region in phase space of the initial voltage of each capacitor. We have never seen these phenomena in any coupled oscillators systems as far as our knowledge goes. We should notice that these phenomena can be observed in the system only with strong nonlinearity.

Because the type of synchronization is different from the case with van der Pol LC oscillators, the number of phase states must be different from previous case. In this system, we can summarize the way of synchronization that when we take one oscillator as a reference signal, the other oscillators the phase of

$$\phi = 0, \pm\frac{2\pi}{3}, \tag{7}$$

while $\phi = 2k\pi/N \quad (k = 1, 2, \cdots, N-1)$ in the system with LC oscillators. So when N oscillators are coupled, we can consider the number of the phase states P_N as follows.



(a) Experimental result. (b) Numerical result.

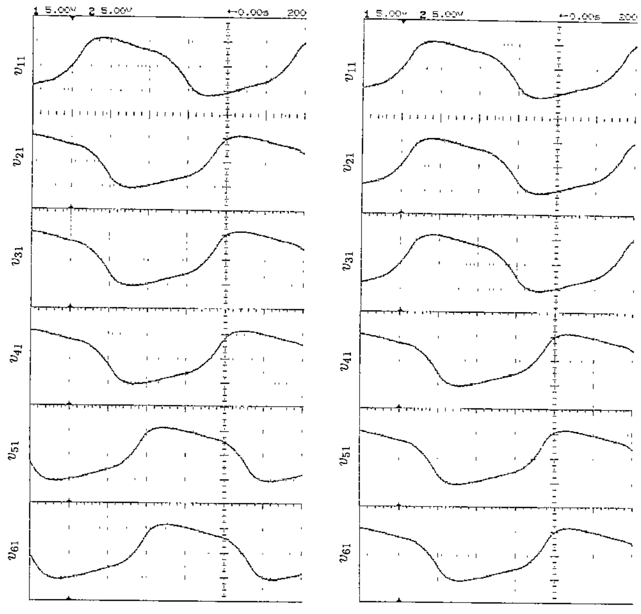
(c) Experimental result. (d) Numerical result.

Fig. 6 Experimental and numerical results for the case of $N = 5$ ($C = 0.015\,\mu$F, $R = 10\,\mathrm{k}\Omega$, $r = 200\,\Omega$, horizontal scale: $200\,\mu$sec/div, vertical scale: 5 V/div, $\alpha = 0.02$, $\varepsilon = 1.5$).

$$P_N = 3^{N-1} \tag{8}$$

From this equation, we can see this system have much more phase states than that of the system with van der Pol LC oscillators when N is not so large. For comparison, we show the number of the phase states of this system and the systems with van der Pol type LC oscillators in Table 1.

When N is large, the number of the phase states P_N becomes smaller than that of the system with van der Pol LC oscillators. But on cellular neural networks, the number of connections should be small because of their feature [9]. So when we use these oscillators systems as a structural element of cellular neural network, the number of coupled oscillators by one resistor should be small. Moreover, the phase differences between the oscillators do not become smaller but keep constant phase differences when the number of oscillators coupled becomes larger, while the phase differences become smaller in the system with LC oscillators. So we can construct the system in which phase errors by noise are hard to occur when we use the system as a memory with coupling many oscillators. Therefore this system is very suitable for a structural element of cellular neural network and a large scale memory.



(a) (b)

Fig. 7 Experimental results for the case of $N = 6$ ($C = 0.015\,\mu$F, $R = 10\,\mathrm{k}\Omega$, $r = 200\,\Omega$, horizontal scale: $200\,\mu$sec/div, vertical scale: 5 V/div).

Table 1 Comparison of the number of phase states.

N	with RC oscillator	with LC oscillator	
		Third-power	Fifth-power
2	3	1	2
3	9	2	6
4	27	3	18
5	81	24	70
6	243	45	280
⋮	⋮	⋮	⋮
13	531,441	479,001,600	792,712,283

4. Conclusions

In this study, we have investigated synchronization phenomena in the Wien-bridge oscillators coupled by one resistor by both of computer calculations and circuit experiments. In this system, we can see only 3-phase and in-phase oscillations in spite of N. This phenomenon has never been seen in any other coupled oscillators systems as far as we know.

Because this system does not include any inductors and is strong against phase error this system is much more suitable for applications on VLSI compared with coupled system of van der Pol type LC oscillators. Our future research is realization of memory or CNN using coupled oscillators system studied in this letter.

References

[1] T. Endo and S. Mori, "Mode analysis of a multimode ladder oscillator," IEEE Trans. Circuits & Syst., vol.CAS-23, no.2, pp.100–113, Feb. 1976.
[2] T. Endo and S. Mori, "Mode analysis of a two-dimensional low-pass multimode oscillator," IEEE Trans. Circuits & Syst., vol.CAS-23, no.9, pp.517–530, Sept. 1976.
[3] T. Endo and S. Mori, "Mode analysis of a ring of a large number of mutually coupled van der Pol oscillators," IEEE Trans. Circuits & Syst., vol.CAS-25, no.1, pp.7–18, Jan. 1978.
[4] H. Kimura and K. Mano, "Some properties of mutually synchronized oscillators coupled by resistances," Trans. IECE, vol.48, no.10, pp.1647–1656, Oct. 1965.
[5] H. Kimura and K. Mano, "Three-phase oscillators by resistive coupling," IECE Tech. Rep. of Nonlinear Theory, NLP64.2-20, 1965.
[6] Y. Nishio and S. Mori, "Mutually coupled oscillators with an extreme large number of steady states," Proc. of ISCAS '92, pp.819–822, May 1992.
[7] S. Moro, Y. Nishio, and S. Mori, "Coupled oscillators with a huge number of steady states —for a structural element of a cellular neural network," Proc. of ECCTD '93, pp.27–32, Aug. 1993.
[8] S. Moro, Y. Nishio, and S. Mori, "Synchronization phenomena in oscillators coupled by one resistor," IEICE Trans. Fundamentals, vol.E78-A, no.2, pp.244–253, Feb. 1995.
[9] Y. Nishio, S. Mori, and A. Ushida, "On coupled oscillators networks —for the cellular neural network," Proc. of ISCAS '93, pp.2327–2330, May 1993.
[10] L.O. Chua and L. Yang, "Cellular neural networks: theory," IEEE Trans. Circuits & Syst., vol.35, no.10, pp.1257–1272, Oct. 1988.