PAPER

On a Ring of Chaotic Circuits Coupled by Inductors

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SUMMARY In this study, a ring of simple chaotic circuits coupled by inductors is investigated. An extremely simple three-dimensional autonomous circuit is considered as a chaotic subcircuit. By carrying out circuit experiments and computer calculations for two, three or four subcircuits case, various synchronization phenomena of chaos are confirmed to be stably generated. For the three subcircuits case, two different synchronization modes coexist, namely in-phase synchronization mode and three-phase synchronization mode. By investigating Poincaré map, we can see that two types of synchronizations bifurcate to quasi-synchronized chaos via different bifurcation route, namely in-phase synchronization undergoes period-doubling route while three-phase synchronization undergoes torus breakdown. Further, we investigate the effect of the values of coupling inductors to bifurcation phenomena of two types of synchronizations.

key words: *chaotic circuit, quasi-synchronization of chaos, coupled oscillator, torus breakdown*

1. Introduction

Coupled oscillators systems are good models to describe various nonlinear phenomena in the field of natural science and a number of excellent studies on mutual synchronization of oscillators have been carried out ([1]–[3] and therein). In Ref. [1], Suezaki and Mori investigated synchronization phenomena observed from two van der Pol oscillators coupled by a capacitor or a resistor and confirmed that for the case of capacitor coupling two synchronization modes coexist, namely one is in-phase synchronization and the other is anti-phase synchronization. Later Endo and Mori investigated a large number of van der Pol oscillators coupled as a ring [3] and confirmed the coexistence of various modes of synchronizations. Because inductor coupling causes the change of oscillation frequencies of synchronization modes and the coexistence of different modes of synchronizations, the investigations on oscillators coupled by inductors may contain a large number of unsolved interesting problems.

On the other hand, many nonlinear dynamical systems in various fields have been clarified to exhibits chaotic oscillations and recently applications of chaos to engineering systems attract many researchers' attentions, for example, chaos noise generator [4]–[6], control of chaos [7]–[9], synchronization of chaos [10]–[16], and so on. Among the studies on such applications, synchronization of chaotic systems or signals is extremely interesting, because the chaotic solution is unstable and small error of initial values must be expanded as time goes. As far as we know, such phenomena have been firstly reported to be generated in simple real circuits by a group of Saito [10]. Since Pecora et al. have investigated such phenomena theoretically [11], many papers have been published until now. Further, the technique of synchronization of chaos is also applied to realize secure communication systems using chaos [17], [18]. However, almost all studies on synchronization of chaos treat only the case that chaotic signals generated from two identical chaotic systems are completely synchronized at the in-phase. Although secure communication systems do not need another types of synchronizations, the investigation of another types of synchronizations of chaotic systems will open the way to another applications of chaos. Moreover, investigating what kind of phenomena are observed from various coupled chaotic systems would contribute to develop the study of nonlinear dynamical systems. As far as we know, another types of synchronizations of chaotic circuits have been firstly reported in Ref. [19]. In Ref. [19], we investigated two or three simple chaotic circuits coupled by one resistor and confirmed the generation of anti-phase quasi-synchronization of chaos and three-phase quasi-synchronization of chaos as well as in-phase. However, as we stated above, coexistence of different types of synchronizations cannot be observed in the case of resistor coupling and we did not pay our attentions on bifurcation route to chaos.

In this study, we investigate various synchronization phenomena observed from a ring of simple chaotic circuits coupled by inductors. We use the term of synchronization for two or more signals having the relation such as $S_1(t) - S_2(t-T) < \varepsilon$ for a constant T and a small ε including $S_1(t) - S_2(t) < \varepsilon$. Also the term quasi-synchronization means ε is replaced by a relatively large constant, which is of course smaller than the average amplitude of each chaotic signal. We consider a simple three-dimensional autonomous circuit as a chaotic subcircuit. This circuit is proposed by Inaba et al. [20] and one of the simplest autonomous chaotic circuits. By carrying out circuit experiments and computer calculations for two, three or four subcircuits case, various quasi-synchronization phenomena of chaos are

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confirmed to be stably generated. Especially we concentrate on the three subcircuits case. In this case, two different quasi-synchronization modes of chaos coexist, namely in-phase quasi-synchronized chaos and three-phase quasi-synchronized chaos. As far as we know this is the first result on the coexistence of different synchronization modes of chaos. We investigate bifurcation route to quasi-synchronization of chaos on which there have been very few discussions. By using Poincaré map, it is confirmed that two types of synchronizations bifurcate to quasi-synchronized chaos via different bifurcation route, namely in-phase synchronization undergoes period-doubling route while three-phase synchronization undergoes torus breakdown [21]–[23]. Further, we investigate the effect of the values of coupling inductors to bifurcation phenomena of the two types of synchronizations.

2. Circuit Model

The circuit model is shown in Fig. 1. In our system N same chaotic circuits are coupled by inductors as a ring. We consider a simple three-dimensional autonomous circuit in Fig. 2 as a chaotic subcircuit. This circuit is proposed by Inaba et al. [20] and is confirmed to exhibit logistic chaos theoretically. This circuit consists of only three memory elements, one linear negative resistor and one diode and is one of the simplest autonomous chaotic circuits.

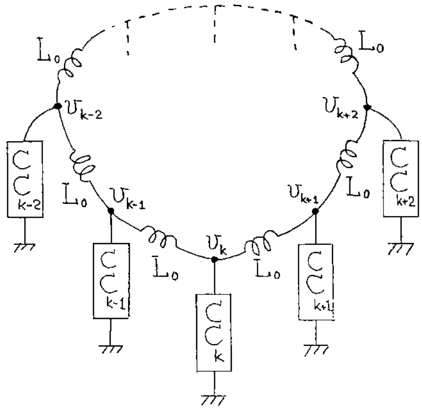

Fig. 1 Circuit model.

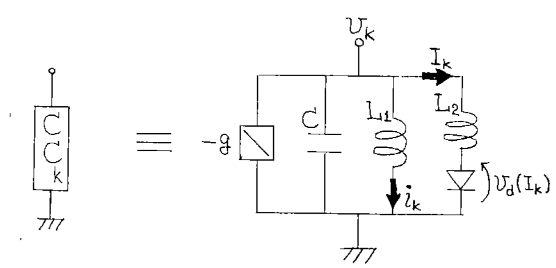

Fig. 2 Chaotic subcircuit.

At first, we approximate the $i-v$ characteristics of the diode by the following two-segment piecewise linear function.

$$v_d(I_k)=\begin{cases} r_d I_k & \cdots \quad (I_k \leqq V/r_d) \\ V & \cdots \quad (I_k > V/r_d) \end{cases} \tag{1}$$

The equation governing the circuit in Fig. 1 is described as follows:

$$\begin{aligned} C\frac{dv_k}{dt} &= gv_k - i_k - I_k + \frac{L_1}{L_0}(i_{k-1} - 2i_k + i_{k+1}) \\ L_1\frac{di_k}{dt} &= v_k \\ L_2\frac{dI_k}{dt} &= v_k - v_d(I_k) \end{aligned} \tag{2}$$

$$(k = 1, 2, \cdots, N)$$

where $i_0 = i_N$ and $i_{N+1} = i_1$. By changing the variables and parameters,

$$\begin{aligned} &v_k = Vx_k, \quad i_k = \sqrt{\frac{C}{L_1}}Vy_k, \quad I_k = \sqrt{\frac{C}{L_1}}Vz_k, \\ &t = \sqrt{L_1 C}\tau, \quad \text{“}\cdot\text{”} = \frac{d}{d\tau}, \\ &\alpha = \frac{L_1}{L_2}, \quad \beta = g\sqrt{\frac{L_1}{C}}, \quad \gamma = r_d\sqrt{\frac{C}{L_1}}, \quad \delta = \frac{L_1}{L_0}, \end{aligned} \tag{3}$$

(2) is normalized as

$$\begin{aligned} \dot{x_k} &= \beta x_k - y_k - z_k + \delta(y_{k-1} - 2y_k + y_{k+1}) \\ \dot{y_k} &= x_k \\ \dot{z_k} &= \alpha\{x_k - f(z_k)\} \end{aligned} \tag{4}$$

$$(k = 1, 2, \cdots, N)$$

where $y_0 = y_N$, $y_{N+1} = y_1$ and

$$f(z_k)=\begin{cases} \gamma z_k & \cdots \quad (z_k \leqq 1/\gamma) \\ 1 & \cdots \quad (z_k > 1/\gamma) \end{cases} \tag{5}$$

For computer calculations, in order to consider the difference of real circuit elements, (4) is rewritten as follows.

$$\begin{aligned} \dot{x_k} &= \beta x_k - y_k - z_k + \delta(y_{k-1} - 2y_k + y_{k+1}) \\ \dot{y_k} &= x_k \\ \dot{z_k} &= \alpha(1 + \Delta\omega_k)\{x_k - f(z_k)\} \end{aligned} \tag{6}$$

Figure 3 shows a typical example of chaotic attractors obtained from the chaotic subcircuit. In the following circuit experiments, the values of the inductors and the capacitor in each chaotic subcircuit are fixed and those values are measured as $L_1 = 66.7\,\text{mH} \pm 0.2\%$, $L_2 = 10.85\,\text{mH} \pm 1.4\%$ and $C = 0.06804\,\mu\text{F} \pm 0.1\%$. Further, we use four diodes connected in series instead of one diode in Fig. 2 to make the $v-i$ characteristics uniform. While in the following computer calculations, the parameter values corresponding to the inductors, the capacitor and the diode are fixed as $\alpha = 6.00$, $\gamma = 100.0$ and $\Delta\omega_k = 0.005(k-1)$ and (6) is calculated by using the Runge-Kutta method with step size $\Delta t = 0.001$.

The circuit in Fig. 1 can be regarded as a chaotic circuit version of the coupled van der Pol oscillators in Ref. [3]. Hence, this coupled chaotic circuits are expected to generate various types of synchronization phenomena.

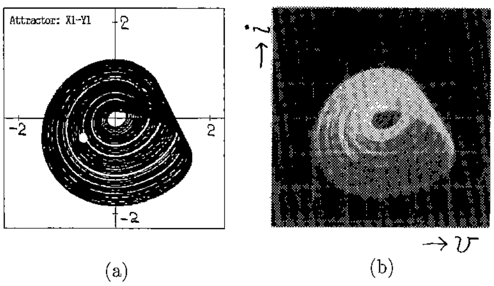

Fig. 3 Typical example of chaotic attractors observed from the chaotic subcircuit. (a) Computer calculation ($\alpha = 6.0$, $\beta = 0.27$, $\gamma = 100.0$). (b) Circuit experiment ($L_1 = 66.7$ mH, $L_2 = 10.7$ mH, $C = 0.0680\,\mu$F, $g = 467\,\mu$S). Horizontal: 2 V/div. Vertical: 2 mA/div.

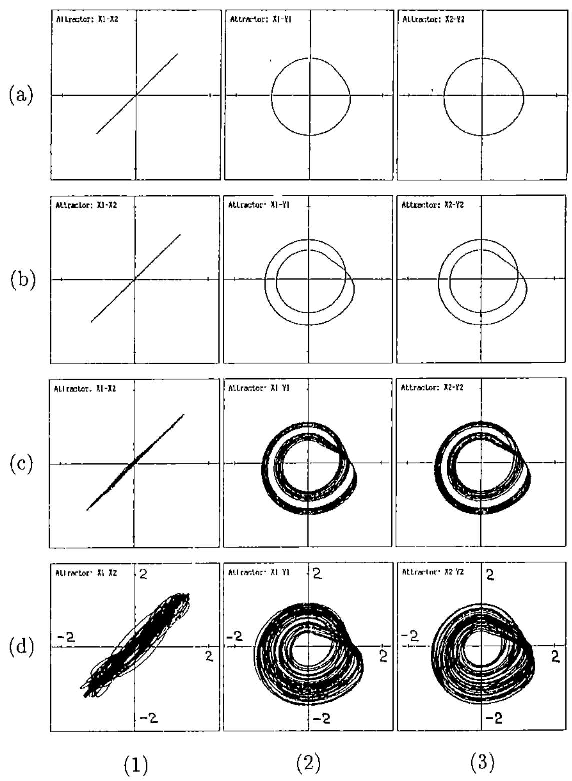

Fig. 4 In-phase synchronization of two chaotic circuits (computer calculation). $\delta = 0.30$. (a) $\beta = 0.07$. (b) $\beta = 0.11$. (c) $\beta = 0.16$. (d) $\beta = 0.22$. (1) x_1 vs. x_2. (2) x_1 vs. y_1. (3) x_2 vs. y_2.

3. Two Subcircuits Case

In this section, we investigate the special case of $N = 2$ briefly, namely only two chaotic subcircuits are coupled by one inductor. For the case of van der Pol oscillators, two coupled system has been confirmed to generate two different synchronization modes, namely in-phase synchronization mode and anti-phase one [1].

We found that both of in-phase and anti-phase synchronization modes were stably excited in the coupled chaotic circuits as well as the van der Pol oscillators case. The two different synchronization modes coexist and we can produce one of two modes by inputting a certain initial conditions. Further, we found that the two types of synchronizations bifurcate from one-periodic to chaotic via different bifurcation routes as a control parameter increases.

Computer calculated results are shown in Figs. 4 and 5. Please note that in-phase synchronizations in Fig. 4 and anti-phase synchronizations in Fig. 5 coex-

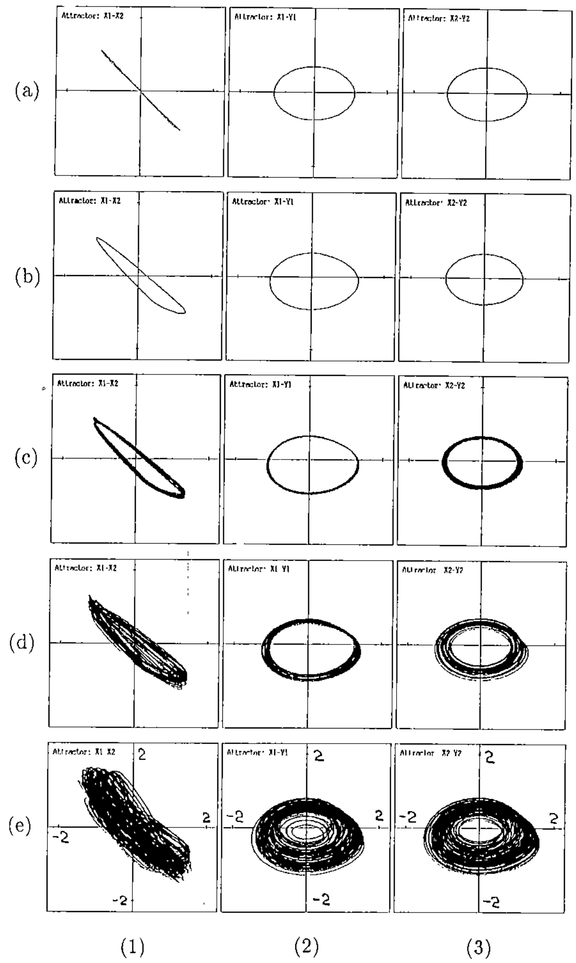

Fig. 5 Anti-phase synchronization of two chaotic circuits (computer calculation). $\delta = 0.30$. (a) $\beta = 0.05$. (b) $\beta = 0.13$. (c) $\beta = 0.16$. (d) $\beta = 0.19$. (e) $\beta = 0.25$. (1) x_1 vs. x_2. (2) x_1 vs. y_1. (3) x_2 vs. y_2.

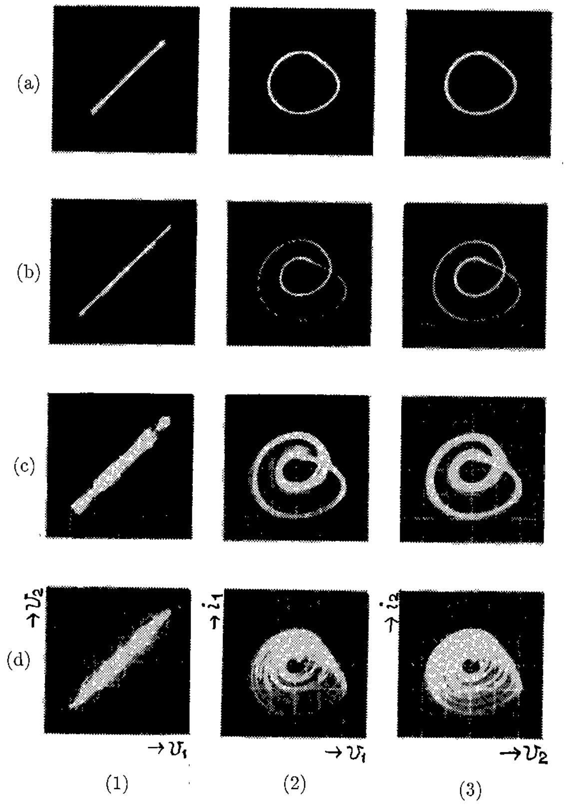

Fig. 6 In-phase synchronization of two chaotic circuits (circuit experiment). $L_0 = 246\,\mathrm{mH}$. (a) $g = 339\,\mu\mathrm{S}$. (b) $g = 388\,\mu\mathrm{S}$. (c) $g = 424\,\mu\mathrm{S}$. (d) $g = 461\,\mu\mathrm{S}$. (1) Horizontal and Vertical: 2 V/div. (2)(3) Horizontal: 2 V/div. Vertical: 2 mA/div.

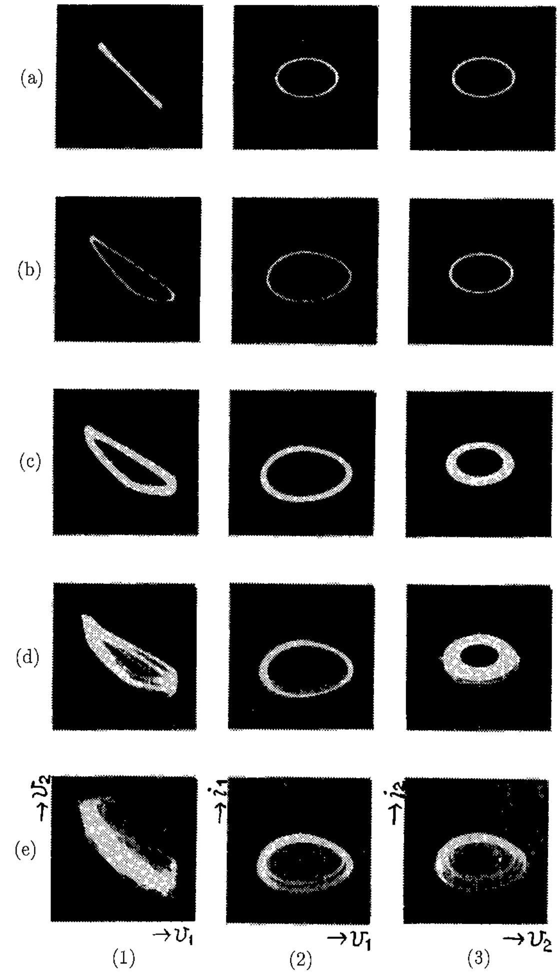

Fig. 7 Anti-phase synchronization of two chaotic circuits (circuit experiment). $L_0 = 246\,\mathrm{mH}$. (a) $g = 164\,\mu\mathrm{S}$. (b) $g = 389\,\mu\mathrm{S}$. (c) $g = 439\,\mu\mathrm{S}$. (d) $g = 461\,\mu\mathrm{S}$. (e) $g = 481\,\mu\mathrm{S}$. (1) Horizontal and Vertical: 2 V/div. (2)(3) Horizontal: 2 V/div. Vertical: 2 mA/div.

ist. Figure 4 shows in-phase synchronization mode. We can see that two circuits are synchronized at the in-phase completely when the attractor is periodic with short period. When the attractor is chaotic, two circuits are not synchronized completely, but are almost synchronized as shown in Fig. 4 (d). We call the situation as quasi-synchronization of chaos. As a parameter β increases, one-periodic attractor bifurcates to chaotic attractor via period-doubling route keeping in-phase synchronization. This is the same as the bifurcation route of the uncoupled subcircuit. Moreover, the shape of the attractors of in-phase synchronization is quite similar to those observed from the uncoupled subcircuit. While, Fig. 5 shows anti-phase synchronization mode. In this case two circuits are not synchronized completely even when the attractor is one-periodic. This is because the subcircuit is not symmetric with respect to the origin. Moreover, the anti-phase synchronization undergoes complicated bifurcation route explained as follows. One-periodic attractor with symmetry on the $x_1 - x_2$ plane in Fig. 5 (a) bifurcates to two one-periodic attractors with asymmetry as Fig. 5 (b). Each asymmetric attractor bifurcates to torus via Hopf bifurcation as Fig. 5 (c) and to chaos vis torus breakdown as Fig. 5 (d). Two asymmetric chaos collide each other and one chaotic attractor with symmetry is generated via symmetry-recovering crisis as Fig. 5 (e). Namely, the anti-phase synchronization exhibits symmetry breaking and recovering and torus via Hopf bifurcation. The corresponding circuit experimental results are shown in Figs. 6 and 7. Both results agree well qualitatively.

4. Three Subcircuits Case

In this section, we consider the case of $N = 3$ in detail, namely three chaotic subcircuits are coupled as a ring. This section is the main part of the present paper.

We found that both of in-phase synchronization and three-phase synchronization are stably generated. Computer calculated results are shown in Figs. 8 and 9. In the figures we omit attractors on the $x_i - y_i$ plane for $i = 2, 3$, because the shape is almost same as the attractors on the $x_1 - y_1$ plane. As well as two subcircuits case, in-phase synchronization in Fig. 8 is confirmed to undergo period-doubling route to chaos. While three-

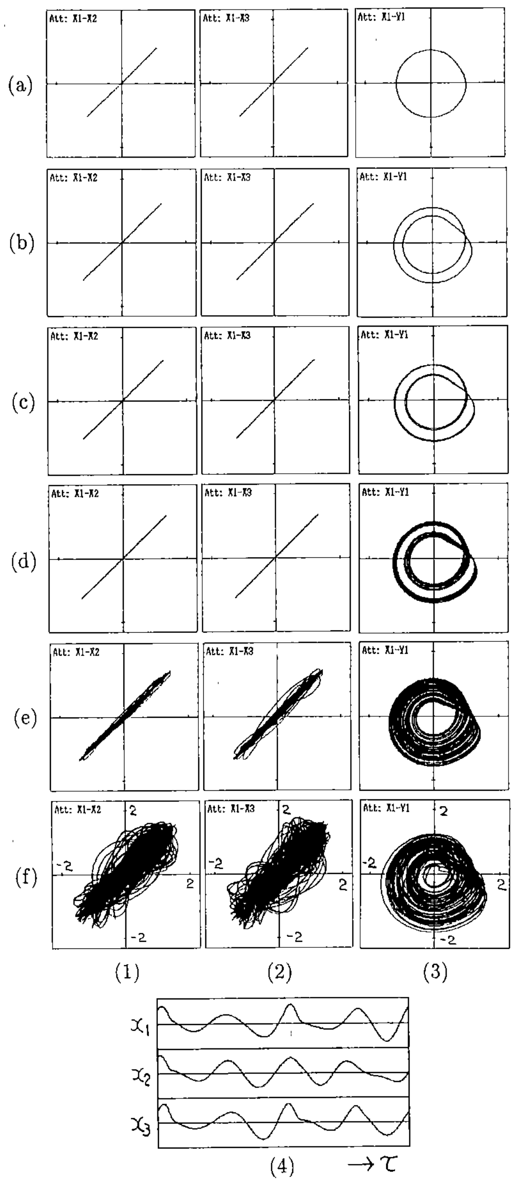

Fig. 8 In-phase synchronization of three chaotic circuits (computer calculation). $\delta = 0.40$. (a) $\beta = 0.06$. (b) $\beta = 0.10$. (c) $\beta = 0.12$. (d) $\beta = 0.15$. (e) $\beta = 0.20$. (f) $\beta = 0.25$. (1) x_1 vs. x_2. (2) x_1 vs. x_3. (3) x_1 vs. y_1. (4) Time waveform for $\beta = 0.25$.

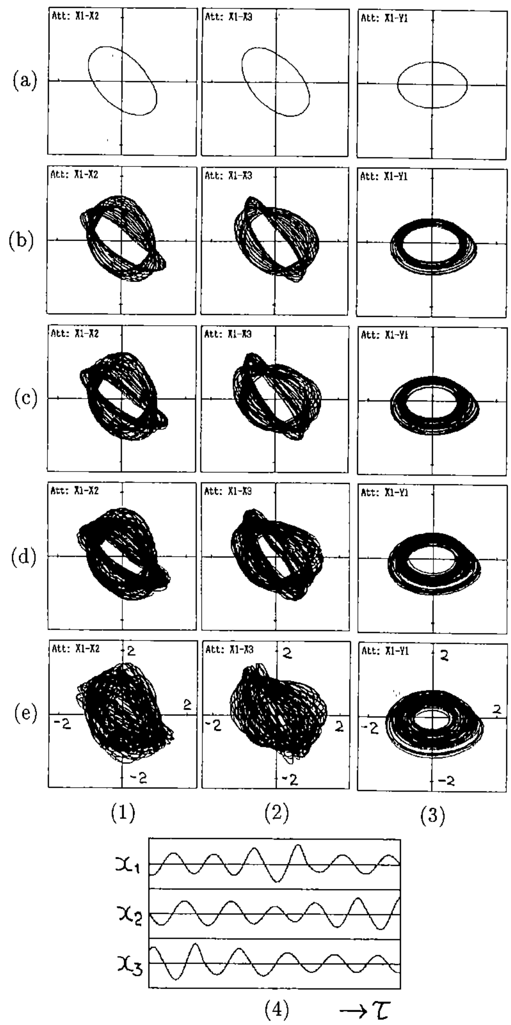

Fig. 9 Three-phase synchronization of three chaotic circuits (computer calculation). $\delta = 0.40$. (a) $\beta = 0.04$. (b) $\beta = 0.102$. (c) $\beta = 0.117$. (d) $\beta = 0.123$. (e) $\beta = 0.15$. (1) x_1 vs. x_2. (2) x_1 vs. x_3. (3) x_1 vs. y_1. (4) Time waveform for $\beta = 0.15$.

phase synchronization in Fig. 9 undergoes torus breakdown. The corresponding circuit experimental results are shown in Figs. 10 and 11. Both results agree well qualitatively. Although we show only the case that the phases of the solution obtained from subcircuits are arranged as $\{x_1, x_2, x_3\}$ in Figs. 9 and 11, we also confirmed the generation of the other phase-state, that is $\{x_1, x_3, x_2\}$.

In order to investigate the bifurcation route in detail, we consider the Poincaré map of each synchronization mode. The Poincaré section is defined as $x_1 = 0$ where $dx_1/dt < 0$. The projections of the Poincaré maps onto $y_1 - y_2$ plane of in-phase synchronization and three-phase synchronization are shown in Figs. 12 and 13, respectively. From Fig. 12 we can see that one-periodic attractor (a) bifurcates to two-periodic (b), four-periodic (c), eight-periodic (d), two-band chaos (e) and one-band chaos (f). This is well-known period doubling route to chaos. As β increase further, synchronization becomes weak as (g)–(i). From Fig. 13 we can see the bifurcation route of three-phase synchronization via torus breakdown. One-periodic attractor (a) bifurcates torus (b) via Hopf bifurcation. As β increases, torus grows as (c)(d). At $\beta \simeq 0.109$, a periodic state (e) appears and after that folded torus and periodic state appear as (f)–(i). As β increases further, Poincaré map has thickness (j)–(l) and area-expanding chaos is considered to be generated.

Moreover, we made one-parameter bifurcation diagram of the Poincaré map as shown in Figs. 14 and 15.

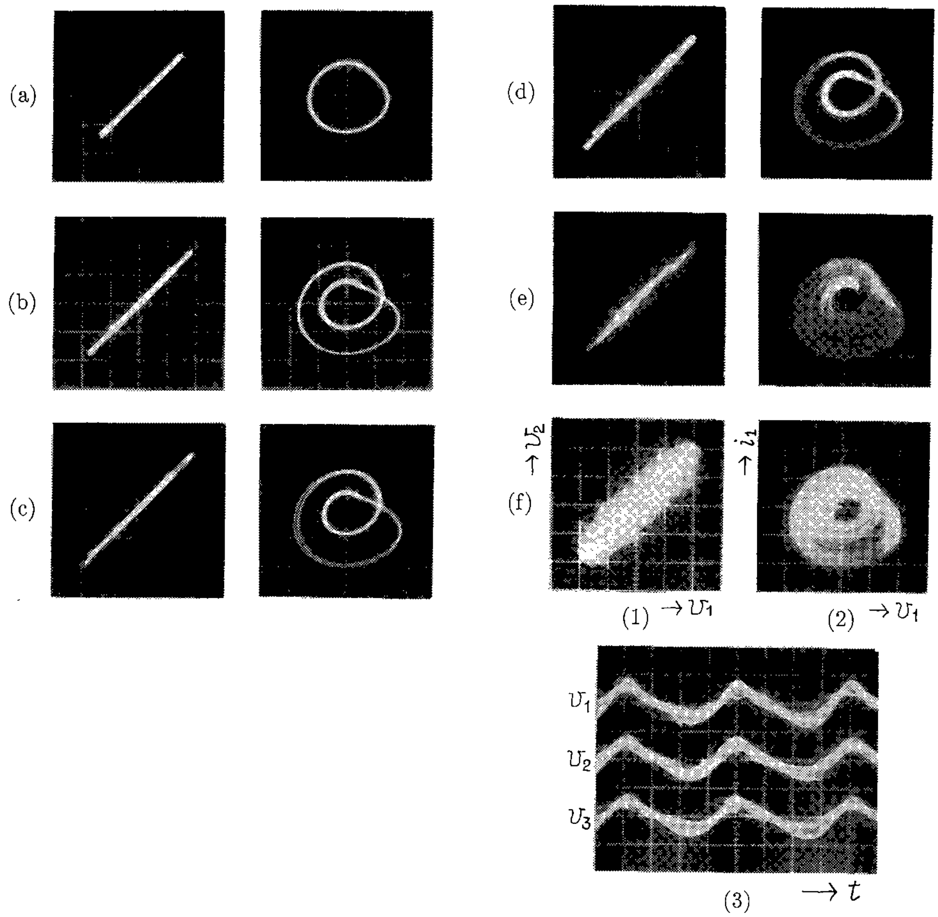

Fig. 10 In-phase synchronization of three chaotic circuits (circuit experiment). L_0 = 138 mH. (a) $g = 295\,\mu$S. (b) $g = 376\,\mu$S. (c) $g = 402\,\mu$S. (d) $g = 418\,\mu$S. (e) $g = 438\,\mu$S. (f) $g = 457\,\mu$S. (1) Horizontal and Vertical: 2 V/div. (2) Horizontal: 2 V/div. Vertical: 2 mA/div. (3) Horizontal: 0.1 ms/div. Vertical: 5 V/div.

Figure 14 shows that in-phase synchronization exhibits logistic chaos. We can also observe the generation of six-periodic window around $\beta = 0.15$. It is clear that the synchronization becomes weak for larger β value. From the Fig. 15 we can confirm the bifurcation route of three-phase synchronization, namely bifurcation of the one-periodic solution to torus around $\beta = 0.05$, the generation of periodic solution around $\beta = 0.10$ and the generation of chaotic solution for β values more than about 0.11. Further, for $\beta > 0.17$ three-phase synchronization disappears and only in-phase synchronization exists.

Next, we consider the effect of the value of the coupling inductor. We carried out both of computer calculations and circuit experiments for another values of L_0 or δ. The results are summarized as follows. For in-phase synchronization, when the coupling is small, periodic solutions having some phase difference are generated. Typical example is shown in Fig. 16 (a). In this case, two-periodic in-phase synchronization is observed, but only the solution of the subcircuit 3 have phase difference corresponding basic one-period. As a result, attractor on $x_1 - x_2$ plane has the strange shape as shown in Fig. 16 (a2). Because of the symmetry of the coupling, three attractors for this type of in-phase synchronization coexist. As β increases, each attractor bifurcates to chaos. At last the three attractors collide each other and one chaotic attractor appears as shown in Fig. 16 (b). This chaotic attractor on $x_1 - x_2$ plane has the shape as Figs. 16 (a1) and (a2) are overlapped. For three-phase synchronization, we found an interesting phenomenon. Namely, parameter region corresponding the generation of three-phase synchronization shifts according to the value of the coupling. Figure 17 shows the one-parameter bifurcation diagram for three-phase synchronization for different δ values. We can see the parameter region shift toward small as δ becomes small.

5. Four Subcircuits Case

In this section, we consider the case of $N = 4$ briefly.

We found that in-phase synchronization and four-

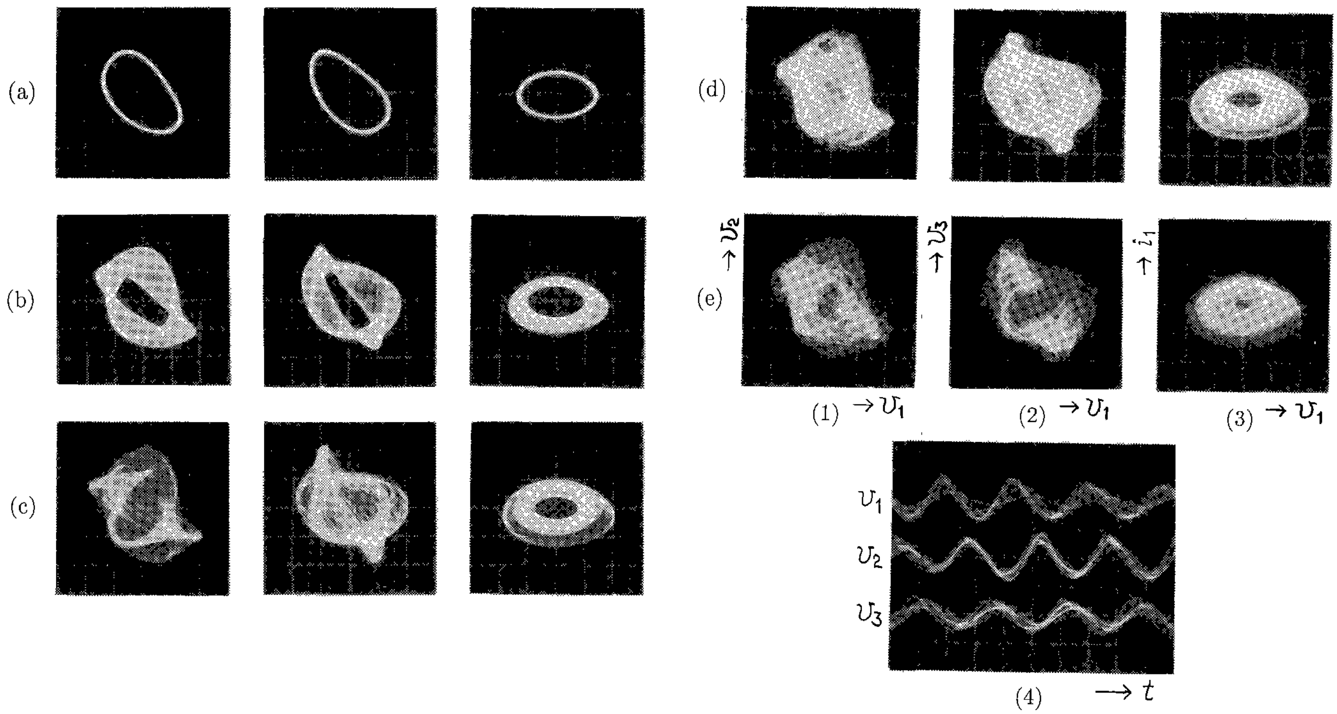

Fig. 11 Three-phase synchronization of three chaotic circuits (circuit experiment). $L_0 = 138$ mH. (a) $g = 249\,\mu$S. (b) $g = 295\,\mu$S. (c) $g = 344\,\mu$S. (d) $g = 356\,\mu$S. (e) $g = 370\,\mu$S. (1)(2) Horizontal and Vertical: 2 V/div. (3) Horizontal: 2 V/div. Vertical: 2 mA/div. (4) Horizontal: 0.1 ms/div. Vertical: 5 V/div.

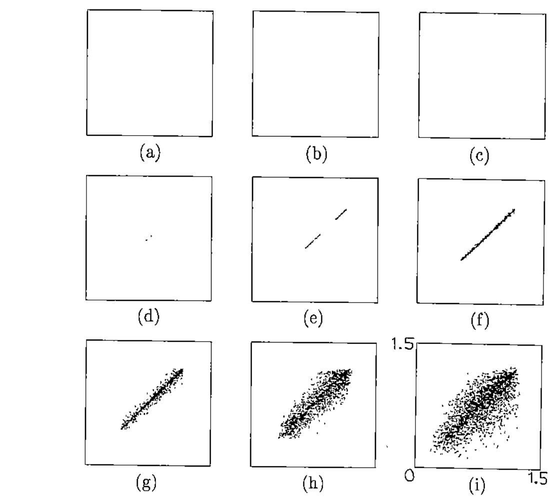

Fig. 12 Poincaré map of the in-phase synchronization. Horizontal: y_1. Vertical y_2. $\delta = 0.40$. (a) $\beta = 0.06$. (b) $\beta = 0.10$. (c) $\beta = 0.12$. (d) $\beta = 0.13$. (e) $\beta = 0.15$. (f) $\beta = 0.18$. (g) $\beta = 0.20$. (h) $\beta = 0.22$. (i) $\beta = 0.25$.

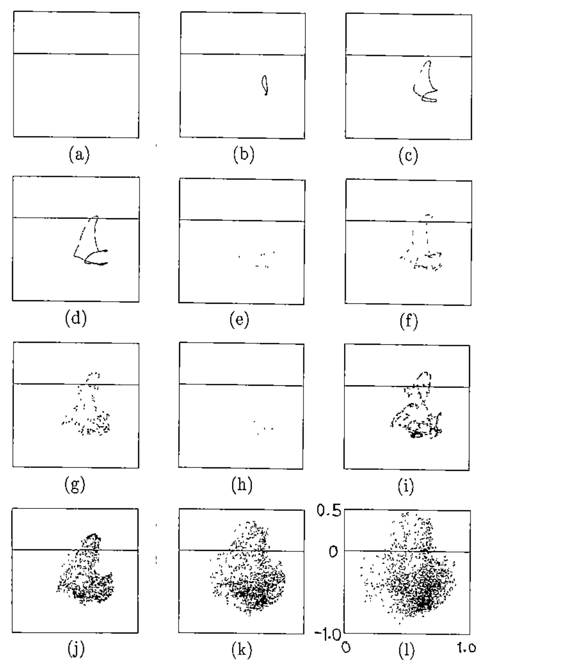

Fig. 13 Poincaré map of the three-phase synchronization. Horizontal: y_1. Vertical y_2. $\delta = 0.40$. (a) $\beta = 0.04$. (b) $\beta = 0.06$. (c) $\beta = 0.09$. (d) $\beta = 0.102$. (e) $\beta = 0.109$. (f) $\beta = 0.1092$. (g) $\beta = 0.117$. (h) $\beta = 0.12$. (i) $\beta = 0.122$. (j) $\beta = 0.123$. (k) $\beta = 0.14$. (l) $\beta = 0.15$.

phase synchronization were stably generated as well as the case of three subcircuits and that these two types of synchronizations bifurcate to chaos via period-doubling route and torus-breakdown route, respectively.

Figure 18 shows in-phase synchronization. In the figure we omit attractors on the $x_i - y_i$ plane for $i = 2, 3, 4$, because the shape is almost same as the attractor on the $x_1 - y_1$ plane shown in Fig. 18 (e). In this case, x_1 and x_2 are almost synchronized as Fig. 18 (a), and x_3

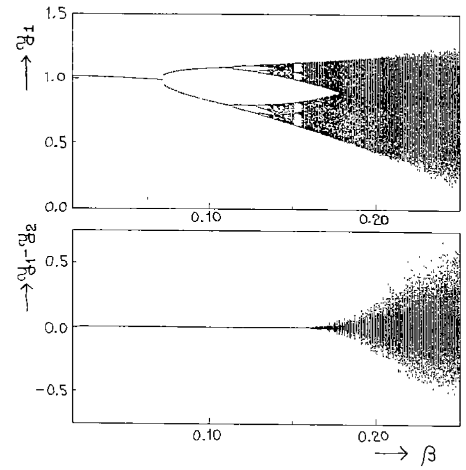

Fig. 14 One-parameter bifurcation diagram of the Poincaré map for the in-phase synchronization. $\delta = 0.40$.

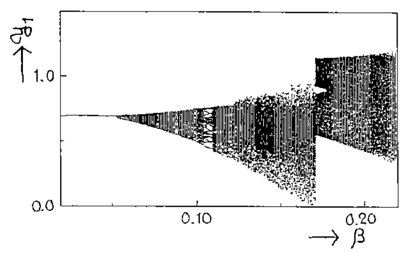

Fig. 15 One-parameter bifurcation diagram of the Poincaré map for the three-phase synchronization. $\delta = 0.40$.

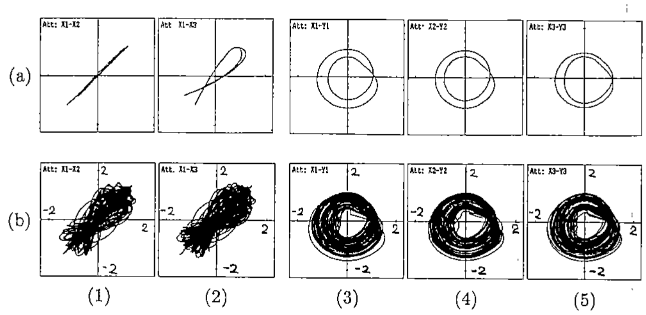

Fig. 16 In-phase synchronization for $\delta = 0.30$. (a) $\beta = 0.10$. (b) $\beta = 0.15$. (1) x_1 vs. x_2. (2) x_1 vs. x_3. (3) x_1 vs. y_1. (4) x_2 vs. y_2. (5) x_3 vs. y_3.

and x_4 are also almost synchronized as Fig. 18 (c), but there seems to be phase shift between these two pairs as Fig. 18 (b). This is the situation similar to the case in Fig. 16. Figure 19 shows four-phase synchronization. This synchronization mode cannot be seen in van der Pol oscillators case [3]. In our circuit model asymmetry of the subcircuit may plays an important role to make the generation of four-phase synchronization possible. Figure 19 shows only the case that the phases of subcircuits are arranged as $\{x_1, x_2, x_3, x_4\}$. However, the other phase-state can be also observed, that is $\{x_1, x_4, x_3, x_2\}$. The corresponding circuit experimental results are shown in Figs. 20 and 21. Both results agree well qualitatively. Especially, the phase shift in the in-phase mode is very clear from Figs. 20 (a)–(c).

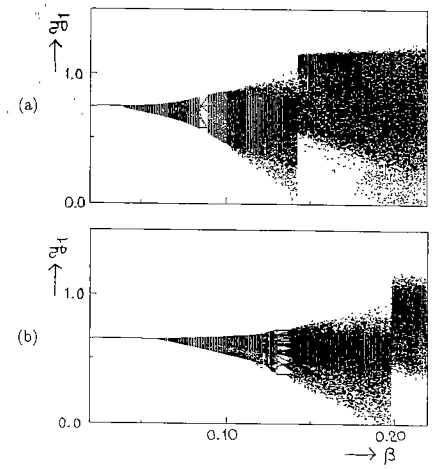

Fig. 17 One-parameter bifurcation diagrams of the Poincaré map for the three-phase synchronization. (a) $\delta = 0.30$. (b) $\delta = 0.50$.

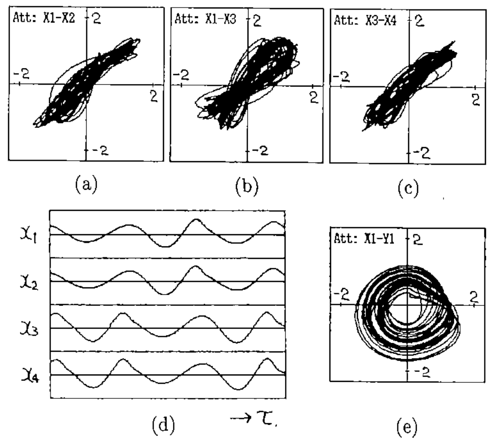

Fig. 18 In-phase synchronization of four chaotic circuits (computer calculation). $\beta = 0.17$ and $\delta = 0.30$. (a) x_1 vs. x_2. (b) x_1 vs. x_3. (c) x_3 vs. x_4. (d) Time waveform. (e) x_1 vs. y_1.

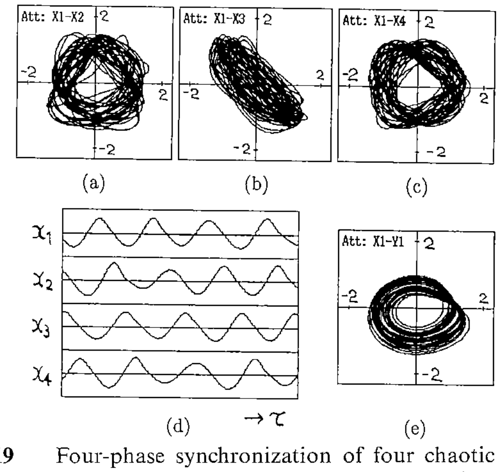

Fig. 19 Four-phase synchronization of four chaotic circuits (computer calculation). $\beta = 0.20$ and $\delta = 0.30$. (a) x_1 vs. x_2. (b) x_1 vs. x_3. (c) x_1 vs. x_4. (d) Time waveform. (e) x_1 vs. y_1.

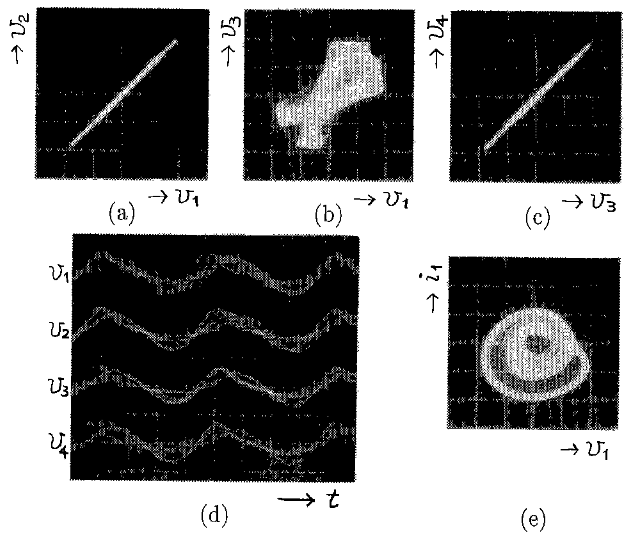

Fig. 20 In-phase synchronization of four chaotic circuits (circuit experiment). $g = 385\,\mu$S and $L_0 = 240$ mH. (a)–(c) Horizontal and Vertical: 2 V/div. (d) Horizontal: 0.1 ms/div. Vertical: 5 V/div. (e) Horizontal: 2 V/div. Vertical: 2 mA/div.

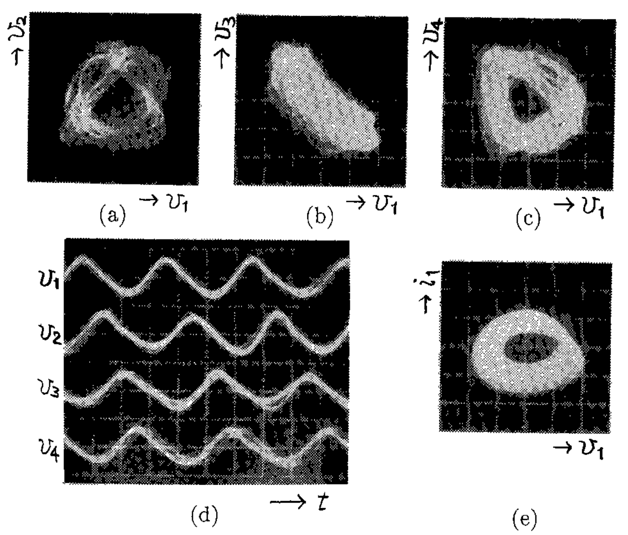

Fig. 21 Four-phase synchronization of four chaotic circuits (circuit experiment). $g = 400\,\mu$S and $L_0 = 240$ mH. (a)–(c) Horizontal and Vertical: 2 V/div. (d) Horizontal: 0.1 ms/div. Vertical: 5 V/div. (e) Horizontal: 2 V/div. Vertical: 2 mA/div.

6. Concluding Remarks

In this study, we investigated quasi-synchronization phenomena observed from simple chaotic circuits coupled by inductors as a ring. By carrying out circuit experiments and computer calculations for two, three or four subcircuits case, we confirmed that various quasi-synchronization phenomena of chaos were stably observed.

We would like to emphasize that there had been very few discussions on the coexistence or bifurcation route of synchronized chaos. Moreover all of interesting phenomena introduced in this paper have been observed from real circuit model made up easily.

Though we omit to introduce some results for another chaotic circuits in this paper, we have carried out circuit experiments for some types of chaotic circuits and have confirmed the generation of similar types of quasi-synchronization of chaos. Hence, various interesting quasi-synchronization phenomena introduced in this paper are considered to be generated in various coupled systems. Namely, quasi-synchronization phenomena of chaos are not special phenomena observed from only a few systems, but common phenomena as well as simple chaos.

We must establish the method for theoretical analysis of the quasi-synchronizations of chaos. Now, we can investigate the stability of completely synchronized chaos by calculating Lyapunov exponents. However, the method cannot be applied for quasi-synchronizations at all. We hope that our study would motivate the establishment of analyzing method for quasi-synchronizations of chaos and that the phenomena in this paper would be applied to various engineering systems.

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