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Dynamic Behavior of a Chaotic Circuit with a Memristor under a Sinusoidal Input

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Abstract

We analyze the dynamics of a chaos generating circuit with a memristor under a sinusoidal input. The effects of input frequency on the circuit dynamics are investigated using time series responses and phase space representations. The results reveal frequency dependent oscillation switching and changes in phase space structures.

1. Introduction

A memristor is a passive circuit element whose resistance changes nonlinearly depending on the history of electric charge or magnetic flux that has passed through it [1]. This remarkable history dependent resistive switching property has attracted significant attention across many research fields as a key component for next generation memory devices and brain-inspired information processing circuits [2]. In particular, oscillators incorporating memristors transform a state variable space constructed with fixed parameters into a history dependent and giving rise to novel oscillation theories. For example, in chaotic circuits with memristors, oscillation switching phenomena of periodic and chaotic oscillations occur depending on the memristor's internal state. This behavior indicates that the bifurcation structure of the chaotic circuit dynamically changes due to the memristor dynamics [3]. This dynamics of the circuit suggest the possibility of realizing self adaptive oscillations that optimally adjust according to past behavior.

In recent years, exploit oscillator dynamics for computational purposes has attracted great attention [4], [5]. Computation can fundamentally be defined as a mapping that generates reproducible outputs from inputs according to a fixed rule. Conventional logic circuits realize such mappings from input bits to output bits. In contrast, when an oscillator is regarded as a computational system, the input is nonlinearly transformed by the oscillator's intrinsic dynamics, and the resulting output is obtained as an observable of the system. Furthermore, oscillators incorporating memristors may produce outputs that depend on the history of the input signals. This history dependent behavior is advantageous in scenarios requiring time series information processing.

Therefore, in this study, inputs are applied to a chaos generating circuit with a memristor, and the resulting outputs are observed from the perspectives of time series responses and phase space representations.

2. Circuit Model

2.1 Memristor model

The operation of a memristor is governed by resistive switching. Under an applied voltage, oxygen vacancies in the insulating layer to redistribute and form conductive paths that change the electrical conduction. In this study, we employ the Hewlett-Packard memristor model. The resistance varies depending on the history of electric charge. When a sinusoidal voltage is applied, this memristor exhibits a nonlinear resistive switching characteristic that forms a pinched hysteresis loop in the v-i plane, as shown in Fig. 1.

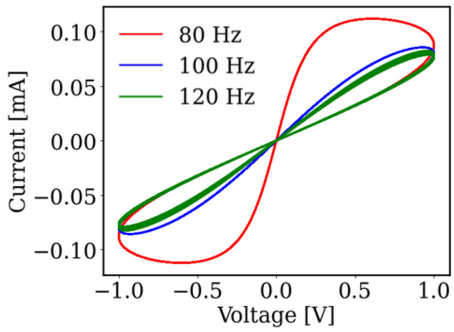

Figure 1: v-i characteristic of the memristor when a sinusoidal voltage is applied.

The memristor resistance is represented as the memristance $M(q)$ in (1). This equation expresses $M(q)$ as a linear function of the electric charge $q(t)$, and the memristance varies continuously with respect to $q(t)$.

$$M(q) = \mu_v \frac{R^2{}_{\text{on}}}{D^2} q(t) + R_{\text{off}} \left(1 - \mu_v \frac{R_{\text{on}}}{D^2} q(t) \right) \quad (1)$$

R_{on} and R_{off} represent the minimum and maximum resistance, respectively. μ_v denotes the dopant mobility, and D is the total thickness of the doped and undoped regions.

2.2 Chaos generating circuit with the memristor under sinusoidal input

To observe the basic response of the circuit to a simple time series input, we will assume a single-frequency sinusoidal input. We will vary the frequency of the applied sinusoidal input and examine the circuit response at each frequency. Figure 2 shows the chaos generating circuit with the memristor under sinusoidal input.

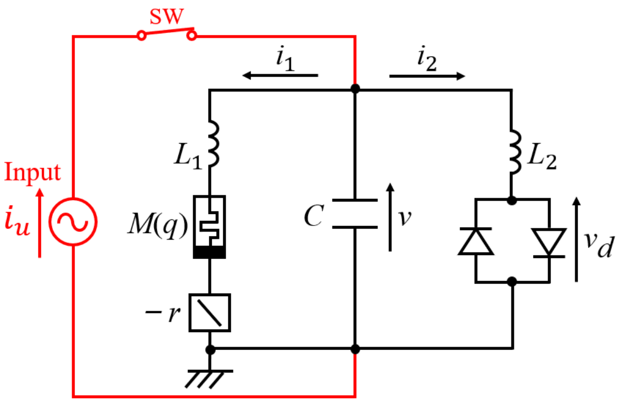

Figure 2: Chaos generating circuit with the memristor under sinusoidal input.

The chaos generating circuit with the memristor consists of a negative resistor $-r$, a capacitor C, two inductors L_1 and L_2, and a dual-directional diodes v_d. The nonlinear element formed by the dual-directional diodes exhibits a piecewise-linear v–i characteristic, as defined in (2).

$$v_d(i_2) = \frac{r_d}{2}\left(\left|i_2 + \frac{V}{r_d}\right| - \left|i_2 - \frac{V}{r_d}\right|\right) \tag{2}$$

Then, the circuit dynamics in Fig. 2 is described by the circuit equation from Kirchihoff's circuit lows as (3).

$$\begin{cases} L_1\dfrac{di_1}{dt} = v + ri_1 - M(q)i_1 \\ L_2\dfrac{di_2}{dt} = v - v_d(i_2) \\ C\dfrac{dv}{dt} = -i_1 - i_2 + i_u \\ \dfrac{dq}{dt} = i_1 \end{cases} \tag{3}$$

The important part in (3) is the $M(q)i_1$ part because it realize a chaotic generating circuit with an additional function to store the history of the passing charges as its memristance value. Next, the circuit equations in (3) is normalized to observe the fundamental response of the circuit output to external inputs by calculating the Runge-Kutta method. We change the variables as follows;

$$i_1 = \sqrt{\frac{C}{L_1}}Vx\,,\ i_2 = \frac{\sqrt{L_1C}}{L_2}Vy\,,\ v = Vz\,,\ q = CVw\,,$$

$$i_u = \sqrt{\frac{C}{L_1}}Vu, t = \sqrt{L_1C}\tau, '\cdot' = \frac{d}{d\tau}, r\sqrt{\frac{C}{L_1}} = \alpha, \frac{L_1}{L_2} = \beta,$$

$$r_d\frac{\sqrt{L_1C}}{L_2} = \gamma, R_{\text{off}}\sqrt{\frac{C}{L_1}} = \eta, \frac{R_{\text{on}}}{R_{\text{off}}} = \zeta, \mu_v\frac{R_{\text{on}}}{D^2}CV = \xi$$

Then, the normalized circuit equations are described as (4).

$$\begin{cases} \dot{x} = z + \alpha x - \eta x(\zeta\xi w + 1 - \xi w) \\ \dot{y} = z - \dfrac{\gamma}{2}\left(\left|y + \dfrac{1}{\gamma}\right| - \left|y - \dfrac{1}{\gamma}\right|\right) \\ \dot{z} = -x - \beta y + u \\ \dot{w} = x \end{cases} \tag{4}$$

Here, τ is the scaling time, α means the negative resistance, β means the inductance, γ means the resistance of the diode when it is off, η means the maximum resistance of the memristor, ξ means the minimum resistance of the memristor and ζ means the ratio of R_{on} and R_{off}. The parameter values are fixed to $\alpha = 0.524$, $\beta = 2.92$, $\gamma = 456$, $\eta = 0.0963$, $\xi = 0.00276$, and $\zeta = 0.125$.

In this study, the step size of the Runge-Kutta method is set to $h = 0.002$. This numerical calculation is performed for τ from 0 to 10,000. The amplitude of the sinusoidal input is set to 0.1. In Fig. 2, the switch (SW) is turned on to apply a sinusoidal input during $0 \le \tau \le 5,000$, and is turned off during $5,000 < \tau \le 10,000$ to observe the circuit behavior after the input is removed.

3. Time Series Waveforms

In this section, the characteristics of the circuit dynamics are examined by observing time series waveforms during the application of the sinusoidal input and after its removal. The responses are compared for different sinusoidal input frequencies.

Figure 3 shows the circuit output without input and the outputs obtained under sinusoidal input at 50 Hz, 100 Hz, 500 Hz, 2.5 kHz, and 25 kHz. Figure 3(a) presents the circuit output without the sinusoidal input, where the oscillation switching of periodic (3-period) and chaotic oscillations is observed. For low and middle frequency sinusoidal inputs as shown in Figs. 3(b), 3(c), 3(d), and 3(e), no periodic oscillations such as those in Fig. 3(a) appear during the input application term ($\tau \le 5000$), and only chaotic oscillations are observed. After the input is removed ($\tau > 5000$), frequency dependent output behaviors emerge, and oscillation

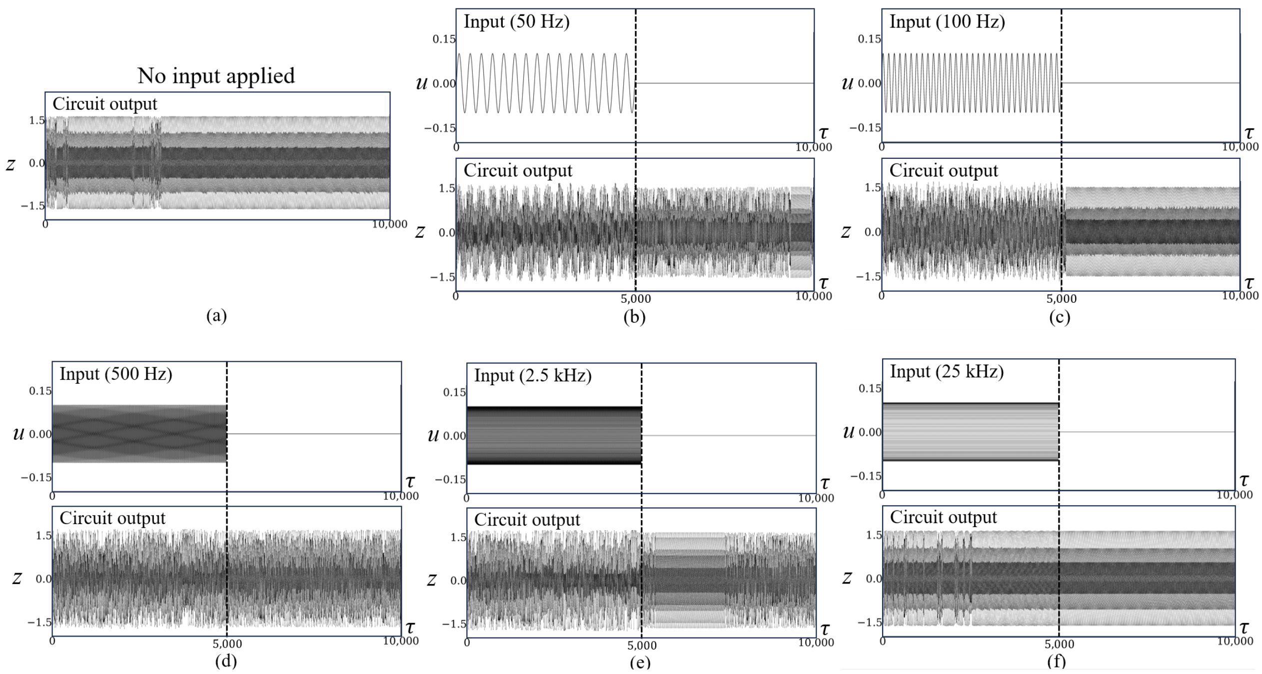

Figure 3: Time series waveform with sinusoidal inputs. (a) No input. (b) 50 Hz. (c) 100 Hz. (d) 500 Hz. (e) 2.5 kHz. (f) 25 kHz.

switching to periodic oscillations is observed in Figs. 3(b), 3(c), and 3(e) in particular. However, in Figs. 3(b), 3(c), and 3(e), the timing at periodic oscillations emerge and their residence durations differ. This observation suggests that differences in the frequency of the applied sinusoidal input influence not only the occurrence of periodic oscillations but also their onset timing and duration. Furthermore, the time series waveform in Fig. 3(f), obtained under a high frequency sinusoidal input, is similar to that in Fig. 3(a) without input. The dominant frequency component of the circuit is originally located around 25 kHz. Therefore, the frequency difference between the applied sinusoidal input and the original oscillation is small, which is considered to result in a limited influence on the circuit dynamics.

These results indicate that when sinusoidal inputs with different frequencies are applied to the chaotic circuit with the memristor, the circuit outputs exhibit distinct temporal characteristics.

4. Phase Spaces

In this section, we investigate the geometric characteristics of the phase spaces constructed by the state variables of the chaotic circuit with the memristor before and after the application of a sinusoidal input. These geometric features help to clarify how the characteristics of the input are transformed into structures in the phase spaces. Figure 4 shows the x-z-w phase spaces before and after the input application for sinusoidal input frequencies of 50 Hz, 100 Hz, 500 Hz, 2.5 kHz, and 25 kHz. From these figures, it can be seen that the geometric structures formed by the state variables in the phase space differ before and after the input. This difference does not simply arise from the removal of the sinusoidal component after the input is turned off. The sinusoidal input influences the dynamics of each state variable, it leads to a modification of the circuit dynamics after the input. In some cases, such as Figs. 4(a), 4(c), and 4(d), the after input geometric structures remain chaotic. In other cases, such as Figs. 4(b) and 4(e), the after input structures exhibit periodic behavior. Moreover, differences associated with the input frequency and the distinction between before and after input states are particularly pronounced along the memristor state variable direction w compared with the other state variables x and z. In some cases, such as Figs. 4(a), 4(c), and 4(d), the state variables form chaotic geometric structures that extend over a wide range along the w direction. In contrast, such as Figs. 4(b) and 4(e), periodic geometric structures are observed within a limited range, although their regions of existence differ.

These observations suggest that differences in input information may be distinctly encoded along the memristor state variable direction, and that this property contributes to the emergence of continuous geometric variations in the phase space corresponding to different inputs. It suggests that input dependent mappings through the chaotic circuit with the memristor can be realized.

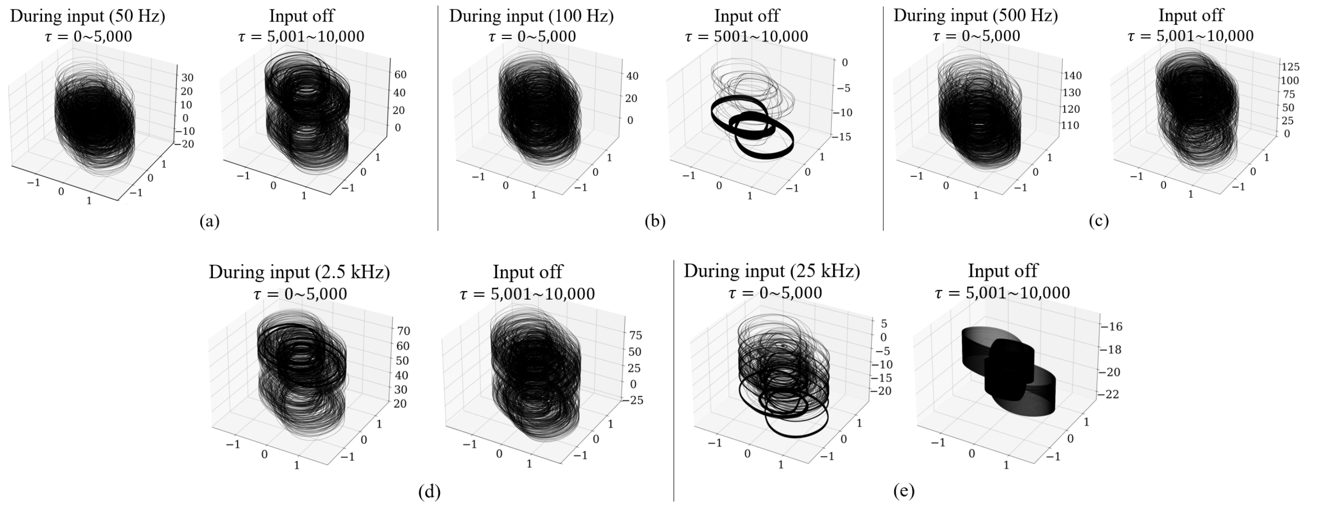

Figure 4: Phase spaces before and after the application of a sinusoidal input. (a) 50 Hz. (b) 100 Hz. (c) 500 Hz. (d) 2.5 kHz. (e) 25 kHz.

5. Conclusions

In this study, the dynamical responses of a chaos generating circuit with a memristor under sinusoidal inputs were investigated from the viewpoints of time series waveforms and phase space structures. By varying the frequency of a single frequency sinusoidal input, we clarified how input characteristics influence the circuit dynamics during and after input application.

The results of the time series analysis demonstrated that the circuit exhibits frequency dependent oscillation switching behaviors. While chaotic oscillations dominate during input application for low and middle frequency inputs, periodic oscillations emerge after input removal with onset timing and residence duration strongly dependent on the input frequency. In contrast, when a high frequency sinusoidal input close to the original dominant frequency of the circuit is applied, the circuit dynamics are weakly affected and oscillation switching similar to the no input case is observed.

Furthermore, phase space analysis revealed that the geometric structures formed by the state variables differ significantly before and after input application. In particular, variations in geometric structures along the memristor state variable direction w were more pronounced than those in the other state variables. These results indicate that the history dependent property of the memristor plays a crucial role in encoding input information.

Those findings suggest that the chaotic circuit with a memristor can realize input dependent dynamical mappings, in which differences in input frequency are reflected in temporal behaviors and phase space structures. This property highlights the potential of a chaotic circuit with a memristor as a physical dynamical system.

Acknowledgment

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