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Effect of a Linear Memristor on Two Types of Synchronization Phenomena in van der Pol Oscillators

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Abstract—This study investigates influences of a linear memristor on two different synchronization phenomena of van der Pol oscillators. Phase difference between x_1 and x_2 and time-series waveforms are observed in the case of in-phase and anti-phase synchronization. In addition, the frequency is calculated from the time-series waveforms. From these results, the frequency does not change when in-phase synchronization. On the other hand, in the case of anti-phase synchronization, the frequency varies depending on the parameters of the linear memristor.

1. Introduction

A memristor is a circuit element whose resistance changes in accordance with the current flowing through it or the magnetic flux penetrating it [1]. The memristor is currently in the development stage and are still difficult to put into practical use [2]. The characteristics of the memristors used in this study are relatively simple. The characteristic of memductance, a conductance value of the memristor, varies in linearly proportion to the magnetic flux. Hereafter, this memristor is referred to as a “linear memristor” in this study. There are memristors assuming various characteristics and synchronization phenomena in coupled systems of oscillators connected by the memristors [3], [4]. However, studies assuming linear memristor is few, so this basic research is important. Elucidating the properties of the linear memristor in this research will lead practical use of memristors in the future, the technological development of coupling systems for oscillators, and the technological development of a neural network.

A neural network is a model of machine learning made to mimic the nervous system of a human brain [5]. The general problems of the neural network are that they take a long time to calculate, consume a lot of power [6]. A possible remedy for these problems is to assume a hardware implementation [7]. Specifically, the structure of the neural network is often not changed, but the components of the neural network would be reproduced in an electric circuit [8]. In this way, the calculation time will be reduced, and power consumption will be decreased. In the simulation of the neural network realized by electric circuits, often nodes are thought of as oscillators, and edges are thought of as circuit elements. From the above, investigating the synchronization phenomena of coupled oscillator systems can be used for basic research on reproducing the parallel processing of the human brain.

In a previous study, we found the coexistence of both in-phase and anti-phase synchronization for the linear memristor connection, depending on the initial values [9]. In this study, we investigate the effect of the linear memristor on the synchronization phenomena of two van der Pol oscillators. Specifically, we observe time-series waveforms of voltage and Lissajous diagrams. Thereby, the influence of the parameters on the synchronization phenomena is considered.

2. Proposed Methods

2.1. Proposed Circuit Model

Figure 1 shows a proposed circuit. This model consists of two van der Pol oscillators coupled by the linear memristor. The van der Pol oscillator is a self-excited oscillator with a limit cycle. It consists of a capacitor C, an inductor L, and a nonlinear resistor G.

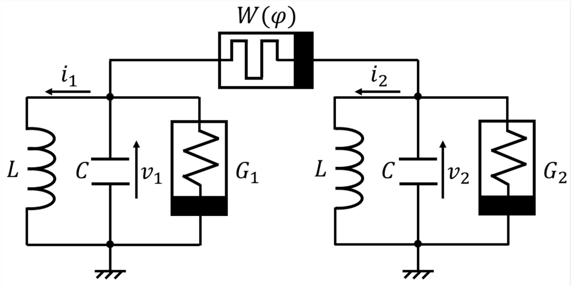

Figure 1: Proposed circuit.

The memductance, conductance value of a memristor, is proportional to the magnetic flux through the memristor.

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The characteristics of the memristor is described as Eq. (1).

$$W(\varphi) = \alpha\varphi \tag{1}$$

The characteristics of the magnetic flux through the memristor is described as Eq. (2).

$$\varphi = L(i_1 - i_2) \tag{2}$$

From Eq. (2), the magnetic flux through the memristor is a constant multiple of the difference between the currents in the left and right van der Pol oscillators in Fig 1.

The I-V characteristic of a nonlinear resistor is described as Eq. (3).

$$i_{Gn} = -g_1 v_n + g_3 v_n^3 \quad (g_1, g_3 > 0) \quad (n = 1, 2) \tag{3}$$

The circuit equations of the proposed circuit come from Kirchhoff's laws. In order to do computer simulation, the variables of the circuit equations are changed by these equations.

$$\begin{gathered} v_n = \sqrt{\frac{g_1}{g_3}} x_n \;; \quad i_n = \sqrt{\frac{g_1 C}{g_3 L}} y_n \quad (n = 1, 2) \;; \\ \varepsilon = g_1 \sqrt{\frac{L}{C}}; \; \zeta = \alpha L \sqrt{\frac{g_1}{g_3}} \;; \\ t = \sqrt{LC}\tau \;; \quad ' \cdot ' = \frac{d}{d\tau} \end{gathered} \tag{4}$$

From the above equations, the circuit equations are normalized as Eqs. (5).

$$\begin{cases} \dot{x_1} = \varepsilon x_1(1 - x_1^2) - \zeta(x_1 - x_2)(y_1 - y_2) - y_1 \\ \dot{x_2} = \varepsilon x_2(1 - x_2^2) + \zeta(x_1 - x_2)(y_1 - y_2) - y_2 \\ \dot{y_1} = x_1 \\ \dot{y_2} = x_2 \end{cases} \tag{5}$$

The parameter ε is related to the nonlinearity of the van der Pol oscillators. The parameter ζ is related to the memristor property. Eqs. (5) are calculated by Runge-Kutta method. Runge-Kutta method is one of the methods for solving ordinary differential equations. It is a method to approximate the coordinates of a point on a curve with known initial values.

2.2. Frequency Calculation

As an example, calculate the frequency of x_1. Extract a data set of waveform peak values (maximals). Calculate the interval between the peak values. The average of the intervals between the peak values is the average period $x_{1ave.}$. The frequency f_{x_1} is calculated by the following Eq. (6).

$$f_{x_1} = \frac{1}{x_{1ave.}} \tag{6}$$

3. Simulation Results

In this section, we observe the change in x_1, x_2, and frequency when the parameter ζ is changed. The parameter ε is 0.1. We compare x_1, x_2, and frequency of the linear memristor connection at different parameter ζ. Phase differences between x_1 and x_2, and the time-series waveforms of x_1 (blue) and x_2 (red) are shown.

3.1. In-Phase Synchronization with Different ζ

We use one of the initial values for which in-phase synchronization can be observed. In this section, initial values of x_1, y_1, x_2 and y_2 are set as 1.9, 0.9, 2.6 and 1.3. the detailed values of ζ is shown in Tab. 1.

Table 1: List of ζ.

	ζ
(a)	0
(b)	0.02
(c)	0.04
(d)	0.06
(e)	0.08
(f)	0.1

Figures 2 to 5 compare the synchronization phenomena.

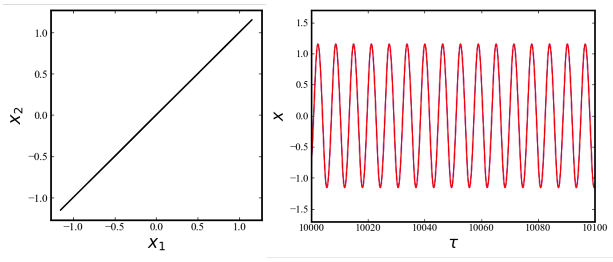

Figure 2: Result of in-phase ($\zeta = 0$).

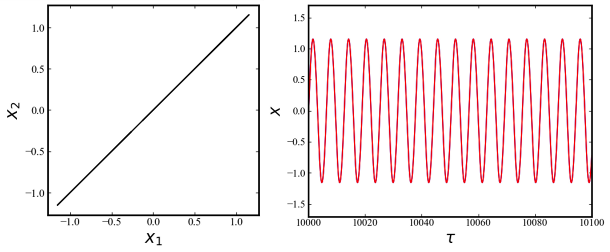

Figure 3: Result of in-phase ($\zeta = 0.02$).

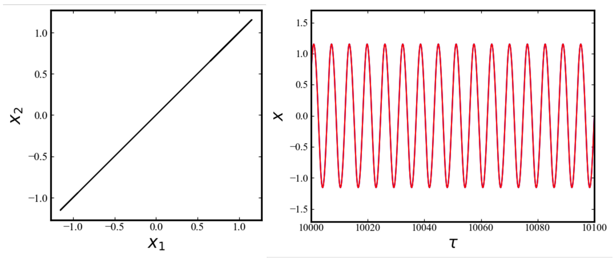

Figure 4: Result of in-phase (ζ = 0.04).

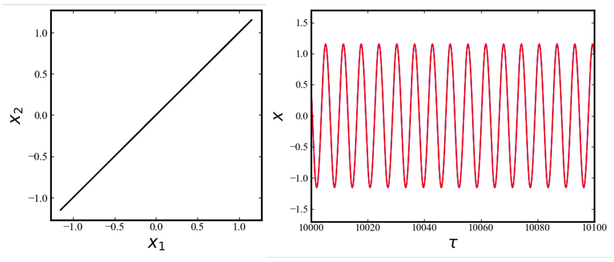

Figure 5: Result of in-phase (ζ = 0.1).

Summarize the x_1 frequency in Tab. 2.

Table 2: x_1 frequency in in-phase synchronization.

	x_1 frequency
(a)	0.15905
(b)	0.15906
(c)	0.15906
(d)	0.15906
(e)	0.15906
(f)	0.15906

From Figs. 2 to 5 and Tab. 2, when in-phase synchronization with the linear memristor connection, the shape of the phase difference between x_1 and x_2 and frequency does not change with the value of ζ. These results are the same synchronization phenomena as for the pure resistor connection.

Since the magnetic flux through the memristor is a constant multiple of the difference between the currents flowing to the left and right van der Pol oscillators, the memductance is proportional to the difference in currents. The figure shows that there is no difference in current because there is no phase difference. This means that the magnetic flux is zero from Eq. (2). The fact that the magnetic flux is zero means that the circuit is uncoupled and the characteristics of the memristor cannot be effectively utilized. In the next section, the simulation will be performed in the reverse phase because there must be a difference in current.

3.2. Anti-Phase Synchronization with Different ζ

We use one of the initial values for which anti-phase synchronization can be observed. In this section, initial values of x_1, y_1, x_2 and y_2 are set as −1.9, 0.9, 2.6 and 1.3. the detailed values of ζ is shown in Tab. 1. Figures 6 to 9 compare the synchronization phenomena.

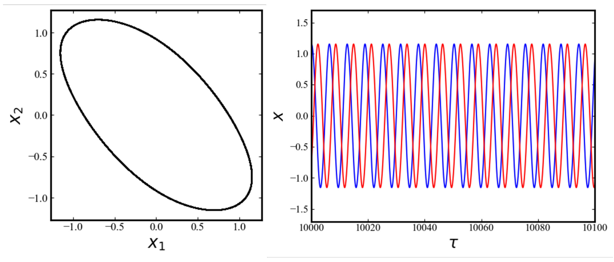

Figure 6: Result of anti-phase (ζ = 0).

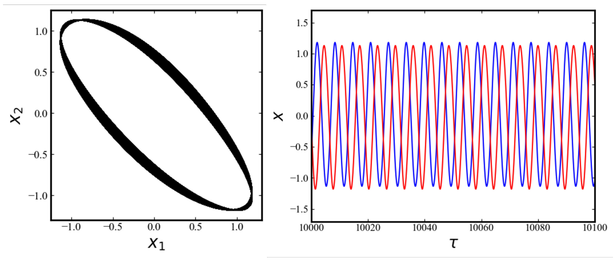

Figure 7: Result of anti-phase (ζ = 0.02).

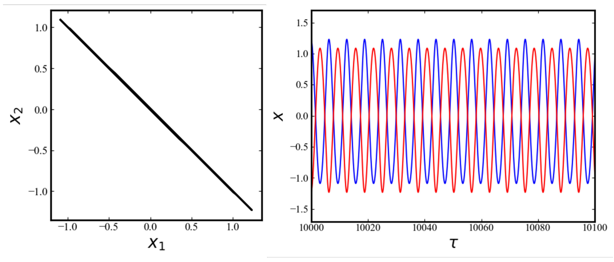

Figure 8: Result of anti-phase (ζ = 0.04).

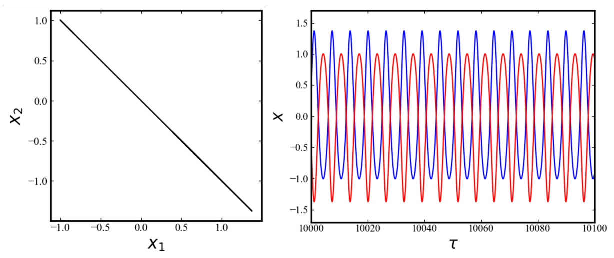

Figure 9: Result of anti-phase (ζ = 0.1).

Summarize the x_1 frequency in Tab. 3.

Table 3: x_1 frequency in anti-phase synchronization.

	x_1 frequency
(a)	0.15905
(b)	0.15901
(c)	0.15882
(d)	0.15853
(e)	0.15811
(f)	0.15756

From Figs. 6 to 9 and Tab. 3, when anti-phase synchronization, the shape of the phase difference between x_1 and x_2 and frequency change with the value of ζ. As ζ increases, the width of the phase difference of x_1 and x_2 decreases and approaches a clear anti-phase. In other words, the larger the ζ, the smaller the phase difference. Correspondingly, the frequency x_1 also decreases.

4. Conclusions

In this study, we proposed a circuit in which two van der Pol oscillators are coupled by a linear memristor. The characteristic of memductance varies in proportion to the magnetic flux. Lissajous diagrams, time-series waveforms, and the frequency of voltage was calculated.

We observed how the memristor parameter ζ affects the synchronization phenomena in each case of in-phase and anti-phase synchronization. To easily evaluate the effect of the linear memristor, we focused on the phase difference diagrams, the time-series waveforms, and the x_1 frequency when the value of ζ for the linear memristor was changed.

In conclusion, neither the figure nor the frequency changed significantly during in-phase synchronization. During anti-phase synchronization, as ζ increases, the phase difference diagram becomes narrower and closer to a clean anti-phase synchronization. Correspondingly, the frequency becomes smaller.

For future work, in order to capture the characteristics of the memristor, we consider that a simulation with continuously varying current should be performed. Furthermore, the simulation should compare the case where the difference in current is constant and the case where it is not. In this simulation, only the parameter ζ is changed. We expect simulations that focus on the difference in currents correctly capture the characteristics of the linear memristor.

In addition to that, this study is a simulation of a circuit in which an oscillator is coupled with a memristor. However, we aim to observe the synchronization phenomena in the inductor and capacitor connection as well. We hope to be able to analyze the characteristics of the linear memristor more accurately by comparing the connection with other elements.

Finally, approximately 3×10^{-2} frequency differences were observed for different parameters in anti-phase synchronization. However, it is not known whether this is a difference of any magnitude. Quantitative examination is needed. Frequency changes usually require the use of inductors or capacitors. It was surprising that the frequency was changed simply by connecting the linear memristor. Theoretical analysis and other quantitative considerations are expected to further expand the application of the memristor.

Acknowledgement

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