Hysteresis Characteristics of a Linear Memristor in van der Pol Oscillators Coupling System

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***Abstract*—In 2008, memristor were developed by the research laboratory of Hewlett-Packard in the United States. However, their characteristics have not been sufficiently analyzed, and structural issues have prevented their practical application. In this study, we analyzed a system in which two van der Pol oscillators were coupled by a memristor, using a device that simplifies the characteristics of the developed memristor. Hysteresis characteristics are the basic principle of memristors. Through numerical simulation, we verified whether synchronization phenomena and hysteresis characteristics could be observed. As a result, it was confirmed that both in-phase and anti-phase synchronization coexist depending on initial values, and the hysteresis characteristics corresponding to each synchronization phenomena were also observed.**

***Index Terms*—Hewlett-Packard memristor, linear memristor, hysteresis characteristic, synchronization phenomena**

I. INTRODUCTION

A memristor is a passive device whose resistance value changes depending on the amount of charge flowing through it and the magnetic flux passing through it. The existence of the memristor was first theoretically proposed in a paper by L.O. Chua in 1971 [1]. The operating principle of the memristor is hysteresis characteristics, which show a history effect between input signals and output responses [2]. However, at that time, the principle behind this phenomenon could not be explained, and the memristor remained unrealized for a long time. Later, in 2008, the memristor was first implemented by the research laboratory of Hewlett-Packard (HP) in the United States [3].

This achievement clarified the principle of hysteresis characteristics. Additionally, this memristor has the characteristics of being implementable at the nanoscale and exhibiting behavior similar to analog components under certain conditions [4]. These properties are similar to the behavior of synapses and may be applicable to the implementation of neural networks. However, the physical properties of the memristor have not yet been fully elucidated, and there are few examples of its implementation.

In this study, we use a "linear memristor" model that simplifies the characteristics of the HP's memristor. We analyze the phenomena and behavior observed in a coupled system of van der Pol oscillators using the linear memristor through numerical simulation. In particular, we focus on how hysteresis characteristics appear depending on the initial state and operating conditions of the circuit, and explore the principles and potential applications of this technology.

II. PROPOSED MODELS

A. Proposed Circuit Model

Figure 1 shows a proposed circuit. This model consists of two van der Pol oscillators coupled by the linear memristor. The van der Pol oscillator is a self-excited oscillator with a limit cycle. It consists of a capacitor C, an inductor L, and a nonlinear resistor G.

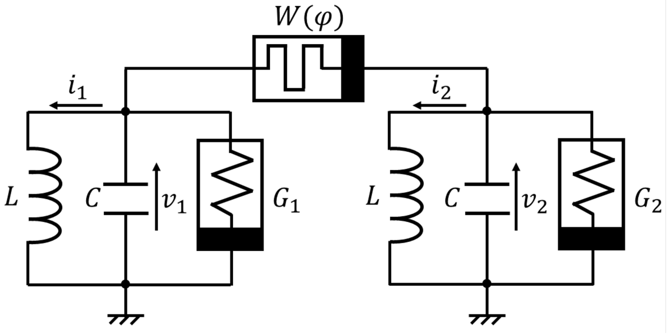


Fig. 1. Proposed circuit.

The memductance, conductance value of a memristor, is proportional to the magnetic flux through the memristor. The characteristics of the memristor is described as Eq. (1).

$$W(\varphi) = \alpha\varphi \tag{1}$$

The characteristics of the magnetic flux through the memristor is described as Eq. (2).

$$\varphi = L(i_1 - i_2) \tag{2}$$

From Eq. (2), the magnetic flux through the memristor is a constant multiple of the difference between the currents in the left and right van der Pol oscillators in Fig 1.

The I-V characteristic of a nonlinear resistor is described as Eq. (3).

$$i_{Gn} = -g_1 v_n + g_3 v_n^3 \quad (g_1, g_3 > 0) \quad (n = 1, 2) \tag{3}$$

The circuit equations of the proposed circuit come from Kirchhoff's laws. In order to do computer simulation, the variables of the circuit equations are changed by these equations.

$$\begin{gathered} v_n = \sqrt{\frac{g_1}{g_3}} V x_n \ ; \quad i_n = \sqrt{\frac{g_1 C}{g_3 L}} V y_n \quad (n = 1, 2) \ ; \\ \varepsilon = g_1 \sqrt{\frac{L}{C}} \ ; \ \zeta = \alpha L \sqrt{\frac{g_1}{g_3}} V \ ; \ t = \sqrt{LC}\tau \ ; \ ' \cdot ' = \frac{d}{d\tau} \end{gathered} \tag{4}$$

From the above equations, the circuit equations are normalized as Eq. (5).

$$\begin{cases} \dot{x_1} = \varepsilon x_1(1-x_1^2) - \zeta(x_1-x_2)(y_1-y_2) - y_1 \\ \dot{x_2} = \varepsilon x_2(1-x_2^2) + \zeta(x_1-x_2)(y_1-y_2) - y_2 \\ \dot{y_1} = x_1 \\ \dot{y_2} = x_2 \end{cases} \tag{5}$$

The parameter ε is related to the nonlinearity of the van der Pol oscillators. The parameter ζ is related to the memristor property. Eq. (5) are calculated by Runge-Kutta method. Runge-Kutta method is one of the methods for solving ordinary differential equations.

III. SIMULATION RESULTS

In this section, we compare simulation results when the initial values of y_1 and x_2 are changed.

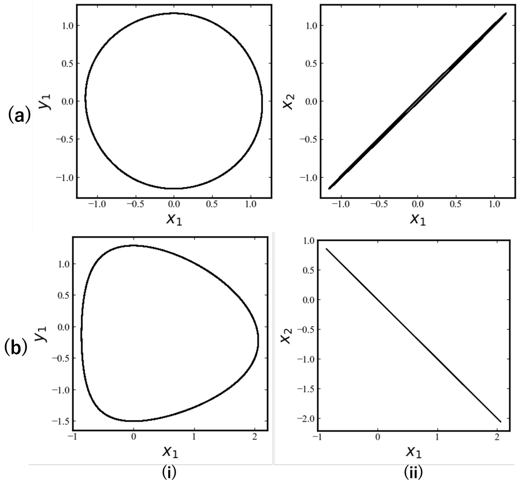
Table 1 Used coditions.

	x_1	y_1	x_2	y_2	ε	ζ
Condition (a)	1.1	2.1	1.2	2.3	0.1	0.24
Condition (b)	1.1	-2.1	-1.2	2.3	0.1	0.1

In each subsection, we compare the phenomena that occur under conditions a and b.

A. Synchronization phenomena

In this subsection, we observe the synchronization phenomena for each condition using (i) a circuit attractor (x_2-y_2) and (ii) a phase difference (x_1-x_2).



(i) (ii)

Fig. 2. Simulation results of synchronization phenomena.

The attractors are distorted by the difference in initial values, and the coexistence of in-phase synchronization and anti-phase synchronization is observed.

B. Hysteresis Characteristic

It is known that the resistance value of the HP's memristor has a hysteresis characteristic. We assumed that the linear memristor, which is simplified model of HP's memristor characteristics, also have the hysteresis characteristic. To examine the hysteresis characteristic of the linear memristor, we observed a i_m-v_m characteristic graph, where i_m is the memristor current and v_m is the memristor voltage. Since v_m is the potential difference of the capacitor, v_m is described as Eq. (6).

$$v_m = v_1 - v_2 \tag{6}$$

Using v_m and Eq. (1), i_m is described as Eq. (7).

$$i_m = v_m W(\varphi) = \alpha L(i_1 - i_2)(v_1 - v_2) \tag{7}$$

We observe the graphs of the i_m-v_m characteristics for each condition.

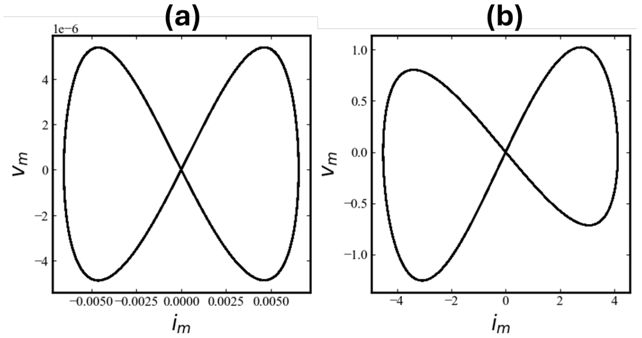


Fig. 3. Simulation results of the hysteresis characteristics.

Hysteresis characteristics similar to those of HP's memristor were observed. Different characteristics were observed in phase and anti-phase synchronization. It was found that the hysteresis characteristics were retained even when HP's memristor were modeled simply.

IV. CONCLUTIONS

The results showed that depending on the initial values, in-phase and anti-phase synchronization phenomena coexistence could be obtained, and that there were hysteresis characteristics corresponding to each synchronization phenomena. It was also found that even when the HP's memristor is simplified into a model, the hysteresis characteristics, which are one of the important features of the memristor, are not lost.

This linear memristor has simpler characteristics and easier calculations compared to the HP's memristor. Furthermore, the results of this study show that it can express the characteristics of the memristor. From the above, it is considered that it can be used in various applications such as numerical simulation, circuit simulation, and circuit implementation.

ACKNOWLEDGMENT

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