Influence of Memristor Dopant Mobility on Chaotic Circuit Dynamics

Taishi Segawa, Yoko Uwate, and Yoshifumi Nishio
Dept. of Electrical and Electronic Engineering, Tokushima University, Japan
Email: {segawa, uwate, nishio}@ee.tokushima-u.ac.jp

***Abstract*—We design a four-dimensional chaotic circuit with a memristor and investigate how changes in the memristor's dopant mobility affect its v-i characteristics and the overall behavior of the chaotic circuit. As a result, we find that the v-i characteristics of the memristor change from linear to various nonlinear forms as the dopant mobility increases. Furthermore, we observe that the dopant mobility influences the behavior of periodic orbits in the chaotic circuit and oscillation switching of periodic and chaotic states.**

I. INTRODUCTION

The working principle of a memristor is based on the resistive switching effect. When a voltage is applied, oxygen vacancies in the insulating layer migrate and form conductive filaments, resulting in a change in current magnitude. Due to this excellent resistance switching behavior, memristors exhibit unique v-i characteristics, such as the pinched hysteresis loop in the Hewlett–Packard (HP) model and piecewise linear v-i characteristics in simplified models. Furthermore, memristors based on various materials, such as TiO_2, HfO_2, and ZnO, have been proposed. Since the dopant mobility varies depending on the material, these devices are expected to exhibit with deiiferent v-i characteristics on each material [1]. Therefore, even if the basic working principle of the memristor remains the same, its response in a circuit may differ depending on the dopant mobility. Accordingly, it is important to investigate the relationship between dopant mobility and circuit behavior.

In this study, we investigate the influence of the memristor dopant mobility on a chaotic circuit dynamics.

II. CIRCUIT MODEL

We use a chaotic circuit with a memristor presented in previous study, shown in Fig. 1 [2].

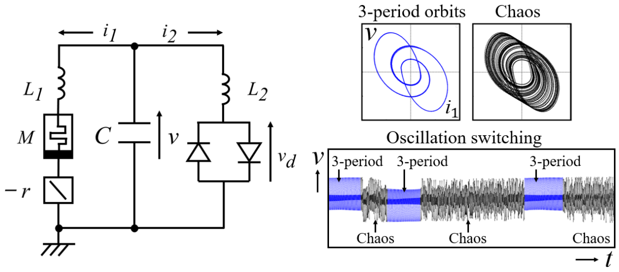

Fig. 1. Chaotic circuit with the memristor

In this circuit, we can observe the oscillation switching phenomena of periodic and chaotic oscillations over time. The original chaotic circuit consists of one negative resistor r, one capacitor C, two inductors L_1 and L_2, and one dual-directional diode as a nonlinear resistor v_d. In this original chaotic circuit, i_1 is usually larger than i_2 when chaos is observed. To take advantage of this property, the memristor is added between the inductor L_1 and the negative resistor $-r$ to increase the influence of the memristor on the chaotic circuit.

In Fig. 1, we use the HP memristor as described in (1).

$$M(q) = \mu_v \frac{R^2{}_{\rm on}}{D^2} q(t) + R_{\rm off}\left(1 - \mu_v \frac{R_{\rm on}}{D^2} q(t)\right) \qquad (1)$$

The resistance of a memristor called memristance and denoted by $M(q)$, is a function of the charge $q(t)$. Here, $R_{\rm on}$ and $R_{\rm off}$ represent the minimum and maximum resistance values, respectively. μ_v denotes the dopant mobility, and D is the total thickness of the doped and undoped regions.

Then, by following the normalizations,

$$i_1 = \sqrt{\frac{C}{L_1}} Vx \, , \; i_2 = \frac{\sqrt{L_1 C}}{L_2} Vy \, , \; v = Vz \, , \; q = CVw \, ,$$

$$t = \sqrt{L_1 C}\tau \, , \; ' \cdot ' = \frac{d}{d\tau} \, , \; r\sqrt{\frac{C}{L_1}} = \alpha \, , \; \frac{L_1}{L_2} = \beta \, ,$$

$$r_d \frac{\sqrt{L_1 C}}{L_2} = \gamma \, , R_{\rm off}\sqrt{\frac{C}{L_1}} = \eta , \; \frac{R_{\rm on}}{R_{\rm off}} = \zeta , \; \mu_v \frac{R_{\rm on}}{D^2} CV = \xi ,$$

the normalized circuit equations are given as follows:

$$\begin{cases} \dot{x} = z + \alpha x - \eta x(\zeta \xi w + 1 - \xi w) \\ \dot{y} = z - \dfrac{\gamma}{2}\left(\left|y + \dfrac{1}{\gamma}\right| - \left|y - \dfrac{1}{\gamma}\right|\right) \\ \dot{z} = -x - \beta y \\ \dot{w} = x. \end{cases} \qquad (2)$$

where α is the negative resistance, β is the ratio of inductance, γ is the resistance of the nonlinear resistor when the diodes are off, η is the maximum memristance, ζ is the minimum memristance, and ξ corresponds to the dopant mobility.

In the computer numerical calculation, the step size of the Runge-Kutta method is set to h = 0.002. This numerical calculation is performed for τ from 0 to 100,000. Some of the parameters are fixed to α = 0.588, β = 2.92, γ = 456, η = 0.171, and ζ = 0.00625. The initial values of each variable is set randomly between -0.6 and 0.6.

III. RESULTS

We observe the v–i characteristics of the memristor while varying the dopant mobility μ_v, with the values of μ_v based on the results reported in [3].

Typically, the v–i characteristics of a memristor are evaluated under sinusoidal excitation. To ensure consistency, we first determine the dominant frequency of the chaotic circuit and then apply a sine wave of the same frequency to the memristor. The resulting v–i characteristics are subsequently compared with those observed during its operation within the chaotic circuit.

The dominant frequency of the chaotic circuit is determined from the power spectrum shown in Fig. 2. As shown in Fig. 2, the primary frequency component is approximately 27.2 kHz. Accordingly, the frequency of the sine wave input for the v–i characteristic comparison is set to 27.2 kHz.

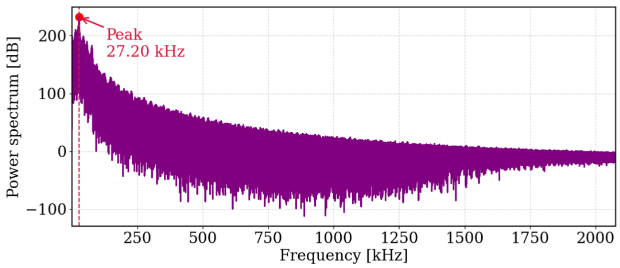

Fig. 2. Frequency of the chaotic circuit

Next, we investigate the v–i characteristics of the memristor under both sine wave excitation and operation within the chaotic circuit, and analyze how the circuit output changes with variations in dopant mobility μ_v. Figure 3 shows the changes in the memristor's v–i characteristics and the corresponding circuit behavior, visualized using a Poincaré map. When $\mu_v = 10^{-14}\mathrm{m}^2\,\mathrm{V}^{-1}\,\mathrm{s}^{-1}$, where a pinched hysteresis loop was reported in previous studies, the v–i characteristic appears nearly linear, resembling that of a pure resistor. This is likely due to the input voltage having a higher frequency than that used in the conventional setup. As μ_v increases, a hysteresis loop gradually emerges. In the range $\mu_v = 10^{-14}$ to $10^{-12}\mathrm{m}^2\,\mathrm{V}^{-1}\,\mathrm{s}^{-1}$, three-period oscillations become more dominant than chaotic oscillations. Moreover, when μ_v increases further to between 10^{-11} and $5 \times 10^{-11}\mathrm{m}^2\,\mathrm{V}^{-1}\,\mathrm{s}^{-1}$, the chaotic oscillations transition to three-period oscillations, which subsequently converge to one-period oscillations. These results indicate that the behavior of the periodic oscillations in the circuit strongly depends on the dopant mobility of the memristor.

Finally, we investigate the change in the oscillation switching of 3-periodic and chaotic oscillations when μ_v is changed. Table I shows the number of times of the oscillation state switches and the ratio of 3-periodic oscillation to the entire simulation time, averaged over five different initial values. From Table I, as μ_v increases, the number of switching events decreases until 1-periodic oscillation occurs, and the ratio of 3-periodic oscillations increases.

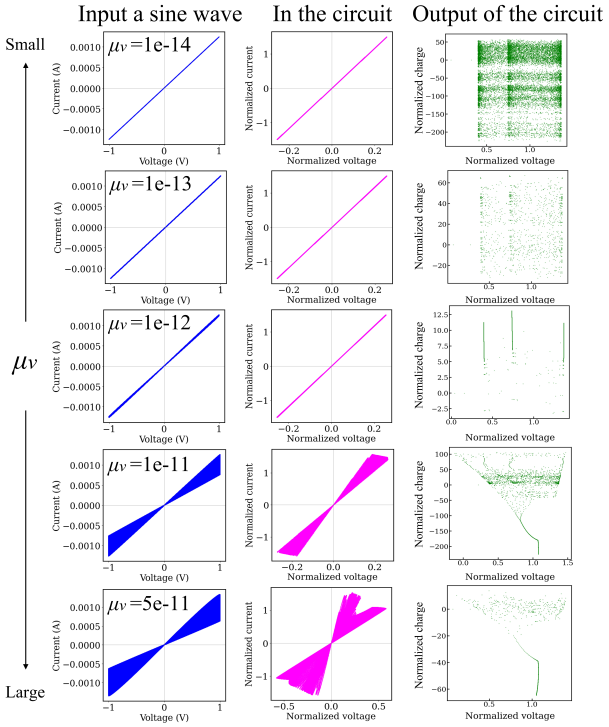

Fig. 3. Changes in v-i characteristics and circuit output due to dopant mobility

TABLE I
OSCILLATION SWITCHING PHENOMENON

	Ave. Oscillation switching [times]	Ave. Period ratio [%]
Pure resistor	0	0.00
μv $[m^2/V \cdot s]$		
1e-14	1071	13.19
1e-13	173	84.18
1e-12	11	99.31
1e-11	124	9.96
5e-11	3	0.02

IV. CONCLUSIONS

By varying the dopant mobility of the memristor in a chaotic circuit, we found that the behavior of periodic oscillations was significantly affected. Furthermore, the number of oscillation switching events and the proportion of periodic oscillations were found to vary with the dopant mobility.

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