# Chaotic Time Series Prediction via ESN with Decorrelated Reservoir Dynamics

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*Abstract*— **This study proposes a new approach to improve reservoir design in Echo State Network (ESN) by combining the orthogonalization of the connection weight matrix with the introduction of variance in the row norm. The evaluation is carried out from three perspectives: prediction accuracy level, reduction in accuracy variance, and convergence time of internal state differences. Experimental results demonstrate that the proposed method shows improved prediction accuracy and improved stability compared to conventional approaches in specific spectral radius regions.**

***Keywords; Echo State Network, Echo State Property, Chaotic Time Series Prediction, Orthogonalization, Row Norm Variance***

## I. INTRODUCTION

Echo State Network (ESN) represents a prominent approach in reservoir computing, demonstrating superior performance in time series prediction tasks. ESN performance relies on the Echo State Property (ESP), which ensures that the network appropriately reflects both current and past input information while guaranteeing that internal state differences converge regardless of initial conditions when identical inputs are provided. This property enables ESN to achieve stable learning for data with complex temporal dependencies [1].

In previous studies on ESN, orthonormalization for the connection weight matrices of reservoir was adopted. Orthonormalization contributes to improved learning performance by ensuring independence among nodes and suppressing information redundancy [2]. However, orthonormalization has the weakness that all row norms in orthonormalization are uniform, which makes the relative importance of reservoir nodes uniform.

In this study, we propose a method to address this weakness by introducing variance in the norm of each row while maintaining the orthogonality of the joint weight matrix. With this method, we aim to improve the representational capability of reservoirs while retaining the advantages of ESP.

## II. SIMULATION SETTINGS

### A. Composition of ESN

The structure of the ESN used in this study is illustrated in Fig. 1. Both input and output are one-dimensional. The reservoir consists of 100 nodes, with a connection probability of 0.1 and a leakage rate of 0.3.

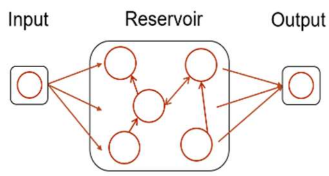


Figure 1: The structure of ESN.

### B. Chaos Time Series

In this study, the Mackey-Glass equation is used as the chaotic time series. The equation is as follows.

$$\frac{dx}{dt} = \beta \frac{x(t-\tau)}{1 + x(t-\tau)^{\gamma}} - \alpha x(t) \tag{1}$$

In Eq.(1), $\beta$=0.2, $\gamma$=10, $\alpha$=0.1, $\tau$=22, $x(0)$=0.7 are set to generate time series that exhibit chaotic behavior. The dataset consists of 600 for training and 200 for testing.

## III. PROPOSED METHOD

### A. Orthogonalization by Givens Rotation

Givens rotation is a method of stepwise orthogonalizing matrices using a rotational transformation in the two-dimensional plane. For an $N \times N$ matrix, the rotation matrix $R$ is defined as follows.

In the rotation matrix $R(i, j, \theta)$, $\cos\theta$ is placed on elements $(i, i)$ and $(j, j)$, $-\sin\theta$ on element $(i, j)$, and $\sin\theta$ on element $(j, i)$, while other diagonal elements are set to 1 and off-diagonal elements are set to 0. The number of rotations $M$ is defined by the following equation using the parameter called density. As in the previous study, we set the density to 0.2. This means that in this study, 990 rotations will be performed on a reservoir consists of 100 nodes.

$$M = density \times \frac{N(N-1)}{2} \tag{2}$$

### B. Introducing Row Norm Variance

In conventional orthogonalization, all row norms are equal to 1, resulting in a fixed spectral radius of 1. In our approach, after orthogonalization via Givens rotation, a row norm variance is introduced. Specifically, the norm of each row norm is deterministically adjusted based on the target variance value to add diversity to the influence among reservoir nodes. In this study, the target variance is varied from 0.1 to 0.5 in increments of 0.1, and experiments are conducted in five different settings.

Since the spectral radius is inherently fixed to 1.0 in the previous study's method, we add a process to adjust the spectral radius in the range of 0.8 to 1.7 for all methods (conventional and proposed) in order to achieve a fair comparison. This method allows us to add diversity to the influence of each node while maintaining the independence of information due to orthogonality. As a result, it is expected to more effectively capture the diverse temporal patterns of chaotic time series.

Furthermore, to ensure the reliability of the experiment, the initial pseudo-random number for the connection weight matrix generation is varied from 1 to 100, and 100 independent trials are conducted.

Performance evaluation is conducted from three perspectives. First, prediction accuracy is quantified using the normalized root mean square error (NRMSE), with lower NRMSE values indicating higher accuracy. Second, for the variability of accuracy, the standard deviation of the NRMSE values over 100 independent trials is calculated to evaluate the stability of the method. Third, for the convergence time of internal state difference, the number of time steps until the average difference between two reservoir states starting from different initial states (randomly generated within the [-0.5, 0.5] interval) becomes less than $1.0 \times 10^{-10}$ is measured as an evaluation of ESP characteristics.

## IV. Simulation Results

The comparison of prediction accuracy is shown in Fig. 2.

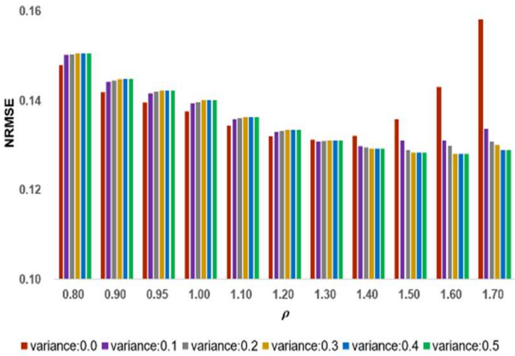


Figure 2: Comparison of prediction accuracy.

In the spectral radius region of $1.3 \leq \rho \leq 1.7$, the proposed method shows a significant improvement in accuracy compared to the previous methods. The most significant improvements were observed at variance values of 0.4 and 0.5, which resulted in an improvement of up to about 18% over the conventional method in NRMSE.

The results show that the introduction of a moderate row norm variance allows each node in the reservoir to efficiently process information on different time scales and to more precisely represent the complex nonlinear interactions due to the delay terms in the Mackey-Glass equation.

Next, the comparison of the variation in prediction accuracy is shown in Fig. 3.

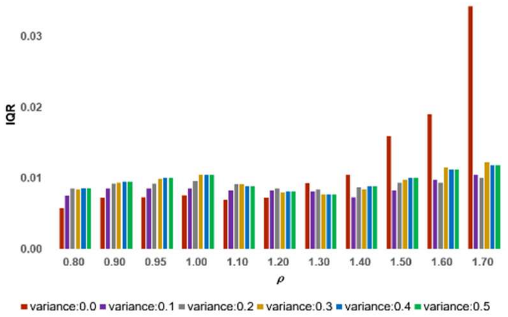


Figure 3: Comparison of variation in prediction accuracy.

In the spectral radius region of $1.3 \leq \rho \leq 1.7$, the proposed method shows a clear reduction in variation compared to the conventional method. In particular, the best results were obtained in settings with variances of 0.1 and 0.2.

Next, the comparison of the convergence time of internal state difference is shown in Fig. 4.

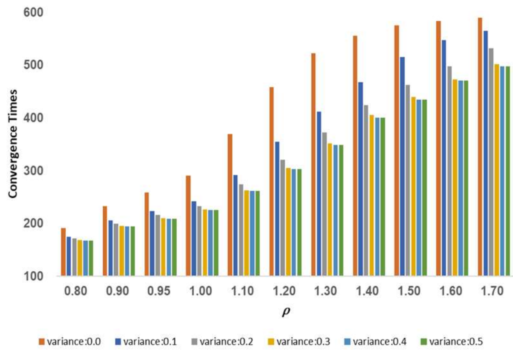


Figure 4: Comparison of convergence time.

In the entire range of spectral radius $0.8 \leq \rho \leq 1.7$, the proposed method achieves earlier convergence than the conventional method.

## V. Conclusion

In this study, we proposed a new reservoir design method that introduces row norm variance into the orthogonalization of the connection weight matrix in ESN and demonstrated its effectiveness in Mackey-Glass time series prediction. The proposed method was shown to simultaneously enhance prediction accuracy, improve stability, and accelerate the convergence of internal state differences within specific spectral radius regions.


## Acknowledgment

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## References


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