Evaluation of Memory Capacity and Performance in Oscillator Reservoir with Different Nonlinearities

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Abstract— **In this research, the results of a waveform classification task are presented. The task is performed using a networked reservoir of van der Pol oscillators interconnected by resistors. The nonlinearity of the oscillators that constitute the reservoir layer is focused, comparing the same nonlinearity for all oscillators, and different nonlinearity for each oscillator. The results show better performance and larger memory capacity when the nonlinearity differs for each oscillator.**

Keywords; coupled oscillator, reservoir, memory capacity

I. Introduction

Deep learning has demonstrated remarkable performance in pattern recognition tasks, however, it requires substantial computational resources and power consumption. As an alternative, reservoir computing (RC) has attracted growing attention due to its potential for low-power and high-speed information processing. In RC, only the output layer is trained, while the internal structure and weights of the reservoir remain fixed. This leads to simpler training procedures and enables flexible system design. Although dynamic reservoirs must exhibit diverse and rich responses to input signals, the requirements are relatively loose, allowing implementation across a wide range of physical systems [1]. In this research, the dynamics of a coupled oscillator system is employed as an implementation of the physical reservoir. Coupled oscillators, known as models for synchronization phenomena in nature, exhibit diverse nonlinear behaviors that make them promising candidates for RC applications [2], [3]. Moreover, recent research on neural-network-based reservoirs has reported that the diversity of activation functions can enhance memory performance [4]. Based on these insights, this research investigates the effect of introducing heterogeneous nonlinear characteristics to individual oscillators, aiming to identify effective design principles for physical reservoirs.

II. Proposed Model and Method

A. Coupled Oscillator Reservoir Model

In this research, van der Pol oscillators are interconnected by resistors to form a network reservoir. Figure 1 shows an overview of the proposed model. Data are input to the oscillator via a current source. The circuit equation of the coupled oscillator is shown in Eq. (1).

$$\begin{cases} C\dfrac{dv_n}{dt} = -i_{L_n} - i_{g_n} + i_u - \sum_{n,\,k=1,\,n\neq k}^{N} \dfrac{1}{R_{nk}}(v_n - v_k) \\ L\dfrac{di_{L_n}}{dt} = v_n, \end{cases} \tag{1}$$

where N is the number of oscillators.

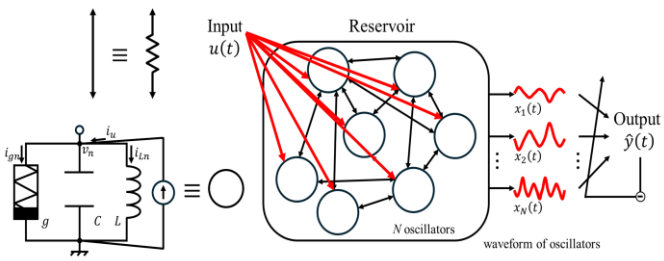


Figure 1. Proposed reservoir model.

The current-voltage characteristic of the nonlinear resistor g used in this research is given as follows:

$$i_g = -g_1 v + g_3 v^3 \quad (g_1, g_3 > 0). \tag{2}$$

The circuit equation in Eq. (1) is normalized using the following normalization parameters:

$$v = \sqrt{\frac{g_1}{g_3}}x,\ i = \sqrt{\frac{g_1 C}{g_3 L}}y,\ i_u = \sqrt{\frac{g_1 C}{g_3 L}}u,\ t = \sqrt{LC}\tau,\ \varepsilon = g_1\sqrt{\frac{L}{C}},\ \gamma = \frac{1}{R}\sqrt{\frac{L}{C}}.$$

The normalized equation is shown in Eq. (3).

$$\begin{cases} \dfrac{dx}{d\tau} = \varepsilon x_n(1 - x_n^2) - y_n + u - \sum_{n,\,k=1,\,n\neq k}^{N} K_{nk}(x_n - x_k) \\ \dfrac{dy}{d\tau} = x_n, \end{cases} \tag{3}$$

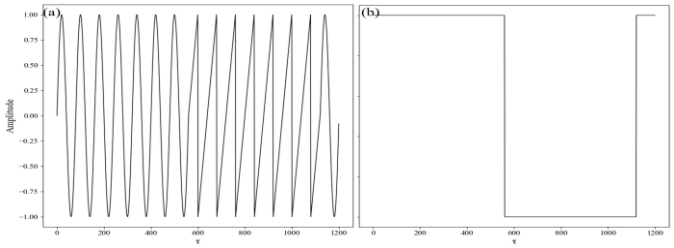
where ε is the nonlinearity of the oscillators and K is the coupling strength between the oscillators. K is given as follows:

$$K_{nk} = E_{nk}\gamma_{nk}, \tag{4}$$

where E represents the adjacency matrix, controlled by the coupling probability p. E = 0 represents no couplings, E = 1 represents couplings. In this research, the normalized equations are simulated using the Runge-Kutta method with a step size h = 0.01.

B. Simulation and Evaluation Methods

In the waveform classification task, sine and sawtooth waves with a period of 80τ per cycle are randomly mixed with periods 1 ~ 10 for each waveform. The mixed waveform is used as the input waveform, and out of a length of 20,000τ (125 periods), the first half 10,000τ is used for training and the second half 10,000τ is used for verification (Fig. 2(a)). The target waveform is a one-hot vector with the sine wave as 1 and the sawtooth wave as 0 (Fig. 2(b)). The output of the reservoir layer is the voltage and current waveform of the oscillator during the period of input to the oscillator, and ridge regression is used.



(a) Input. (b) Target.

Figure 2. Waveform of input and target (0~1200τ).

The evaluation method is based on the normalized root mean square error (NRMSE) and the accuracy (ACC). The model output that approximates the target (NRMSE) is binarized with a threshold value of 0.5 to obtain a binary label. It compares it with the correct answer label and calculates the degree of agreement (ACC).

It also calculates the memory capacity (MC) and clarifies the relationship between accuracy and MC. MC is expressed in terms of the maximum delay length k such that the delay task succeeds, MC is shown in Eq. (5).

$$\mathrm{MC} = \sum_{k=1}^{\infty} r^2(k)\,, \tag{5}$$

however, $r^2(k)$ is given as follows:

$$r^2(k) = \frac{\mathrm{Cov}^2\left(u(n-k), \hat{y}_k(n)\right)}{\mathrm{Var}\left(u(n)\right)\mathrm{Var}\left(\hat{y}_k(n)\right)}, \tag{6}$$

where $\hat{y}_k(n)$ is the model output at delay length k, $u(k-n)$ is the time series input data delayed by time k, $\mathrm{Cov}(\cdot,\cdot)$ is the covariance and $\mathrm{Var}(\cdot)$ is the variance.

III. Simulation Results

Simulations are performed when all oscillators have the same value of ε (ε = 0.05, 0.1, 0.15) and, when each oscillator has a different ε within a defined range (ε = 0.09 ~ 0.11, 0.07 ~ 0.13, 0.05 ~ 0.15). The value of ε is a random number that follows a uniform distribution in the above range and is randomly set for each oscillator. Other parameters are N = 25, K = 0 ~ 0.5, p = 0.1, and the results are the average of 10 simulations.

A. Results of Waveform Classification and MC

The results of the waveform classification task are shown in Fig. 3. The horizontal axis represents the nonlinearity of the oscillator ε, and the left side of the vertical axis represents NRMSE and the right side ACC.

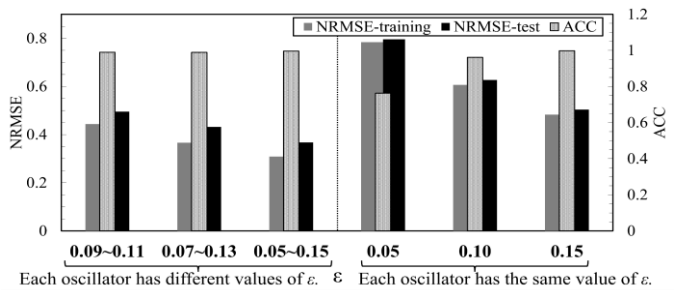


Figure 3. Result of waveform classification.

When the value of ε differs for each oscillator, the NRMSE becomes smaller and ACC is found to be close to 1.

Figure 4 shows the results for MC. The horizontal axis is ε and the vertical axis is MC.

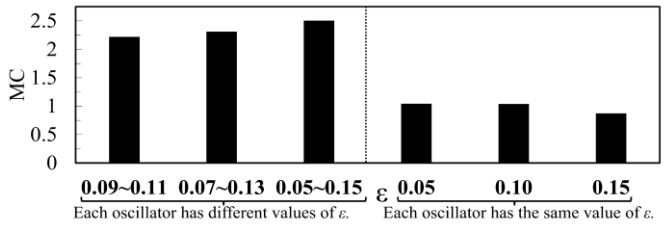


Figure 4. Memory capacity.

The different value of ε for each oscillator more than doubles the MC, and an increased ε range further enhances it.

B. Comparison of Proposed Model and Echo State Network

Table I shows the comparison of the ESN and the proposed model (with the same value and different values of ε).

TABLE I. Comparison of proposed model and ESN.

	Reservoir Type		
	Proposed model		ESN
ε	0.15	0.05~0.15	
NRMSE(test)	0.529	0.276	0.706
ACC	1.00	1.00	1.00

Although the ACC is close to 1 for all reservoir types, it is the smallest NRMSE in the proposed model with different values of ε.

IV. Conclusion

In this research, the effect of different nonlinearity ε for each oscillator on waveform classification was investigated. The results showed that waveform closer to the target function can be output, improving MC. This indicates that strict parameter tuning is unnecessary for implementation and that the approach is effective for designing reservoirs in memory-critical tasks.

Acknowledgement

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