Comparative Study of Physical Reservoir Computing with Periodic and Chaotic Circuits for Chaotic Time Series Prediction

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Abstract— **Reservoir computing is suitable for fast machine learning and consists of a reservoir layer and a readout layer. In this study, we implement physical reservoir computing in which the nodes of the input and reservoir layers are replaced by chaotic circuits and compare and evaluate the results when the parameters of the chaotic circuits are varied in time series data prediction.**

Keywords; reservoir computing, chaos circuit, chaos prediction

I. INTRODUCTION

Deep learning, which has attracted much attention in recent years, is a type of machine learning based on population neural network models. However, problems such as computational cost in learning exist. Therefore, reservoir computing has been proposed as a model that aims for sufficiently high computational performance while reducing the amount of computation required for learning [1]. In reservoir computing, the composite weights of the input and reservoir layers are fixed, and only the output weights are used for learning. This allows for a reduction in computational costs. In this study, we implement physical reservoir computing in which nodes are replaced by chaotic circuits and evaluate the prediction of chaotic data.

II. PROPOSED MODEL

Physical reservoir computing is a hardware implementation of the reservoir layer. The basic structure is the same as the Echo State Network.

In this research, physical reservoir computing is realized by replacing the nodes of the reservoir layer with chaotic circuits. Figure 1 shows physical reservoir computing used in this study. It is expected to enable low power consumption and real-time information processing on individual terminals without using a cloud environment. A current source is attached only to the input layer circuit and the chaos data is input there. The chaos data was created from the Mackey-Glass equation. The circuits in the reservoir layer do not have a current source but are connected to each other by resistors.

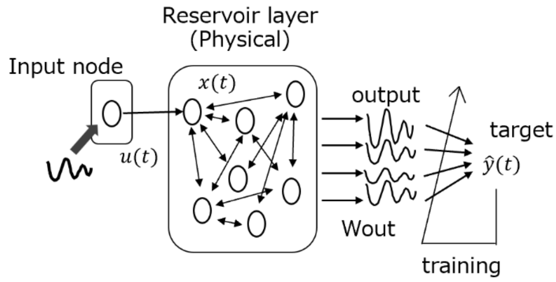

Fig. 1: Reservoir computing composed chaotic circuit.

First, time series data are input to the chaos circuit in the input layer. Then, the time-series data is weighted and propagated to one chaos circuit in the reservoir layer. The chaos circuit in the reservoir layer receives the waveform and propagates the outgoing waveform to the circuits in the other reservoir layers. Then, the system receives all the outgoing waveforms from the chaos circuits in the reservoir layer and determines the optimal weights. Then, the degree of agreement between the output waveform and the target waveform is evaluated.

Figure 2 shows the Nishio-Inaba circuit, the chaotic circuit used in this study. This circuit can output periodic waveforms, and chaotic waveforms by adjusting parameters. This circuit consists of two inductors, a capacitor, a negative resistance, and a diode.

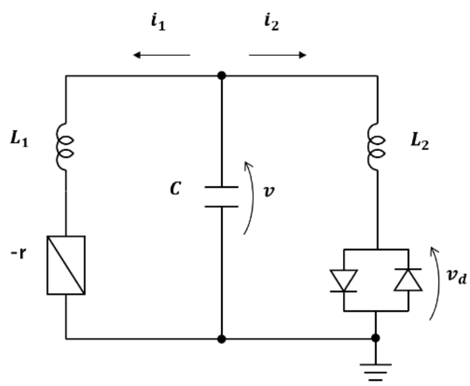

Fig. 2: Circuit model of chaotic circuit.

Equation (1) shows the circuit equation for the Nishio-Inaba circuit.

$$\begin{cases} L_1 \frac{di_1}{dt} = v + ri_1 \\ L_2 \frac{di_2}{dt} = v - v_d \\ C \frac{dv}{dt} = -i_1 - i_2 \end{cases} \quad (1)$$

Equation (2) shows the normalized equation of the circuit equation for the Nishio-Inaba circuit. Equation (3) shows for $f(y_i)$ included in the normalization equation.

$$\begin{cases} \dot{x}_i = \alpha x_i + z_i \\ \dot{y}_i = z_i - f(y_i) \\ \dot{z}_i = -x_i - \beta y_i - \sum_{j=0}^{n} \gamma_{i,j}(z_i - z_j) \end{cases} \quad (2)$$

$$f(y_i) = \frac{\delta}{2}\left(\left|y_i + \frac{1}{\delta}\right| - \left|y_i - \frac{1}{\delta}\right|\right) \quad (3)$$

The normalization parameters are shown below.

$$i_1 = \sqrt{\frac{C}{L_1}} Vx,\ i_2 = \frac{\sqrt{L_1 C}}{L_2} Vy, v = Vz, \alpha = r\sqrt{\frac{C}{L_1}},\ \beta = \frac{L_1}{L_2}$$

$$\delta = r_d \frac{\sqrt{L_1 C}}{L_2},\ \gamma = \frac{1}{R}, t = \sqrt{L_1 C}\tau$$

The Nishio-Inaba circuit can output periodic or chaotic waveforms by adjusting the value of α.

Figure 3 shows the input chaotic data.

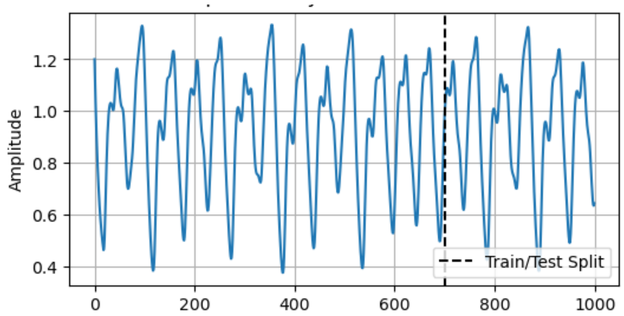

Fig. 3: Chaotic data created from the Mackey-Glass equation.

III. SIMULATION RESULT

RMSE is a measure of the error between the predicted and actual values. Equation (4) shows the equation of RMSE.

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2} \quad (4)$$

In this case, 10 chaos circuits are prepared in the reservoir layer, and only one circuit in the input layer and one chaos in the reservoir layer are connected.

Experiments were conducted with 4 patterns for each α parameter, 1-periodic solution, 2-periodic solution, 3-periodic solution, and chaotic solution. And each pattern coupling probability varied from 0.1~1.0. Figure 4 shows the simulation results.

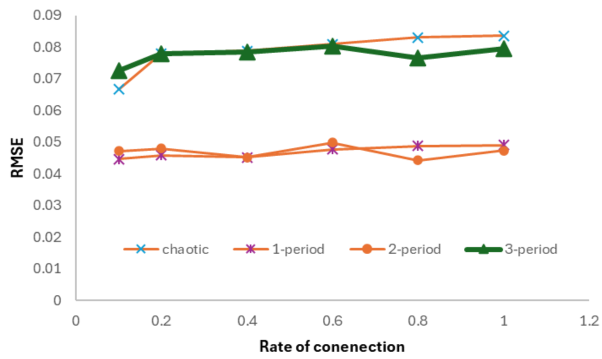

Fig. 4: Simulation results.

As a result, the periodic solution was more accurate than the chaotic solution. Also, the prediction accuracy was higher when the coupling probability was smaller. The topology within the reservoir layer was found to be more suitable for sparse prediction. In addition, the chaotic data input in this case was not too complex and periodic, as shown in Figure 3. This suggests that the periodic solution was more suitable for prediction.

IV. CONCLUSIONS

In this study, we proposed physical reservoir computing, in which the nodes of reservoir computing are replaced by chaotic circuits. In this study, we used the Nishio-Inaba circuit, which exhibits the characteristics of periodic and chaotic solutions by varying the parameters of the chaotic circuit. In this study, we evaluated which solution is better at predicting chaotic data. As a result, the periodic solution was more accurate than the chaotic solution.

ACKNOWLEDGMENT

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