Complex Network of Chaotic Circuits with Memristors Switching Oscillation States

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***Abstract*—In this study, we design and test a complex network using our chaotic circuits with memristors that switch between periodic and chaotic oscillations. As a result, we observe the simultaneous presence of both periodic and chaotic oscillations, alongside instances where switching between these oscillation types occurs within the proposed complex network. In addition, we confirm that the synchronization state and the dynamics of the memristor change with coupling strength. Furthermore, this study investigates the ratio of periodic and chaotic oscillations by changing the memristor parameters to clarify the effect of memristors in the complex network.**

I. Introduction

Complex networks have attracted a great deal of attention from various fields, including biology, engineering, and social systems. In particular, understanding the dynamic behavior of complex networks is crucial because even systems that exhibit simple behavior individually can demonstrate complex behavior when coupled within networks [1]–[3]. Among the dynamical behaviors of complex networks, synchronization is a typical and significant phenomenon. In addition, synchronization in coupled non-linear circuits is excellent for modeling high-dimensional nonlinear phenomena [4]. In order to realize more flexible non-von neumann hardware, it is important to analyze synchronization phenomena in reaction-diffusion systems realized in networks including nonlinear elements such as diodes [5]–[12].

The memristor is a two-terminal nonlinear circuit element. It has also attracted great attention because its memory characteristics make it possible to realize calculations on a processing block basis in the system [13], [14]. Due to this property, it is expected to be used for a variety of applications such as advanced memory computing systems [15]–[18].

In our previous study, we found that a chaotic circuit with a memristor exhibited switching phenomena between periodic and chaotic oscillations [19], [20].

In this study, we design a fully-coupled network with resistors using our chaotic circuits that switch between periodic and chaotic oscillations as nodes. We analyze the dynamic behavior by observing the time variation of voltages and memristors, attractors, the Lissajous diagram, and the synchronization rate. In addition, we examine the effect of changing the memristor parameters on the ratio of periodic and chaotic oscillations to clear the effect of each memristor resistance in the proposed network model.

II. Materials and methods

A. Chaotic Circuits with Memristors

In this study, we use the $Hewlett\text{-}Packard$ memristor model. The resistance value of it is called memristance $M(q)$. The i-v characteristics of $M(q)$ is shaped as a pinched hysteresis loop, as shown in Fig. 1(b).

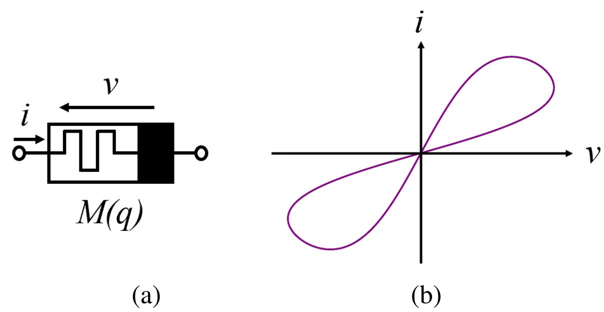


Fig. 1: Memristor model. (a) Memristor symbol, (b) i-v characteristics.

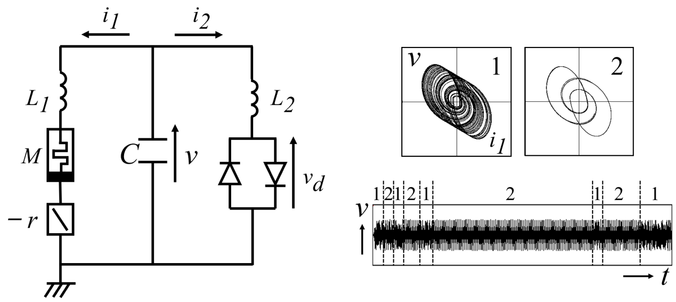
Memristance $M(q)$ is defined as in (1). A function of the charge $q(t)$ that passes through the memristor.

$$M(q) = \mu_v \frac{R^2{}_{\mathrm{on}}}{D^2} q(t) + R_{\mathrm{off}} \left(1 - \mu_v \frac{R_{\mathrm{on}}}{D^2} q(t)\right) \quad (1)$$

In the proposed network, each node is equipped with a chaotic circuit as shown in Fig. 2(a). This circuit is composed of two inductors, a memristor, a linear negative resistor, a capacitor, and the dual-directional diodes. The I-V characteristics of the dual-directional diodes are approximated using the three-segment piecewise-linear function defined in (2). The parameter r_d is resistance when it is off. In this circuit, the switching phenomenon between periodic and chaotic oscillations occurs as shown in Fig. 2(b).

$$v_d(i_2) = \frac{r_d}{2} \left(\left| i_2 + \frac{V}{r_d} \right| - \left| i_2 - \frac{V}{r_d} \right| \right) \quad (2)$$

In order to do computer calculation, the variables of the circuit equations are normalized such that:



(a) (b)

Fig. 2: Chaotic circuit model. (a) Chaotic circuit. (b) Attractors and time-series of voltage while switching.

$$i_{1_n} = \sqrt{\frac{C}{L_1}} V x_n \,, \quad i_{2_n} = \frac{\sqrt{L_1 C}}{L_2} V y_n \,, \quad v_n = V z_n \,,$$

$$q_n = CV w_n \,, \quad t = \sqrt{L_1 C}\tau \,, \quad ' \cdot ' = \frac{d}{d\tau} \,, \quad r\sqrt{\frac{C}{L_1}} = \alpha \,,$$

$$\frac{L_1}{L_2} = \beta \,, \quad r_d \frac{\sqrt{L_1 C}}{L_2} = \gamma \,, \quad R_{\text{off}} \sqrt{\frac{C}{L_1}} = \eta \,, \quad \frac{R_{\text{on}}}{R_{\text{off}}} = \zeta \,,$$

$$\mu_v \frac{R_{\text{on}}}{D^2} CV = \xi \,, \quad \frac{1}{R}\sqrt{\frac{L_1}{C}} = \delta$$

When the all chaotic circuits are coupled fully with each other via resistors, the normalized equations are given as follows:

$$\begin{cases} \dot{x}_n = z_n + \alpha x_n - \eta x_n(\zeta \xi w_n + 1 - \xi w_n) \\ \dot{y}_n = z_n - \dfrac{\gamma}{2}\left(\left| y_n + \dfrac{1}{\gamma} \right| - \left| y_n - \dfrac{1}{\gamma} \right| \right) \\ \dot{z}_n = -x_n - \beta y_n - \sum_{k=1}^{N} \delta(z_n - z_k) \\ \dot{w}_n = x_n \end{cases} \quad (3)$$

In (3), x_n and y_n are the scaling currents, z_n is the scaling voltage, τ is the scaling time, α is the negative resistance, β is the ratio of inductance, γ is the resistance of the dual-directional diodes when it is off, η is the maximum memristance, ξ is the minimum memristance, ζ is the ratio of maximum to minimum memristance and δ is the coupling strength. For computer simulations, we calculated (3) using the fourth-order Runge-Kutta method with the step size h = 0.01 and simulated τ from 0 to 70,000.

B. Network Model

As shown in Fig. 3, we proposed a fully-coupled network model with N = 10 chaotic circuits connected by resistors R. In addition, the synchronization state of the voltage and the memristance at each connection is calculated by (4) for the voltage and (5) for the memristance.

$$|z_n - z_k| < 0.1 \qquad (n, k = 1, \cdots, 10) \quad (4)$$

$$|\xi(\zeta - 1)(w_n - w_k)| < 0.01 \quad (n, k = 1, \cdots, 10) \quad (5)$$

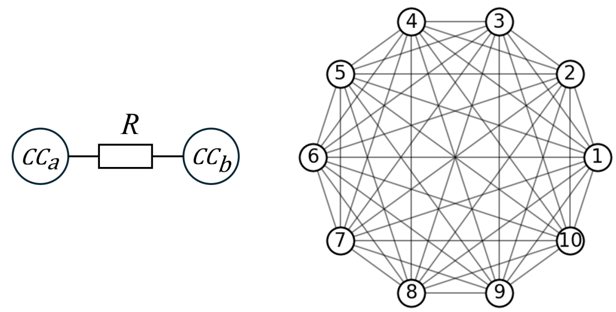


Fig. 3: Fully-coupled chaotic circuits.

III. Results

In this section, the initial values of x_n, y_n, z_n, and w_n were set by shifting by 0.01, starting from x_1. The parameters of chaotic circuits were set at α = 0.524, β = 2.92, γ = 456, η = 0.0963, ζ = 0.00276 and ξ = 0.0111. In this section, we analyze the dynamic behavior of complex networks, such as synchronization phenomena and fluctuations in memristance as changing the coupling strength δ and the maximum memristance η.

A. Dynamic Behavior

In this subsection, we study attractors to analyze the dynamic behavior of each chaotic circuit in the proposed network. In addition, we investigate the time-series of z and the Lissajous diagram for reference chaotic circuit to confirm the synchronization state. Furthermore, we observe the time-series of memristance to make clear the effect of memristor on switching phenomena.

Figure 4 shows attractors, Lissajous diagrams and the time-series for each z when the coupling strength δ = 0.00100. In the Lissajous diagrams, the reference chaotic circuit is the chaotic circuit at node 1. From the attractors and Lissajous diagrams in Fig. 4, we can confirm the coexistence of synchronization due to periodic oscillations and asynchronism due to chaotic oscillations. Also, when each time series z is zoomed in, the oscillation state changes between constant and irregular, so that the switching between periodic and chaotic oscillations occurs even in the proposed complex network, as in the case of a single chaotic circuit.

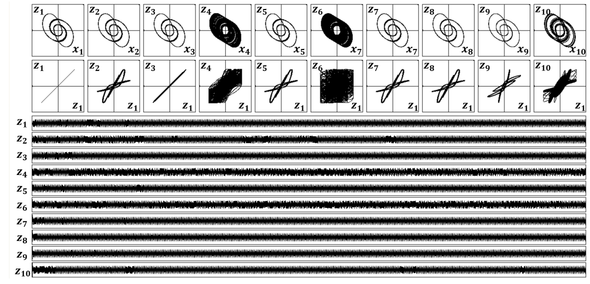


Fig. 4: Dynamic behavior of proposed network model, with δ = 0.00100, attractor and Lissajous diagrams with τ = 60,000 to 70,000, and time-series of z with τ = 0 to 70,000.

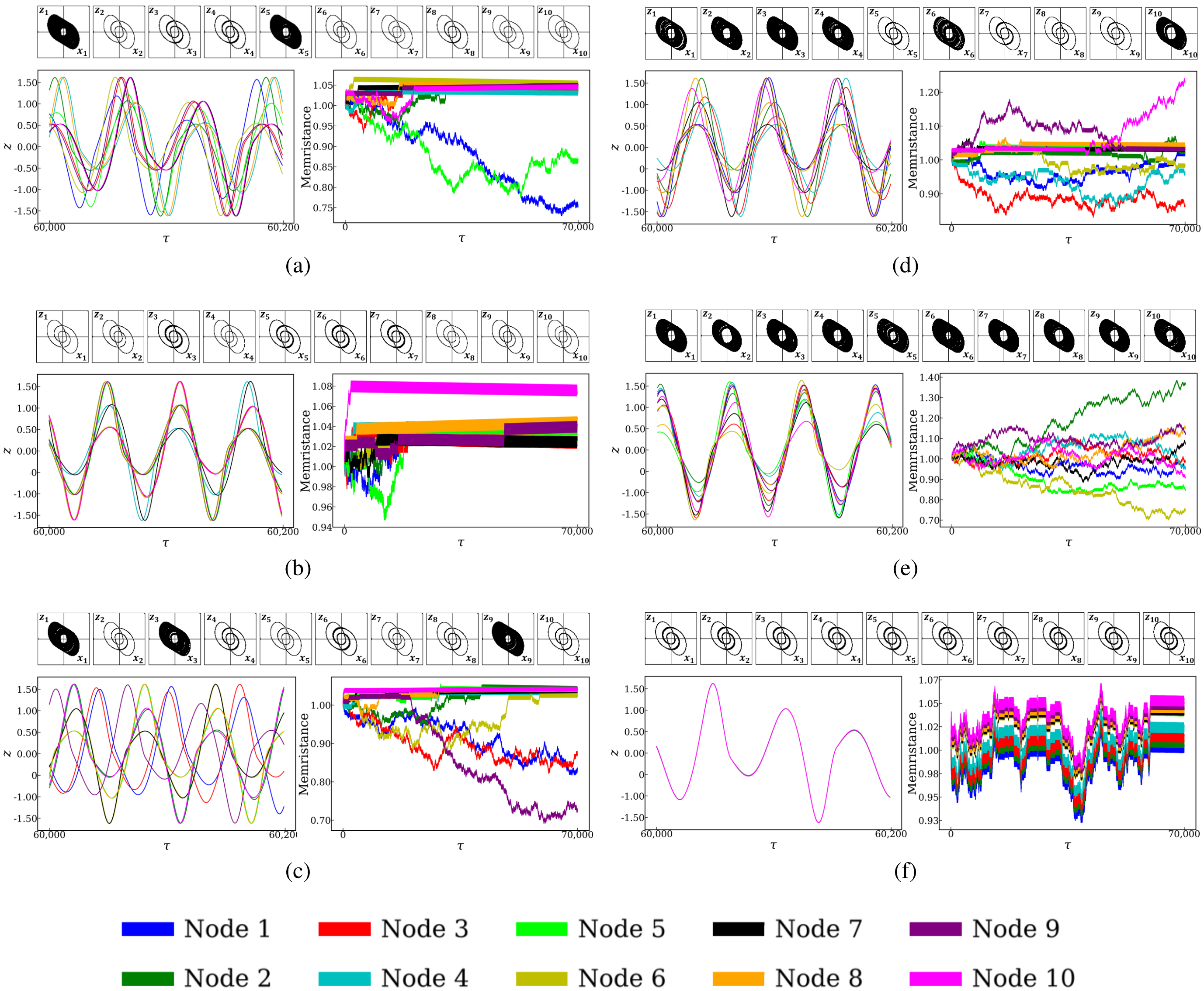


Fig. 5: Time-series of z and the memristance at each coupling strength. (a) δ = 0.000100, (b) δ = 0.000215, (c) δ = 0.000464, (d) δ = 0.00215, (e) δ = 0.0100, (f) δ = 0.100 (Attractor with τ = 60,000 to 70,000, time-series of z with τ = 60,000 to 60,200, time-series of memristance with τ = 0 to 70,000).

In order to analyze the effect of the coupling strength δ on synchronization state changes and oscillation switching, we also studied time-series of z and the memristance at each coupling strength. Figure 5 shows the simulation results. The results of time series z of in Figs. 5(a) and (b) show that many chaotic circuits are not amplitude-synchronized, but are phase-synchronized. However, Figs. 5(c), (d), (e) and (f) show that the time series z in most chaotic circuits not only phase-synchronized, but also changes so that the amplitude difference at the same τ becomes smaller, indicating that the time series z is approaching amplitude-synchronized as δ is increased. This transition highlights the complex relationship between coupling strength and synchronization mechanisms. In addition, the time series of memristance in Fig. 5 shows that the change in memristance varies greatly depending on δ. Furthermore, the time series of memristance in Fig. 5 shows that the change in memristance varies greatly depending on whether the oscillations of the chaotic circuits to which the memristors belong are periodic or chaotic. Even if the type of each oscillation is the same, the memristance values and the behavior are different. We can confirm that the memristor behavior depends not only on the coupling strength but is also sensitive to the type of oscillation. From these results, we consider that the memristors could be playing a crucial role in the transition from phase-synchronization to amplitude-synchronization in the proposed complex network composed of chaotic circuits with memristors, possibly acting as a catalyst for the synchronization process.

B. Synchronization Rate

We study the average synchronization rate of z and memristance when the coupling strength δ is changed from 0.000100 to 0.100 with logarithmically equal intervals. The synchronization state of z and the memristance are calculated for all chaotic circuits between $\tau = 0$ and 70,000 according to (4) and (5).

Figure 6 shows the average synchronization rate of z and memristance. Based on this finding, we can conclude that the average synchronization rate for z rises with an increase in δ value. Here, this average synchronization rate is defined by using the difference in the voltage variable z. However, Fig. 5 shows that many chaotic circuits are phase-synchronized state rather than amplitude-synchronization state when δ is small. Therefore, we consider that increasing δ causes a transition from phase-synchronized to amplitude-synchronized state in the proposed complex network. On the other hand, we can confirm that the average synchronization rate of memristance tends to decrease slightly with increasing δ until $\delta = 0.100$. In addition, the synchronization state of memristance is asynchronism at any δ. This indicates that each memristor is running at different values, even though the parameter settings for each memristor are the same.

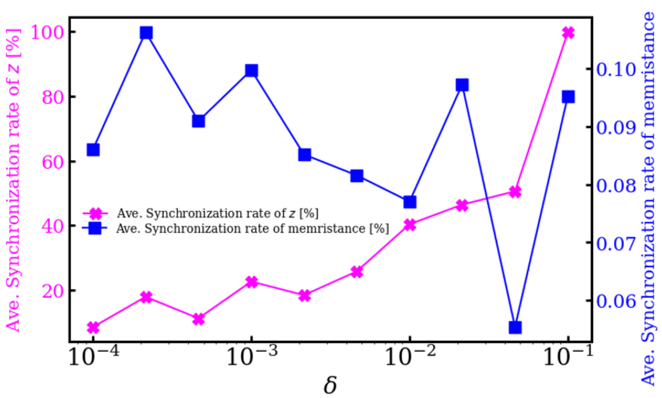


Fig. 6: Synchronization rate depending on the coupling strength δ (δ = 0.000100, 0.000215, 0.000464, 0.00215, 0.00100, 0.0464, 0.0215 and 0.100).

C. Ratio of Periodic and Chaotic Oscillations

In this subsection, we study the ratio of periodic and chaotic oscillations by changing the memristor parameter to clarify the effect of each memristor in the proposed network. We change the memristor parametr η, which is related to the maximum resistance value of the memristor, by 0.01 from 0.05 to 0.15. Other parameters are fixed as $\alpha = 0.524$, $\beta = 2.92$, $\gamma = 456$, $\zeta = 0.00276$, $\xi = 0.0111$, $\delta = 0.00215$ and $\tau = 0$ to 70000. Furthermore, we use Poincaré maps to count the periodic and chaotic oscillations of each chaotic circuit. In each chaotic circuit, a Poincaré section is set in the $z = 0$ plane. When the orbit passes through the Poincaré section, it is determined whether it is a periodic or chaotic oscillation from the values of x and y. A total of 5 sets of different initial values are tested for each variable to determine the average ratio of periodic to chaotic oscillations.

As a result, as it can be seen in Fig. 7, the ratio of periodic and chaotic oscillations has an extreme value distribution. The ratio of periodic oscillations tends to increase with increasing η, with a maximum value of 68.5 % at $\eta = 0.10$. On the other hand, the ratio of chaotic oscillations tends to decrease with increasing η, with a minimum value of 31.5 % at $\eta = 0.10$. This result suggests that the ratio of periodic to chaotic oscillations can be tuned by the memristor parameter.

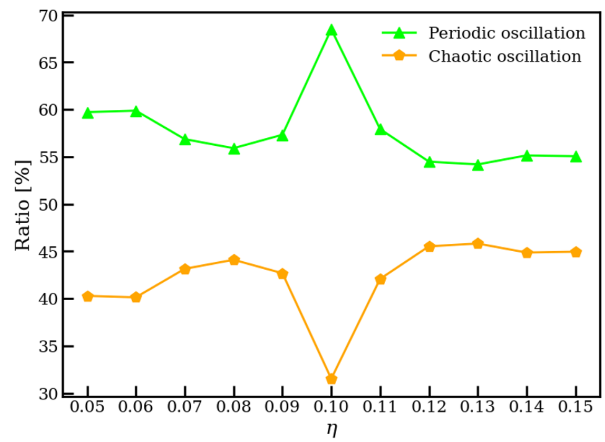


Fig. 7: Ratio of periodic and chaotic oscillations.

IV. Conclusions

In this study, we designed and tested a complex network using chaotic circuits with memristors that switch between periodic and chaotic oscillations. As a result, we verified that the switching between periodic and chaotic oscillations also occurs in the proposed complex network. In addition, we obtained that the synchronization state and memristance behavior differed when the coupling strength is modified. When coupling strength is small, many chaotic circuits are only phase-synchronized and the average amplitude synchronization rate is low. With the rise in coupling strength, there is an increase in both the number of chaotic circuits that exhibit amplitude-synchronization and the average rate of amplitude-synchronization. On the other hand, the average memristance synchronization rate is very small at any coupling strength. This suggests that complex network interactions have complex effects on memristor dynamics. Also, we changed the memristor parameter to study changes in the ratio of periodic to chaotic oscillations. As a result, we observed that the ratio of periodic to chaotic oscillations can be tuned by the memristor parameter.

These results provide useful insights for realizing reaction-diffusion systems in hardware. We can adjust not only the synchronization state of the system by changing the coupling strength between nodes, but also add complexity to the dynamics of memristors inserted at the node level. This is expected to lead to the realization of hardware that incorporates complexity and nonlinearity closer to that of the natural world.

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