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Phase Difference Patterns in Two Coupled Chaotic Circuits Including Memristors

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Abstract

We designed and simulated a 4-dimensional chaotic circuit with $Hewlett$-$Packard$ memristor. We investigated attractors and time series waveforms. As a results, our proposed circuit could switch the oscillation state over time. Furthermore, we developed two coupled system of chaotic circuits with memristors. As a results, we obtained nonlinear phenomena of phase deference from the coupled system.

1. Introduction

Neumann-type computer, which is indispensable to the modern information society, converts all continuous information into 01 discrete information. The processing is also synchronized to an operating frequency of several GHz. On the other hand, neurons or neural networks in the human brain are said to be nonlinear oscillators that process continuous information [1]. To realize more complex, flexible, and adaptive information processing technologies in the future, it is useful to gain insight from our brain function. In other words, a better understanding of nonlinear oscillators, one of the models of brain function, is useful for both scientific and engineering purposes. Furthermore, the abundant dynamics of human brain are said to have chaotic properties [2].

Chaos has unpredictability. It has the property that small differences in initial values will spread exponentially in the future. Chaos is attracting a great deal of attention because this nonlinearity is expected to deepen our understanding of science and to have engineering applications. One of the most famous applications is the oscillatory system chaotic neurons. In order to realize more flexible information processing systems such as learning, association, memory, and pattern recognition, it is necessary to analyze the chaotic oscillators that define chaotic neurons. These characteristics make chaos a keyword theory in the future information society. Furthermore, memory devices that enable brain reversibility are attracting attention. In this study, we focused on memristor.

A memristor has attracted a great deal of attention because of this excellent resistance change characteristics by the history of the previous charge or flux flow through it [3].

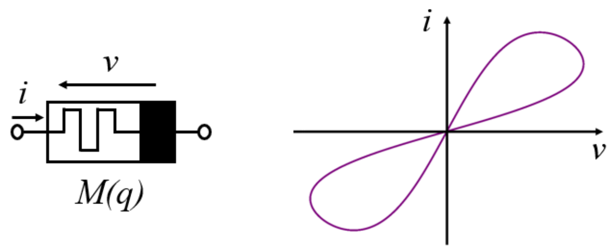
By combining the continuity of oscillation, the nonlinearity of chaos, and the memory performance of memory elements, oscillators with abundant nonlinearities can be created.

In this study, we design a chaotic circuit with a memristor. The purpose of this research is investigating fundamental dynamics of the proposed model. In order to achieve our goal, we analyze time series waveform for each variables and attractor. In addition, we design two coupled circuit model of chaotic circuits with memristors to investigate the interaction on other circuits.

2. Materials and Methods

2.1 Memristor Mathematical Model

In this study, we use $Hewlett$-$Packard$ memristor mathematical model. This memristor has the i-v characteristic as a pinched hysteresis loop. Figure 1 shows the schematic symbol and the i-v characteristic when a sin curve is input.



(a) Schematic symbol. (b) i-v characteristic.

Figure 1: Memristor model. (a) Memristor symbol. (b) i-v characteristic when a sin curve is input.

The resistance of the memristor is called memristance $M(q)$. $M(q)$ is defined in (1) as a function of the charge $q(t)$ which passed through the memristor.

$$M(q) = \mu_v \frac{R^2{}_{\rm on}}{D^2} q(t) + R_{\rm off}\left(1 - \mu_v \frac{R_{\rm on}}{D^2} q(t)\right) \quad (1)$$

where $R_{\rm on}$ is the minimum resistance, $R_{\rm off}$ is the maximum resistance, μ_v is the average drift mobility of the charges, D is the length of doped and undoped parts combined.

2.2 Chaotic Circuit Model with the Memristor

The proposed chaotic circuit with the memristor is shown in Fig. 2. It is a self-exciting oscillator. The original chaotic circuit consists of one negative resistor r, one capacitor C, two inductors L_1 and L_2, and one dual-directional diode v_d [4]. In this original chaotic circuit, i_1 is usually larger than i_2. To take advantage of this property, the memristor is added between the inductor L_1 and the negative resistor $-r$ to increase the influence of the memristor on the chaotic circuit.

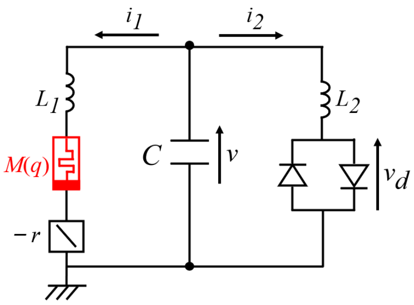


Figure 2: Proposed chaotic circuit with the memristor.

The dual-directional diode acts as a nonlinear resistor and is responsible for maintaining stable oscillation. The i-v characteristics of the dual-directional diodes are approximated as the three-segment piecewise-linear function defined in (2). The parameter r_d is resistance when the diodes are off.

$$v_d(i_2) = \frac{r_d}{2}\left(\left|i_2 + \frac{V}{r_d}\right| - \left|i_2 - \frac{V}{r_d}\right|\right) \tag{2}$$

Then, the circuit dynamics in Fig. 2 is described by the circuit equation from Kirchihoff's circuit lows as (3).

$$\begin{cases} L_1\dfrac{di_1}{dt} = v + ri_1 - M(q)i_1 \\ L_2\dfrac{di_2}{dt} = v - v_d(i_2) \\ C\dfrac{dv}{dt} = -i_1 - i_2 \\ \dfrac{dq}{dt} = i_1 \end{cases} \tag{3}$$

The important part in (3) is the $M(q)i_1$ part. Because the most significant difference between the proposed model and the original chaotic circuit is that the proposed model has a variable multiplication term, while the original chaotic circuit has no variable multiplication term. The multiplicative terms of these independent variables are important in the sense that they give the proposed model new nonlinear characteristics.

Next, the circuit equations in (3) is normalized to investigate the dynamics by calculating the Runge-Kutta method. We change the variables as follows;

$$i_{1_n} = \sqrt{\frac{C}{L_1}}Vx_n, i_{2_n} = \frac{\sqrt{L_1C}}{L_2}Vy_n, v_n = Vz_n, q_n = CVw_n,$$

$$t = \sqrt{L_1C}\tau, `\cdot' = \frac{d}{d\tau}, r\sqrt{\frac{C}{L_1}} = \alpha, \frac{L_1}{L_2} = \beta, r_d\frac{\sqrt{L_1C}}{L_2} = \gamma$$

$$R_{\text{off}}\sqrt{\frac{C}{L_1}} = \eta\ , \ \frac{R_{\text{on}}}{R_{\text{off}}} = \zeta\ , \ \mu_v\frac{R_{\text{on}}}{D^2}CV = \xi\ , \ \frac{1}{R}\sqrt{\frac{L_1}{C}} = \delta$$

When the proposed circuit is coupled with another proposed circuit via a resistor R, the normalized circuit equations are described as (4). To consider a single circuit, we substitute N = 1 to (4).

$$\begin{cases} \dot{x}_n = z_n + \alpha x_n - \eta x_n(\zeta\xi w_n + 1 - \xi w_n) \\ \dot{y}_n = z_n - \dfrac{\gamma}{2}\left(\left|y_n + \dfrac{1}{\gamma}\right| - \left|y_n - \dfrac{1}{\gamma}\right|\right) \\ \dot{z}_n = -x_n - \beta y_n - \sum_{k=1}^{N}\delta(\mathrm{z}_n - z_k) \\ \dot{w}_n = x_n \qquad\qquad (n, k = 1, 2) \end{cases} \tag{4}$$

where τ is the scaling time, α means the negative resistance, β means the inductance, γ means the resistance of the diode when it is off, η means the maximum resistance of the memristor, ξ means the minimum resistance of the memristor and ζ means the ratio of R_{on} and R_{off}

3. Simulation Results

In this study, the step size of the Runge-Kutta method is set to h = 0.002. This numerical calculation is performed for τ from 0 to 10,000. Some of the parameters are fixed to β = 2.92, γ = 456 and ξ = 0.00276. The initial values of the variables are shifted by 0.02 for each variable.

3.1 Oscillation Switching Phenomena

The purpose of this subsection is to discover new dynamics in a single chaotic circuit by adding a memristor. Figure 3 shows the time series waveforms of z and w in (4).

As shown in Figs. 3, we discover the oscillation switching phenomena between periodic and chaotic oscillations over time. In a conventional chaotic circuit, once parameters are set, either periodic or chaotic oscillations can only be observed. Therefore, the natural occurrence of oscillation switching phenomena as dynamics are new phenomena in the chaotic circuit.

Figure 3 shows one of example of the oscillation switching. This oscillation switching is between 2-periodic, 5-periodic and chaotic oscillations when the parameters α = 0.588, η = 0.0856 and ξ = 0.125. The number of periods of periodic

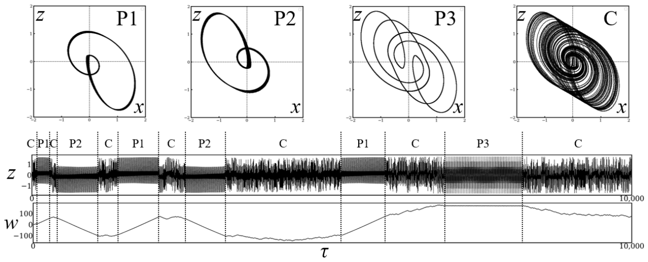


Figure 3: 2-periodic, 5-periodic and chaotic oscillations (α = 0.588, η = 0.0856 and ξ = 0.125).

oscillation is determined by the number of orbits that make up the closed orbit. In this parameter settings, Two types of periodic oscillations appear with different orbital numbers, 2-period and 5-period. In addition, the same 2-periodic oscillation has a point symmetric attractor. The ratio of periodic to chaotic oscillations is 40.5% for periodic oscillations and 59.5% for chaotic oscillations.

3.2 Attractors on x-z-w Space

The purpose of this subsection is to investigate the relationship between the oscillation switching and the memristor. The variable w in (4) represents the charge of the memristor. Therefore, we observe 3D phase spaces with w as one of the axes. Figure 4 shows attractors on x-z-w space. Black represents chaotic oscillation and other colors represent periodic oscillations.

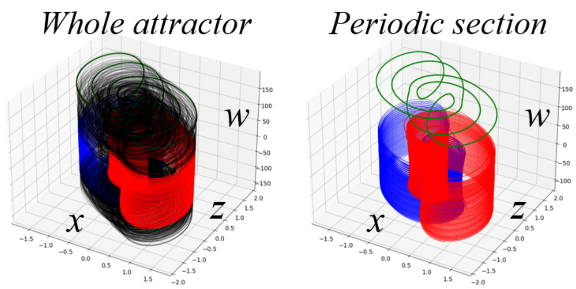


Figure 4: Attractor on x-z-w space of 2-periodic, 5-periodic and chaotic oscillations.

As shown in Fig. 4, we can see that the change in w plays a role in moving back and forth between periodic and chaotic oscillations. Furthermore, we can see that 2-periodic oscillations partially overlap in Fig. 4, but there is no direct switch from one 2-periodic oscillation to the another from Fig. 3. In other words, all periodic oscillations are connected via chaotic oscillation, and chaotic oscillation always appear after periodic oscillation. These suggests that the variable w associated with the memristor acts as a vector for the switching between periodic and chaotic oscillations and influences the behavior of the proposed chaotic circuit.

4. Two Coupled Circuit Model

The purpose of this section is to investigate the dynamics of coupled circuit model that switch between periodic and chaotic oscillations over time and the interaction on each other.

Figure 5 shows the two coupled circuit model connected by a resistor R. As shown in Fig. 5, circuit 1 and circuit 2 are chaotic circuits which are chaotic circuit with a memristor, as shown in Fig. 2.

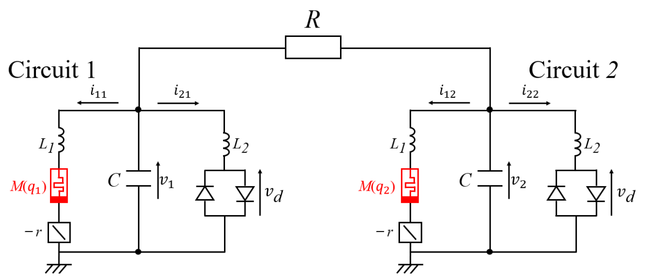


Figure 5: Coupled circuit model.

In the coupled circuit model, the parameters settings for each circuit are equal, and the initial values of the variables are inverted for circuit 1 and circuit 2 in Fig. 5. Hence, the behavior of the coupled circuit model is obtained by calculating (4).

Figure 6 shows attractors on x-z plane and time series waveforms of z for each circuit in Fig. 5.

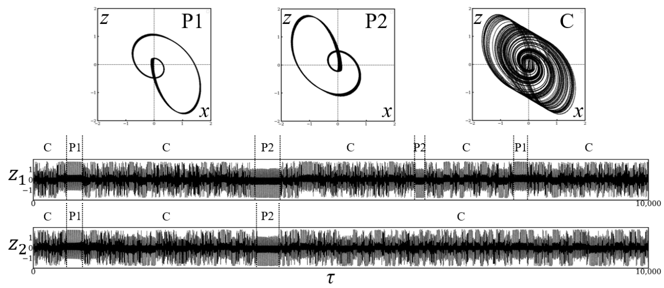


Figure 6: Oscillation switching between circuit 1 and circuit 2 in Fig. 5 (δ = 0.005).

As shown in Fig. 6, we discover that in two coupled circuit model, oscillation switching occur in each circuit with periodic and chaotic oscillations over time as in a single circuit. However, the timing of switching is different on each circuit. Therefore, the behavior of the coupled circuit model is also different at each point in time. Hence, we also investigated the phase difference between circuits. Figure 7 shows the phase differences for each type of oscillation.

As shown in Fig. 7, when both circuits are chaotic oscillations, the range of phase difference change is large. When both circuits are 2-periodic oscillations of the same shape,

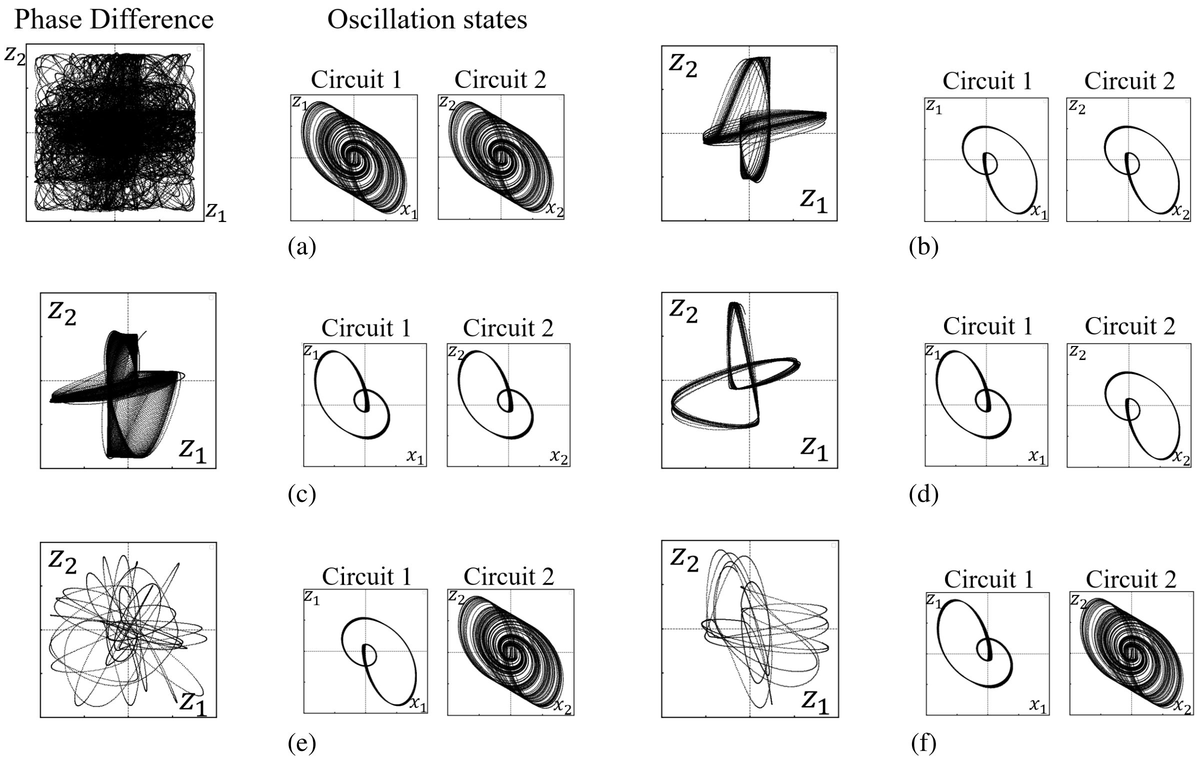


Figure 7: Phase differences for each type of oscillation. (a) Chaos and chaos, (b) 2-period of the same shape, (c) Another 2-period of the same shape, (d) Point symmetry, (e) 2-period and chaos, (f) Another 2-period and chaos.

the phase difference is found to change within a certain range. When both circuits are periodic oscillators and point-symmetric attractors, the phase difference is nearly fixed. When one circuit is in periodic oscillation and the other is in chaotic oscillation, the phase difference changes within a more limited range than when both circuits are in chaotic oscillation.

5. Conclusions

We have proposed a chaotic circuit with a memristor. In this model, we have obtained the oscillation switching phenomena between periodic and chaotic oscillations over time. From attractors on three variables space including the memristor variable, we have observed that changes in memristor variable has a possibility to contribute to switching between periodic and chaotic oscillations. In addition, we have proposed two coupled circuit model using our proposed chaotic circuit including memristor. In the two coupled circuit model, we have obtained the oscillation switching phenomena between periodic and chaotic oscillations over time as well as a single circuit. Furthermore, we have obtained the phase differences of the coupled model changes as the oscillation type changes, and the characteristic of the phase differences change depend on the combination of oscillation types.

References

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