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Observation of Linear Memristor's Effect on Synchronization Phenomena in van der Pol Oscillators

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Abstract

This study is about memristor's effect in synchronization phenomena of two van der Pol oscillators. We compare The case connected by a memristor and the case connected by a pure resistor. From the results, we observe and verify the influence of a parameter of the van der Pol oscillator's nonlinearity and a parameter of the memristor on the synchronization phenomena.

1. Introduction

A memristor is an element whose resistance changes in accordance with the current flowing through it or the magnetic flux penetrating it [1]. The memristor is currently in the development stage and is still difficult to put into practical use [2]. The characteristics of the memristor used in this research are assumed to be relatively simple and change "in proportion" to the magnetic flux penetrating the memristor. Hereafter, this memristor is referred to as a "linear memristor".

Previous studies have investigated synchronization phenomena in coupled systems of memristors and oscillators assuming various characteristics [3]-[4]. However, studies assuming linear memristors are few and insufficient, so this basic research is important.

A neural network is a model of machine learning made to mimic the nervous system of the brain [5]. In the simulation of neural networks reproduced by electric circuits, nodes are often assumed to be oscillators and edges are assumed to be circuit elements. In other words, investigating the synchronization phenomena of the coupled system of oscillators can be used for basic research on reproducing parallel processing of a human brain in circuits.

This study is about effect of linear memristor on the synchronization phenomenon of two van der Pol oscillators. Specifically, the parameters of nonlinearity of van der Pol oscillators and the memristor memductance are varied. We observe how time series waveforms of current and voltage, attractors, and Lissajous diagrams change depending on parameters. Then, we consider influence of the parameters. In order to clearly show the influence and effect of the linear memristor, we also simulate the synchronization phenomena in a circuit in which the linear memristor is replaced by a pure resistor.

2. Proposed Model

Figure 1 shows the memristor model. The current and voltage of the memristor are denoted by i and v, and Ohm's law is described as Eq. (1).

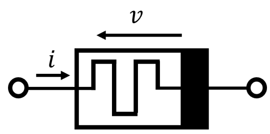

Figure 1: Memristor model.

$$i = W(\varphi)v \tag{1}$$

The definition of memductance $W(\varphi)$ is assumed to be described in Eq. (2) with α as the proportionality constant.

$$W(\varphi) = \alpha\varphi \tag{2}$$

It is called a linear memristor because the memductance increases monotonically in proportion to the magnetic flux. Figure 2 shows the van der Pol oscillator. This oscillator consists of a capacitor, an inductor, and a nonlinear resistor.

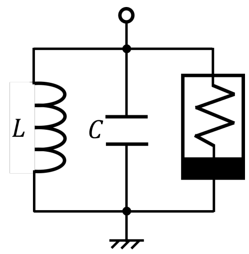

Figure 2: van der Pol Oscillator.

Figure 3 shows the proposed circuit model. This model consists of two van der Pol oscillators coupled by the memristor.

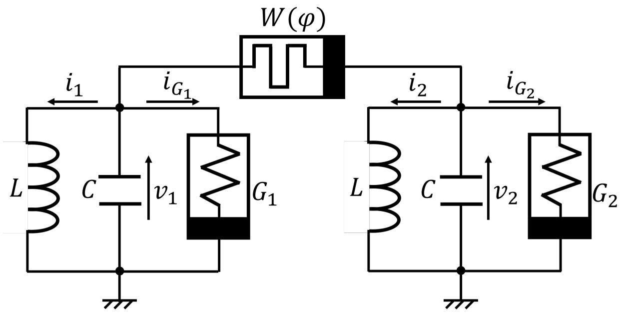

Figure 3: Proposed circuit model.

The circuit equations are given as Eq. (3).

$$\begin{cases} C\dfrac{dv_1}{dt} = -i_{G1} - \alpha L(i_1 - i_2)(v_1 - v_2) - i_1 \\ C\dfrac{dv_2}{dt} = -i_{G2} + \alpha L(i_1 - i_2)(v_1 - v_2) - i_2 \\ L\dfrac{di_1}{dt} = v_1 \\ L\dfrac{di_2}{dt} = v_2 \end{cases} \quad (3)$$

where the current and voltage of the nonlinear resistor are denoted by i_{G1} and i_{G2}, and the $i - v$ characteristics are described in Eq. (4).

$$i_{Gn} = -g_1 v_n + g_3 v_n^3 \quad (g_1, g_3 > 0) \quad (n = 1, 2) \quad (4)$$

The circuit equations are normalized by the following normalization parameters.

$$v_n = \sqrt{\frac{g_1}{g_3}} x_n, \quad i_n = \sqrt{\frac{g_1 C}{g_3 L}} y_n \quad (n = 1, 2)$$
$$\varepsilon = g_1 \sqrt{\frac{L}{C}}, \quad \zeta = \alpha L \sqrt{\frac{g_1}{g_3}}, \quad t = \sqrt{LC}\tau \quad (5)$$

From these parameters, we see that the parameter for nonlinearity of the van der Pol oscillator is ε and the parameter for memristor is ζ. The normalized circuit equations is shown in Eq. (6).

$$\begin{cases} \dfrac{dx_1}{d\tau} = \varepsilon x_1(1 - x_1^2) - \zeta(x_1 - x_2)(y_1 - y_2) - y_1 \\ \dfrac{dx_2}{d\tau} = \varepsilon x_2(1 - x_2^2) + \zeta(x_1 - x_2)(y_1 - y_2) - y_2 \\ \dfrac{dy_1}{d\tau} = x_1 \\ \dfrac{dy_2}{d\tau} = x_2 \end{cases} \quad (6)$$

In the simulation, ε takes different values for each van der Pol oscillators. So ε for the oscillator on the left side of the circuit model is ε_1, and ε on the other side is ε_2 (Fig. 3).

3. Simulation Results

Simulation are performed by changing the four initial values, the parameter ε of the nonlinearity of the van der Pol oscillator and the parameter ζ of the memristor. The simulation operates using the Runge-Kutta method. We compare the results when coupled by the linear memristor with the results when coupled by a pure resistor. Two different results with different conditions are shown for each section. Also, each result includes (i) attractor on x_2-y_2 plane and (ii) phase difference of x_1-x_2.

3.1 Simulation 1 (ε_1=ε_2)

In this subsection, we compare simulation results when the initial values are changed without changing ε and ζ. Table I shows list of used conditions.

Table 1: Used conditions.

	x_1	y_1	x_2	y_2	ε_1	ε_2	ζ
Condition 1	1.1	2.1	1.2	2.3	0.1	0.1	0.24
Condition 2	1.1	-2.1	-1.2	2.3	0.1	0.1	0.24

3.1.1 Pure Resistor

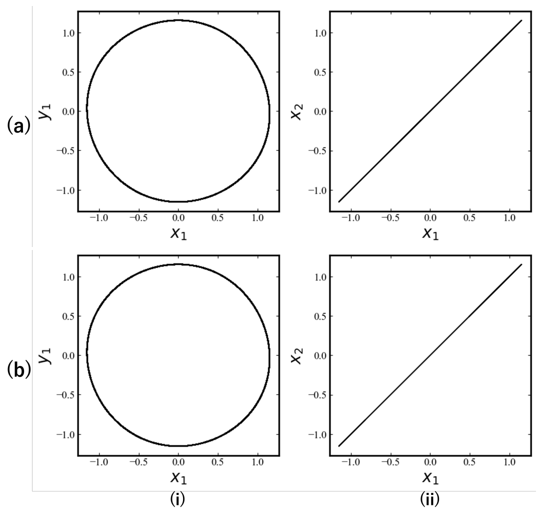

Figure 4: Simulation result using condition 2 in Tab. I with pure resistor. (a) condition 1, (b) condition 2, (i) a circuit attractor (x_2-y_2), (ii) a phase difference (x_1-x_2).

When connected with pure resistors, there is no difference in synchronization between the different initial values.

3.1.2 Linear Memristor

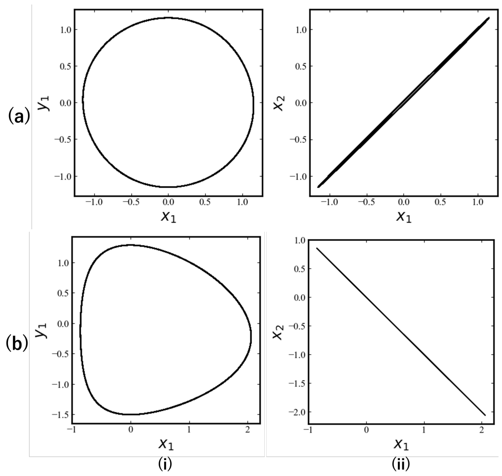

Figure 5: Simulation result using condition 2 in Tab. I with linear memristor. (a) condition 1, (b) condition 2, (i) a circuit attractor (x_2-y_2), (ii) a phase difference (x_1-x_2).

When connected with the memristor, the attractors are distorted by the difference in initial values, and the coexistence of in-phase synchronization and anti-phase synchronization is observed.

In the next results, expecting to be able to observe synchronization phenomena other than coexistence, the normalized equations are adjusted so that the parameters of the left and right nonlinearities can be changed independently.

3.2 Simulation 2 ($\varepsilon_1 \neq \varepsilon_2$)

In this subsection, we compare simulation results when the initial values are changed without changing ε and ζ, but with ε_1 and ε_2 being different. Table II shows list of used conditions.

Table 2: Used conditions.

	x_1	y_1	x_2	y_2	ε_1	ε_2	ζ
Condition 3	1.1	2.1	1.2	2.3	0.6	0.1	0.24
Condition 4	1.1	-2.1	-1.2	2.3	0.6	0.1	0.24

3.2.1 Pure Resistor

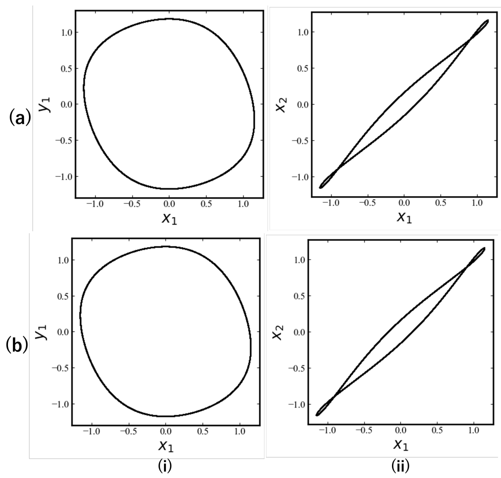

Figure 6: Simulation result using condition 4 in Tab. II with pure resistor. (a) condition 3, (b) condition 4, (i) a circuit attractor (x_2-y_2), (ii) a phase difference (x_1-x_2).

When connected with pure resistors, there is no difference in synchronization between the different initial values.

3.2.2 Linear Memristor

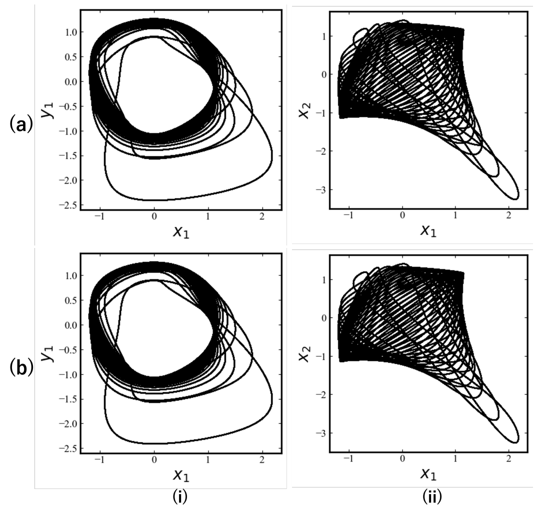

Figure 7: Simulation result using condition 4 in Tab. II with linear memristor. (a) condition 3, (b) condition 4, (i) a circuit attractor (x_2-y_2), (ii) a phase difference (x_1-x_2).

From Simulaition 1, the coexistence of in-phase and anti-phase synchronization is observed for the same values of ε_1 and ε_2, depending on the difference in initial values.

However, when ε_1 and ε_2 are different, i.e., the left and right nonlinearities are different, the difference in initial values had no effect on the results.

From the above, we considered that the cause of this complicated attractor and Lissajous diagram might be due to the parameter ζ of the memristor. In the next results, we observed how the attractor and Lissajous diagram change depending on the value of ζ.

3.3 Simulation 3 (When the ζ is changed)

In this subsection, we compare simulation results when ζ is changed without changing the initial values and ε. ε_1 and ε_2 are different. The simulations are performed only on circuits connected by linear memristor.Table III shows list of used conditions.

Table 3: Used condition.

	x_1	y_1	x_2	y_2	ε_1	ε_2
Condition 5	1.1	-2.1	-1.2	2.3	0.6	0.1

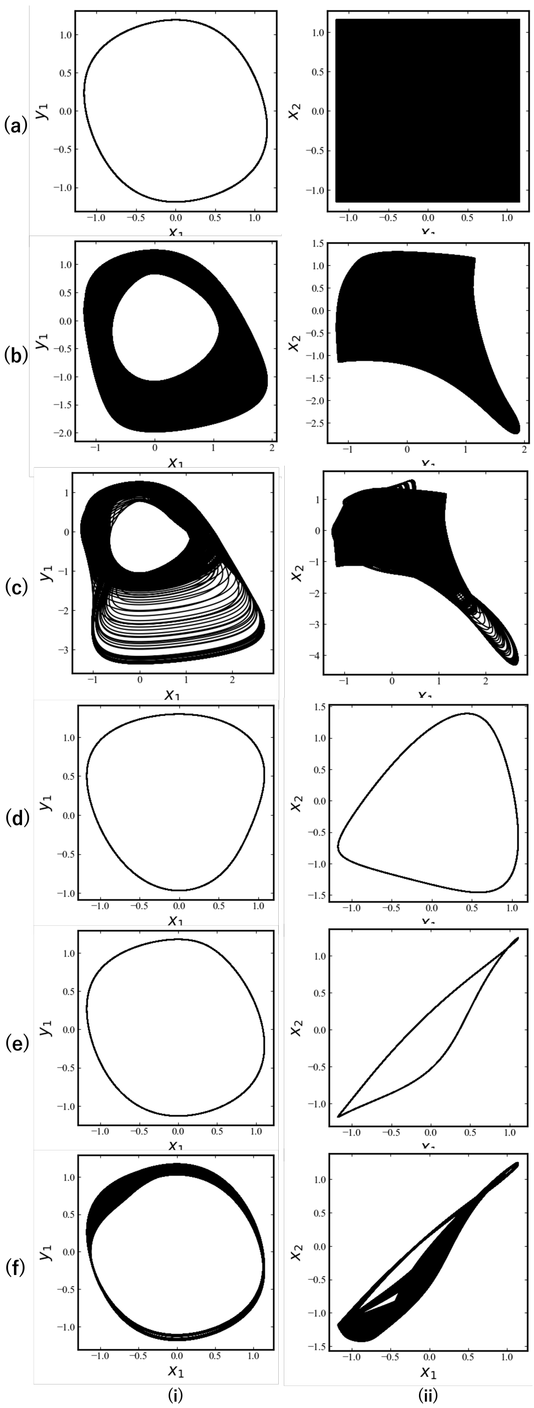

Figure 8: Simulation result using condition 5 in Tab. Ⅲ. (a) ζ = 0.00, (b) ζ = 0.22, (c) ζ = 0.25, (d) ζ = 0.39, (e) ζ = 3.00, (f) ζ = 4.90, (i) a circuit attractor (x_2-y_2), (ii) a phase difference (x_1-x_2).

About $0 \leq \zeta \leq 0.250$, complex attractors and Lissajous diagrams are observed. About $0.252 \leq \zeta \leq 0.382$, attractor and Lissajous diagram are not output. About $0.390 \leq \zeta \leq 4.930$, the graph is simpler than $0 \leq \zeta \leq 0.250$, however graph become more complex as ζ increased.

4. Conclusions

In this study, we observed the synchronization phenomena of van der Pol oscillators coupled by memristors. When ε_1 and ε_2 are the same, clearly different synchronization phenomena, in-phase synchronization and anti-phase synchronization, are observed depending on the difference in initial values. However, when ε_1 and ε_2 are different, there is no difference in synchronization phenomena depending on the initial value.

When the value of ζ is changed while ε_1 and ε_2 are different, the attractor and Lissajous diagram changed. In conclusion, the linear memristor has a complex effect on the synchronization phenomena of the left and right van der Pol oscillators with different nonlinearities.

In the future, we aim to corroborate the output results with a theoretical analysis. The advantage of assuming a linear memristor is that theoretical analysis is relatively easy and detailed. Theoretical analysis using the averaging method for coupled systems including memristors is a novelty, because conventional memristors have complex properties that make theoretical analysis difficult. Theoretical analysis facilitates comparison of the dynamics of both memristors and pure resistors, and makes it possible to evaluate the properties of memristors in an easy-to-understand rule. Specifically, we aim to calculate the range of initial values at which synchronization phenomena occur for in-phase synchronization and anti-phase synchronization, respectively, using the averaging method. In addition, there is a range of values of ζ for which the results are not output in Simulation 3, and we aim to clarify why the results are not output. Also, we aim to observe changes in the attractor and Lissajous diagram by changing ζ over time.

References

[1] L. O. Chua, "Memristor-The Missing Circuit Element", IEEE Trans. on Circuit Theory, vol. CT-18, no. 5, pp. 507-519, September. 1971.

[2] Y. Kotani, Y. Uwate, Y. Nishio, "Synchronization in Three Coupled van der Pol Oscillators with Different Memristor Coupling Strengths", Journal of Signal Processing, vol. 28, no. 4, pp. 99-102, July. 2024.

[3] M. Di Ventra, Y. V. Pershin and L. O. Chua, "Circuit Elements with Memory: Memristors, Memcapacitors, and Meminductors," Proc. of the IEEE, vol. 97, no. 10, pp. 1717-1724, October. 2009.

[4] B. Muthuswamy, "IMPLEMENTING MEMRISTOR BASED CHAOTIC CIRCUITS", International Journal of Bifurcation and Chaos, vol. 20, no. 5, pp. 1335-1350, November 2009.

[5] X. Yang, B. Taylor, A. Wu, Y. Chen and L. O. Chua, "Research Progress on Memristor: From Synapses to Computing Systems," in IEEE Transactions on Circuits and Systems I: Regular Papers, vol. 69, no. 5, pp. 1845-1857, May 2022.