Effect of Coupling Strength on Synchronization Phenomena in Chaotic Circuits with Unidirectional Coupling

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Abstract— **In this study, a chaotic circuit called the Nishio-Inaba circuit was coupled with unidirectional coupling. Therefore, we investigated changes in the synchronization phenomenon due to differences in coupling strength between circuits. When three Nishio-Inaba circuits are connected, if the coupling strengths are equal, the synchronization rate will be higher for pairs with equal parameters. On the other hand, we found that varying the coupling strength changes the synchronization phenomenon between circuits.**

Keywords; chaos Nishio-Inaba Synchronize (key words)

I. Introduction

Synchronization is a natural phenomenon in which oscillations of different periods affect each other, resulting in oscillations of exactly the same period [1]. Such phenomena are observed in limit-cycle oscillators, such as the collective firing of neurons, the nerve cells of the human brain. Chaos is aperiodic, non-linear and sensitive to initial conditions, resulting in values that are highly dependent on the initial conditions. While chaos has boundaries and synchronous phenomena can be observed, aspects of chaos that are unpredictable and difficult to control have attracted attention. Furthermore, in recent years, non-linear neural networks have increasingly been used to process more complex information. However, these neural networks are known to be costly and time-consuming for more complex processing.

Therefore, in this study, in order to create a neural network in an electrical circuit, a chaos circuit called the Nishio-Inaba circuit is used [2]. The coupling of this circuit is set to a state of complete unidirectional coupling by setting one direction of coupling to zero coupling strength. In this case, synchronization phenomena of the chaos circuit are investigated by simulation [3][4].

II. Proposed Method

A. Circuits Model

First, Fig. 1 shows Nishio-Inaba circuit model. It is one of the chaos circuit models. In this study, we consider the Nishio-Inaba circuit as a single node. The circuit equation of this circuit is represented by Eq. (1). Here, v_d is the characteristic equation of the bidirectional diode. Additionally, the normalized circuit equation using normalized parameters for computer simulation is shown in Eq. (2).

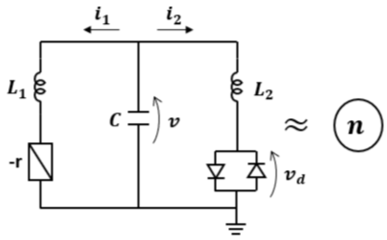

Figure 1. Nishio-Inaba Circuit Model

$$\begin{cases} L_1 \dfrac{di_1}{dt} = v + r i_1 \\ L_2 \dfrac{di_2}{dt} = v - v_d \\ C \dfrac{dv}{dt} = -i_1 - i_2 \\ v_d = \dfrac{r_d}{2}\left(\left| i_2 + \dfrac{V}{r_d} \right| - \left| i_2 - \dfrac{V}{r_d} \right| \right) \end{cases} \tag{1}$$

$$\begin{cases} \dot{x}_i = \alpha x_i + z_i \\ \dot{y}_i = z_i - f(y_i) \\ \dot{z}_i = -x_i - \beta y_i \\ f(y_i) = \dfrac{\delta}{2}\left(\left| y_i + \dfrac{1}{\delta} \right| - \left| y_i - \dfrac{1}{\delta} \right| \right) \end{cases} \tag{2}$$

B. System Model

Fig. 2 shows the system model which is proposed in this study. There are three Nishio-Inaba circuits. The coupling between node 1 and node 2 is unidirectional from node 1 to node 2, and the coupling between node 2 and node 3 is unidirectional from node 3 to node 2. The coupling strength between node 1 and node 2 is denoted as γ_1, and the coupling strength between node 3 and node 2 is denoted as γ_2. The circuit parameter α is set to α_1=0.410 for node 1, and α_2=0.400 for node 2 and node 3. The other circuit parameters are β=3.0 and δ=470.

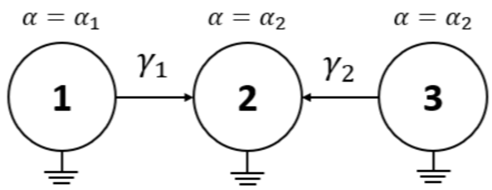

Figure 2. System Model

In this study, the synchronization rates between node 1 and node 2, as well as between node 2 and node 3, are measured while varying the ratio of coupling strengths between circuits in the following four patterns.

Case 1 $\gamma_1{:}\gamma_2{=}1{:}1$, Case 2 $\gamma_1{:}\gamma_2{=}2{:}1$

Case 3 $\gamma_1{:}\gamma_2{=}3{:}1$, Case 4 $\gamma_1{:}\gamma_2{=}4{:}1$

III. Result of Simulation

The average synchronization rates between circuits obtained from the simulation are shown in Fig. 3 to Fig. 6. In the simulation of this study, while keeping the ratio of coupling strengths fixed, the value of coupling strength γ_1 is varied from 0 to 3.0. The initial values are modified in 5 different ways during the simulation. In the figures, S1 represents the synchronization rate between node 1 and node 2, while S3 represents the synchronization rate between node 2 and node 3.

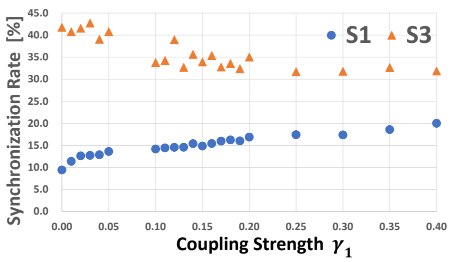

Figure 3. Result of Case 1

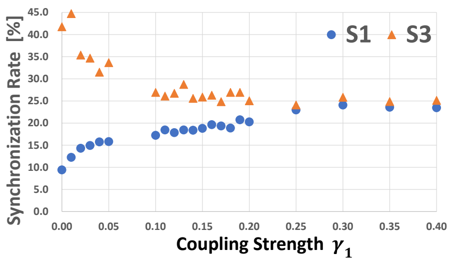

Figure 4. Result of Case 2

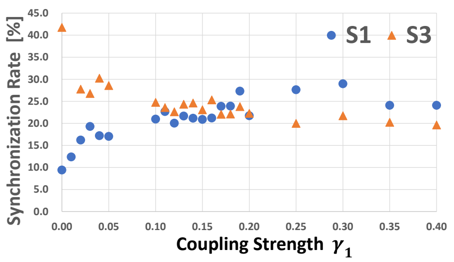

Figure 5. Result of Case 3

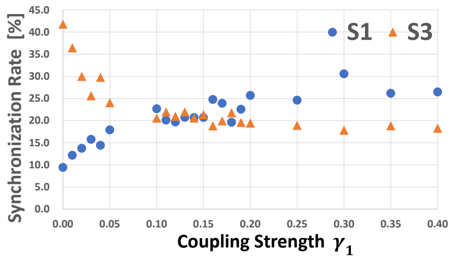

Figure 6. Result of Case 4

From Figures 3 to 6, it is found that when the ratio of coupling strengths is equal, the synchronization rate between nodes is higher when the parameters of the nodes are the same, and lower when the parameters are different. However, the difference in synchronization rates is not very large. Furthermore, when varying the ratio of coupling strengths, the synchronization rate between node 1 and node 2 remains relatively constant regardless of the ratio of coupling strengths, while the synchronization rate between node 2 and node 3 increases as the ratio of coupling strengths becomes more unequal.

From the results of this study, it is clear that in the Nishio-Inaba circuit, a ratio of coupling strengths of 4:1 is most ideal, and when the coupling strength is smaller, the synchronization rate between node 2 and node 3 is higher, while when the coupling strength is larger, the synchronization rate between node 1 and node 2 is higher.

IV. Conclusion

This study investigated the relationship between parameters and synchronization rates when coupling the Nishio-Inaba circuit with unidirectional connections, as well as the relationship between the ratio of coupling strengths and synchronization rates. It was observed that when parameters are different, the impact on nodes by unidirectional connections is reduced, while the impact is increased when parameters are the same. Furthermore, it was evident that the switching of synchronization is most clearly manifested when the ratio of coupling strengths is 4:1.

References

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