Associative Memory Function Using Coupled Oscillators with Sparse Coupling

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***Abstract*—In this study, we applied sparse coupling to the coupling system of van der Pol oscillators and investigated the realization of associative memory. The phase difference of oscillator waveforms and the difference of output patterns by disconnecting the coupling between oscillators was clarified. As a result, it was confirmed that the realization of associative memory is possible using sparse coupling, however the maximum rate of coupling disconnection for which associative memory is possible depends on the combination of oscillators in the coupling system.**

I. Introduction

Synchronization phenomena are abundant in nature, and there has been extensive research on this phenomena observed from the coupling system of oscillators. It has been proven that coupling system of oscillators is able to realize pattern recognition and associative memory by using the synchronization phenomenon [1],[2]. This is known as the Oscillatory Neural Network (ONN) where the oscillators mimic the neurons of the brain [3],[4]. And it is being studied with the goal of bringing pattern recognition and associative memory capabilities closer to those of humans.

In previous studies, associative memory was achieved by using an associatron in the coupling system of oscillators [5]. This is because the application of the asociatron enabled the definition of the coupling strength between oscillators and the calling of a storage matrix. Furthermore, the relationship between the coupling strength between oscillators and associative memory was investigated [6].

In this study, sparse coupling is used in the coupling system of oscillators to disconnect the coupling. The couplings to be disconnected are divided into three cases, and the phase difference and recall patterns are compared by the computer simulations. We also investigate the relationship between the rate of disconnection and associative memory.

II. System Model of Associative memory using the coupling oscillators

Figure 1 (a) shows the circuit diagram of the van der Pol (VDP) oscillator used in this study. A model of a mutually coupled system consisting of 6 × 10 VDPs is shown in Fig. 1 (b). All oscillators are coupled to each other via resistors, and in this study, they are partially disconnected.

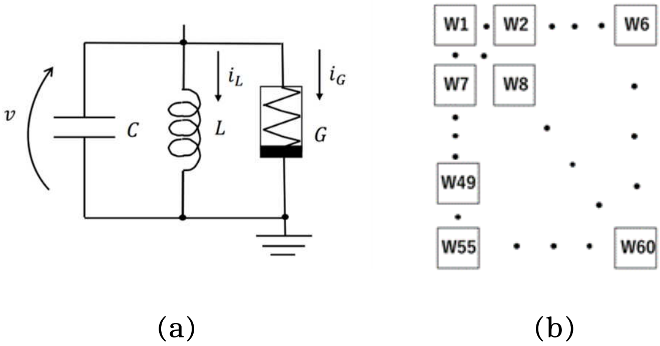


Fig. 1. System model ((a) The circuit diagram of the VDP, (b) A model of a globally coupled system consisting.).

The normalized circuit equations are given as follows:

$$\begin{cases} \dfrac{dx_n}{d\tau} = \varepsilon x_n(1 - x_n^2) - y_n + \alpha K \displaystyle\sum_{k=1}^{60}(x_n - x_k), \\ \dfrac{dy_n}{d\tau} = x_n \end{cases} \quad (1)$$

Where α is the connectivity matrix, it represents the state of coupling between oscillators, and it is possible to disconnect the coupling between the specified oscillators.

Here, the method is explained based on the proposed method in previous study. For Eq. (1) K is defined as

$$K = \begin{cases} E_0 \times s \\ E \times s \end{cases}$$

s is the coupling strength and E_0 and E are based on the asociatron. Additionally E_0 is the storage matrix for the input pattern and E is the storage matrix for the stored patterns. f^0 and f^p are image data and there are the same number of oscillators W1 to W60 in the model. They are "-1" when W1 ∼ W60 in Fig. 1 (b) represents black and "1" when W1 ∼ W60 represents white.

$$\begin{cases} f^0 = (f^0_{W1} \;\; f^0_{W2} \;\; f^0_{W3} \;\; \cdots \;\; f^0_{W60}) \\ f^p = (f^p_{W1} \;\; f^p_{W2} \;\; f^p_{W3} \;\; \cdots \;\; f^p_{W60}) \end{cases}$$

E_0 and E are defined using the transpose matrix f^T as follows:

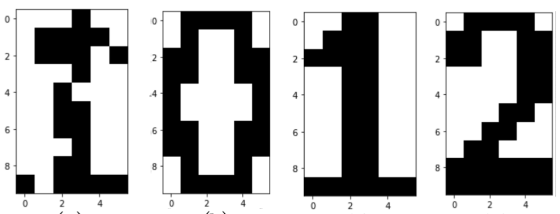
$$\begin{aligned} E_0 &= f^{0T} \; f^0 \\ &= \begin{pmatrix} f^0_{W1}f^0_{W1} & f^0_{W1}f^0_{W2} & \cdots & f^0_{W1}f^0_{W60} \\ f^0_{W2}f^0_{W1} & f^0_{W2}f^0_{W2} & \cdots & f^0_{W2}f^0_{W60} \\ \vdots & \vdots & \ddots & \vdots \\ f^0_{W60}f^0_{W1} & f^0_{W60}f^0_{W2} & \cdots & f^0_{W60}f^0_{W60} \end{pmatrix} \end{aligned} \quad (2)$$

$$\begin{aligned} E &= \sum_{k=1}^{p} f^{kT} \; f^k \\ &= \sum_{k=1}^{p} \begin{pmatrix} f^k_{W1}f^k_{W1} & f^k_{W1}f^k_{W2} & \cdots & f^k_{W1}f^k_{W60} \\ f^k_{W2}f^k_{W1} & f^k_{W2}f^k_{W2} & \cdots & f^k_{W2}f^k_{W60} \\ \vdots & \vdots & \ddots & \vdots \\ f^k_{W60}f^k_{W1} & f^k_{W60}f^k_{W2} & \cdots & f^k_{W60}f^k_{W60} \end{pmatrix} \end{aligned} \quad (3)$$

To realize associative memory using these storage matrices E_0 and E, first synchronous simulation of the oscillator with E is performed, and then the simulation is switched to simulation with E_0. The output pattern is determined by the phase difference of the reference oscillator W1, and the color of the pixels is determined by the separation between 0 and 180 degrees.

III. Simulation Results

The input pattern is shown in Fig. 2 (a) and The stored patterns are shown in Fig. 2 (b), (c) and (d).



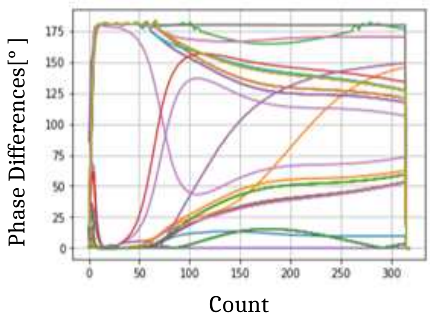
(a) (b) (c) (d)

Fig. 2. 6 × 10 patterns ((a) is input pattern, (b), (c) and (d) are stored patterns.).

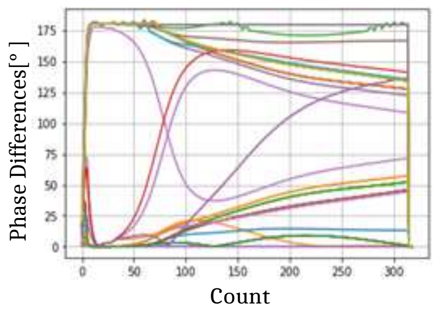
The values of f^0 and f^p are determined from Fig. 2, and E_0 and E are created by Eps. (2) and (3). In the case of patterns in Fig. 2, the input pattern Fig. 2 (a) similar to the stored pattern Fig. 2 (c). If Fig. 2 (c) is a recall pattern, associative memory is considered to be realized. Equation (1) is calculated using Runge-Kutta method with the step size h = 0.1 for τ = 0 $\sim$ 2000. The initial values of x and y are random between 0 and 0.1, the parameters are ε = 0.1, s = 0.005 in Eq. (1). The storage matrix was changed from E_0 to E at τ =100 for recalling a stored pattern. Count is the number of times the reference oscillator passed through the Poincar section.

A. Coupling Disconnection of Noisy Part of Oscillators

Oscillators in the noisy part of the model are those whose pixels have the wrong color when compared to the input pattern in Fig. 2 (a), and the correct stored pattern in Fig. 2 (c). Figure 3 shows the phase difference when the oscillator in the noisy part of Fig. 2 (a) is disconnected from the 38 randomly selected oscillators in Fig. 3 (a), and the 40 randomly selected oscillators in Fig. 3 (b).



(a)



(b)

Fig. 3. Phase difference ((a) 8 × 38 disconnect coupling between oscillators, (b) 8 × 40 disconnect coupling between oscillators.).

The results of the recall patterns at count = 300 are shown in Fig. 4.

The correct recall pattern was obtained in Fig. 4 (a). However, it can be seen from Fig. 2 (c) and Fig. 4 (b) that the color of W13 has changed from the stored pattern. Therefore, the associative memory function was impaired when more than 38 oscillators were disconnected.

B. Coupling Disconnection of Randomly Selected Oscillators

Here, the coupling between some randomly selected oscillators is disconnected from an equal number of other oscillators in this model. The phase difference of disconnected couplings of 29 randomly selected oscillators for each are shown in Fig.

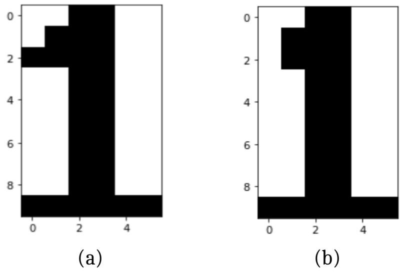


Fig. 4. Recall patterns ((a) 8×38 disconnect coupling between oscillators, (b) 8×40 disconnect coupling between oscillators.).

5 (a), and 30 randomly selected oscillators for each are shown in Fig. 5 (b).

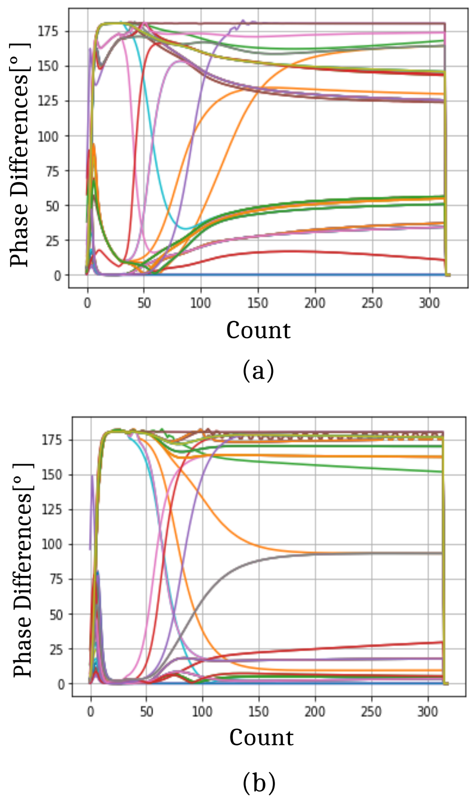


Fig. 5. Phase difference ((a) 29×29 disconnect coupling between oscillators, (b) 30×30 disconnect coupling between oscillators.).

The results of the recall patterns at count = 300 are shown in Fig. 6.

It was impossible to realize associative memory when 30×30 couplings were disconnected in Fig. 6 (b). Therefore, the correct recall pattern could be obtained by disconnecting 28×28 connections in Fig. 7 (a), and the maximum value at which associative memory could be realized was 28 when the same number of oscillator connections were disconnected.

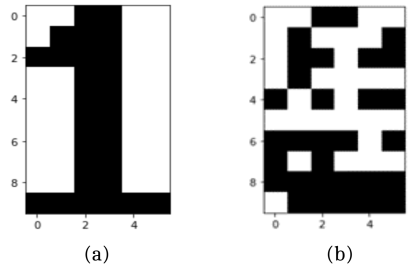


Fig. 6. Recall patterns ((a) 29×29 disconnect coupling between oscillators, (b) 30×30 disconnect coupling between oscillators.).

Next, to compare with subsection A, which uses 8 oscillators in the noisy part, we use the same number of randomly selected oscillators. These oscillators are disconnect the coupling of 8 and 51 or 52 randomly selected oscillators. The phase difference of disconnecting the coupling of 51 oscillators is shown in Fig. 7 (a) and the coupling of 52 oscillators is shown in Fig. 7 (b).

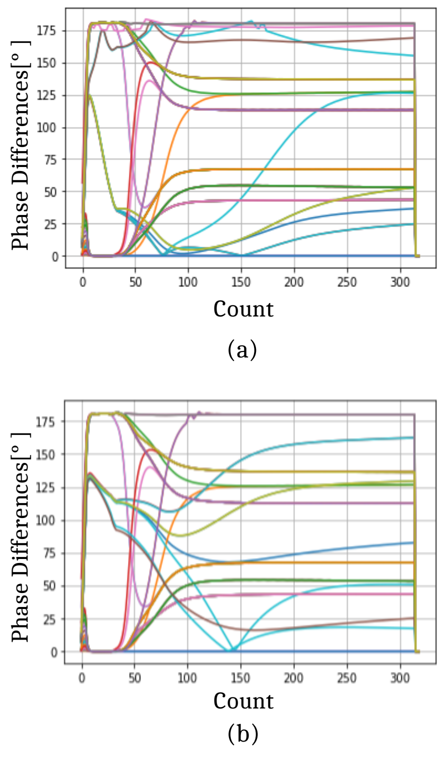
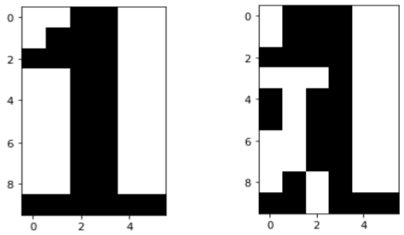


Fig. 7. Phase difference ((a) 8×51 disconnect coupling between oscillators, (b) 8×52 disconnect coupling between oscillators.).

TABLE I
THE RATE OF COUPLING DISCONNECTION

	The number of disconnections in simulation					
	8 × 38	***8 × 40***	***8 × 51***	***8 × 52***	***29 × 29***	***30 × 30***
The rate of disconnected couplings[%]	17.2	18.0	23.0	23.5	47.5	50.8
Realization of associative memory	○	×	○	×	○	×

The results of the recall patterns at count = 300 are shown in Fig. 8.



(a) (b)

Fig. 8. Recall patterns ((a) 8 × 51 disconnect coupling between oscillators, (b) 8 × 52 disconnect coupling between oscillators.).

The correct recall pattern was obtained by disconnecting 8 × 51 connections in Fig. 8 (a). Associative memory was not realized when 8 × 52 couplings were disconnected.

Finally, Summarize whether associative memory is realized in each of the simulation, the number of disconnections in the simulations and the rate of coupling disconnections in the model are shown in Table I.

When disconnecting the same number of oscillator connections, the model had a 50% connection disconnection rate, and associative memory was not realized. The same number of disconnected oscillators can realize associative memory even though the rate of coupling disconnection is higher. Compared to disconnecting the couplings in the noisy parts of Fig.1 (a), disconnecting the couplings of 8 randomly selected oscillators resulted in a higher rate of coupling disconnection, and associative memory was realized. When the coupling of the noisy parts was disconnected, the recall pattern was not broken as abruptly as the 30 × 30 or the 8 randomly selected oscillators. Therefore, it can be observed that the larger the rate of coupling disconnection, the larger the recall pattern is broken.

IV. CONCLUSIONS

In this study, we proposed to use sparse coupling for the coupling system of van der Pol oscillators to disconnect the coupling. The realization of associative memory was investigated based on the phase difference of oscillation waveforms and recall patterns.

As a result, it was possible to realize associative memory using sparse coupling, but associative memory was not realized when the rate of coupling disconnection was high. In addition, there were differences in the recall pattern breaking. When the oscillator and the coupling of the noisy part were disconnected, the recall pattern was not broken and the coupling disconnection rate was small, however when the 30 × 30 couplings were disconnected, the recall pattern was broken significantly. In addition, although the oscillators in the noisy part and the 8 oscillators selected at random had the same number of oscillators, the realization of associative memory was realized even if many oscillators and couplings were disconnected. In summary, it was possible to confirm that the rate of coupling disconnection varied depending on the combination of the 60 oscillators.

For the future work, we would like to investigate the couplings that can be preferentially disconnected when disconnecting couplings. In addition, a circuit simulation is to be performed to confirm the difference from this study.

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