Synchronization Patterns in Nonlinear Circuit Networks Altered by Learning Rules

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Abstract

This study investigates the synchronization phenomena when the weights of coupled chaotic circuit networks are learned. In contrast to previous approaches using voltage differences of capacitors for synchronization detection, we employ phase differences in this study. This enables more accurate weight learning based on precise synchronization detection. We explore how synchronization phenomena and weights change with and without weight learning.

1. Introduction

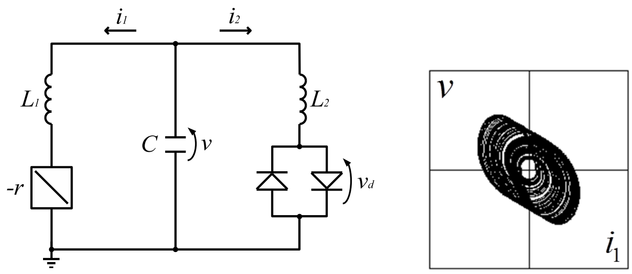
Synchronization phenomena in nonlinear circuit networks exhibit intriguing phenomena such as N-phase synchronization, oscillation quenching, and independent oscillation. These phenomena hold promise for applications in modeling and controlling high-dimensional nonlinear systems. In particular, chaos theory is applied in biomedical engineering for the analysis of physiological signals such as heart rate variability and brain waves. This contributes to the understanding of the complex dynamics of biological systems and plays a role in the development of medical diagnostics and treatment methods. The fundamentals of chaotic synchronization have been studied with regard to the geometry and stability of synchronization [1], [2]. Examples of applications using chaos synchronization include secure communication and image encryption [3], [4].

As part of our previous research, we have investigated clustering using synchronization in coupled chaotic circuit networks [5]-[7]. Placing chaotic circuits on a two-dimensional plane and reflecting the distance between chaotic circuits in the magnitude of coupling allows for efficient clustering. Additionally, for more advanced clustering, we have confirmed the effectiveness of altering coupling weights using learning rules. In Refs. [6] , [7], the method for synchronization detection involves using the voltage difference between coupled chaotic circuits. However, in synchronization detection for chaotic coupling systems, using phase rather than voltage difference enables more accurate synchronization measurement.

Therefore, in this study, we investigate the behavior of coupled chaotic circuit networks by employing phase-based synchronization detection. Take one chaotic circuit as the reference circuit and calculate the phase difference using a Poincare section. First, we examine the synchronization rate of the entire network, and furthermore, we investigate how the distribution of weights changes by utilizing learning rules.

2. Chaotic Circuits Model

Figure 1 shows the chaotic circuit model that has been investigated in the literature [8]-[10]. This circuit consists of three memory elements, one linear negative resistance element, and one nonlinear resistance element consisting of two diodes. The negative resistance is realized using the linear region of a negative impedance converter made from an operational amplifier.



(a) Chaotic circuit. (b) Chaotic attractor.

Figure 1: Chaotic circuit model.

The approximate $I - V$ characteristics of the nonlinear resistance element are indicated by the following equation, where the parameter r_d is the slope of the nonlinear resistance.

$$v_d(i_2) = \frac{r_d}{2}\left(\left|i_2 + \frac{V}{r_d}\right| - \left|i_2 - \frac{V}{r_d}\right|\right). \qquad (1)$$

By changing the variables, such that

$$i_1 = \sqrt{\frac{C}{L_1}}Vx;\ i_2 = \frac{\sqrt{L_1 C}}{L_2}Vy;\ v = Vz;$$

$$r\sqrt{\frac{C}{L_1}} = \alpha;\ \frac{L_1}{L_2} = \beta;\ r_d\frac{\sqrt{L_1 C}}{L_2} = \delta; t = \sqrt{L_1 C}\tau \tag{2}$$

The normalzied equations represent the circuit equations when all the chaotic circuits are coupled globally with each other (all-to-all coupling).

$$\begin{cases} \dfrac{dx_i}{d\tau} = \alpha x_i + z_i \\ \dfrac{dy_i}{d\tau} = z_i + f(y) \\ \dfrac{dz_i}{d\tau} = -x_i - \beta y_i - \displaystyle\sum_{j=1}^{N} \gamma_{ij}(z_i - z_j) \\ \qquad (i, j = 1, 2, \cdots, N) \end{cases} \tag{3}$$

where $f(y)$ is described as follows:

$$f(y) = \frac{\delta}{2}\left(\left|y + \frac{1}{\delta}\right| - \left|y - \frac{1}{\delta}\right|\right). \tag{4}$$

In the computer simulations, we set the parameters to be $\alpha = 0.460$, $\beta = 3.0$ and $\delta = 470$. The characteristic function $f(y)$ can be described as a three-segment piecewise-linear function. In this study, the value of γ_{ij} reflects the distance between the circuits in an inverse manner, as described using the following equation:

$$\gamma_{ij} = \frac{g}{(d_{ij})^2}. \tag{5}$$

Here, d_{ij} denotes the Euclidean distance between the $i-th$ circuit and the $j-th$ circuit, while g is a scaling parameter that determines the coupling strengths.

3. Circuit Arrangement and Learning Process

Figure 2 shows an example of the arrangement of 100 chaos circuits. This is generated by a Gaussian distribution with a parameter $\sigma = 0.7$. In the simulation, investigations are conducted for five different networks.

Next, we explain about learning process of coupled chaotic circuits networks. A flowchart of these steps is shown in Fig. 3.

We apply this Hebbian rule to chaotic circuit synchronization. In other words, the coupling between the synchronized chaotic circuits is made stronger, and the coupling between the un-synchronized chaotic circuits is made weaker. The Hebbian rule is applied to the chaotic circuits network as following steps.

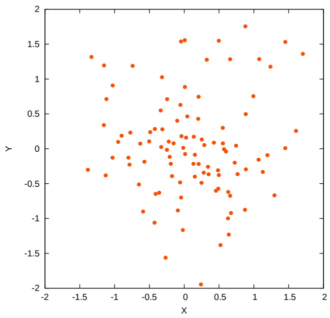


Figure 2: One example of circuit arrangement in 2-dimensional space ($N = 100$, σ=0.7).

[step-1] At the initial state, all nodes are fully connected with coupling strengths depending on distance.
[step-2] After a transient phase, we apply two rules for a sequence of generations. Each generation has length $G = 10$.

- **(Determination of synchronization:)** In order to determine whether two nodes are alike, we calculate the synchronization ratio for every pair of oscillators. If the synchronization ratio is larger than 80%, the corresponding coupling strength becomes stronger with $\Delta\gamma = 1e^{-6}$.

  In order to analyze the synchronization ratio, we define a synchronization state by calculating phase difference as

  $$|\theta_k - \theta_n| < 20^\circ \quad (k \in S_n)$$

[step-3] Step-2 is repeated until iterations are reached ($G = 2000$).
[step-4] At the final state ($G = 2000$), we check the synchronization ratio for every pair of oscillators.

Here, two learning methods are considered. One method involves increasing the weights for synchronized connections and decreasing the weights for unsynchronized connections ($\mathbf{Learning - PM}$). The other method involves increasing the weights only for synchronized connections without decreasing the weights ($\mathbf{Learning - P}$). The differences between the two learning methods is also investigated.

4. Simulation Results

First, we investigate the average synchronization rate of the entire circuit and the number of connections with synchronization exceeding 80 percent. The average results for five different networks are summarized in Table 1. Focusing on the average synchronization rate, it is observed that without weight learning, the rate is 67.7 %. On the other hand, both methods with learning show an improvement in synchronization rates. Furthermore, when comparing Learning-PM and Learning-P, it is evident that Learning-P achieves a higher

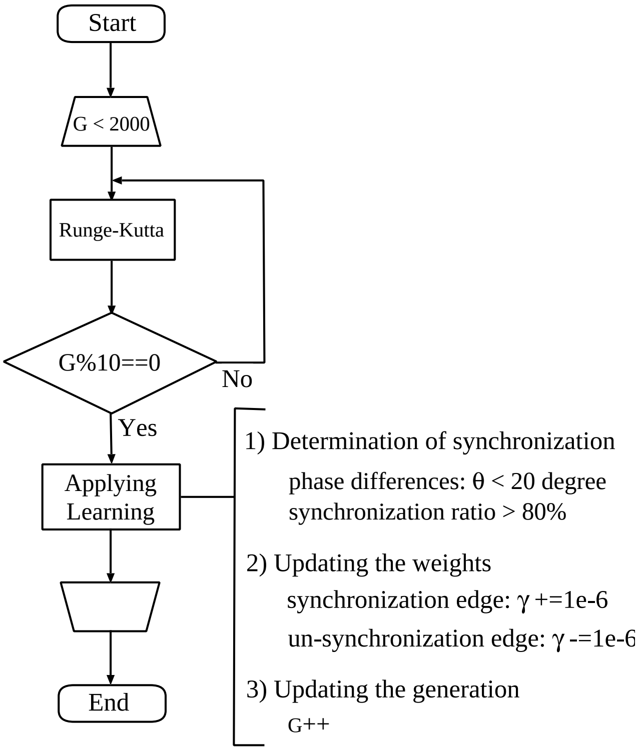


Figure 3: Flowchart of learning process.

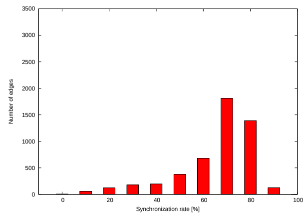
synchronization rate. Similarly, the number of highly synchronized connections is more than three times greater with weight learning compared to no learning. The results indicate that learning weights enhances the synchronization rate. The higher synchronization rate observed with Learning-P is attributed to its lack of an effect reducing the size of connections with low synchronization rates.

Table 1: Synchronization rate and high-synchronized edges

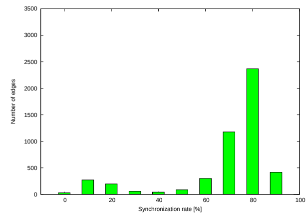
	Average of sync. rate [%]	Number of high-sync. edges
No-learning	67.70	1301.4
Learning-PM	75.13	3122.6
Learning-P	83.91	3862.6

Next, we investigated the synchronization rate distribution of each edge within the circuit network. The simulation results are presented in Fig. 4, where the horizontal axis represents the synchronization rate, and the vertical axis represents the number of edges. In the case of no learning, it can be observed that there is a peak at 70 $\%$ synchronization rate. For Learning-PM, the peak has increased to 80 $\%$ and it is notable that the synchronization rates around 10 are higher compared to the case with no learning. In the case of Learning-P, the peak is at 80 $\%$, indicating the absence of edges with low synchronization rates.

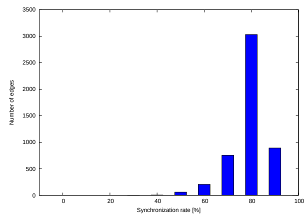
Figure 5 shows the distribution of weights value. In this proposed circuit system, the weight value is determined by the distance between two chaotic circuits. We investigate the effect of learning by checking the change of distribution of weights. In the case of no-learning, most of weights has small values. By using learning process, the peak of distribution is shifted to around 0.002. The difference between Learning-PM and Leraning-P is that there are not may small weights for Learning-P.



(a) No-learning.

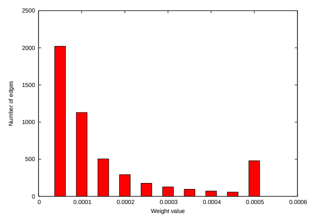


(b) Learning-PM.

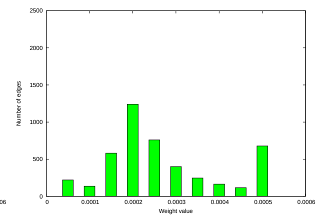


(c) Learning-P.

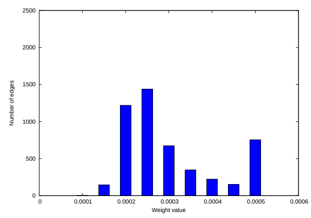
Figure 4: Distribution of synchronized edges.



(a) No-learning.



(b) Learning-PM.



(c) Learning-P.

Figure 5: Distribution of weight value.

Next, we consider the timing of learning process. The simulation results of synchronization rate and number of high-synchronized edges depending on timing of learning process are shown in Figs. 6 and 7, respectively. This is the average result of five different networks. From these figures, we can confirm that if number of applying learning is large, the synchronization rate and the number of high-synchronized edges are also high value.

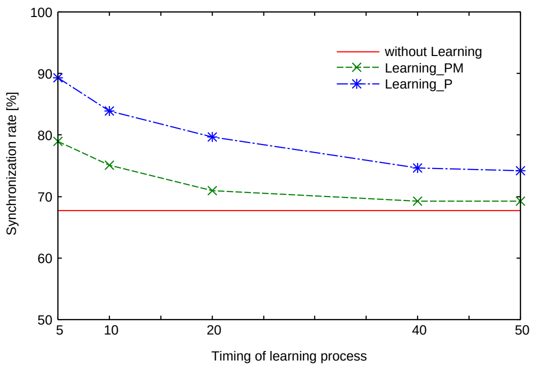


Figure 6: Synchronization rate depending on timing of learning process.

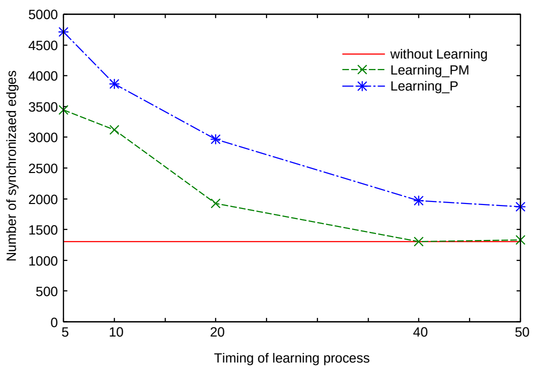


Figure 7: High-synchronized edges depending on timing of learning process.

5. Conclusion

In this study, we have investigated the characteristics of the proposed chaotic circuits networks with learning by focusing on synchronization rate and weight value. By using the computer simulations, we have confirmed that in the case of synchronization rate, the peak of distribution becomes two peaks by applying Learning-PM. In the case of weight value, the distribution form is changed from power distribution to normal distribution.

As future works, we would like to propose more effective learning method by using neuro science phenomena such as STDP coupling.

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