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Synchronization in Three Coupled van der Pol Oscillators with Different Memristor Coupling Strength

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Abstract

In this study, we propose three coupled van der Pol oscillators with different memristor couplings strength. We investigate synchronization state changing the one memristor coupling strength. Compared with different resistor coupling strength, amplitude death as new synchronization state is obtained. In addition, we research synchronization in dependence on the one memristor parameter to analyze the relative phase difference, the amplitude and the power consumption.

1. Introduction

Human brain is composed of over one hundred billion neurons and synapses. Information transmission in the human brain involves the transmission of electrical signals between neurons through synapses. At this time, neurons are synchronized with each neuron. In electrical engineering, human brain is modeled by coupled oscillators, and synchronization phenomena are investigated in various coupled oscillators for development of brain science [1], [2].

Many researchers investigated synchronization phenomena on coupled oscillators via resistors [3], inductors [4] and time-varying resistors [5]. These circuit elements are characterized by a current. Recent years, a memristor has been focused on as the synapse because resistance value of the memristor is characterized by a charge [6], [7]. In general, the charge is defined as the integral of the current. In human brain, the weight of the synapse is changed during transmitting information. Hence the memristor has ability of realizing the synapse of human brain. Research on synchronization phenomena on coupled oscillators with memristor synapses provides the realization of human brain behaviors.

In previous study, three-phase synchronization state and in-phase and anti-phase synchronization state have been observed in three coupled van der Pol oscillators with different resistor coupling strength [2]. Also, our research group proposed three coupled van der Pol oscillators with memristor coupling strength, and confirmed three synchronization types: three-phase synchronization state, anti-phase synchronization and amplitude death [8]. However, the characteristics of all memristors are same. Synchronization phenomena on coupled van der Pol oscillators with different memristor coupling strength have not been studied.

In this study, we propose three coupled van der Pol oscillators with different memristor coupling strength. First, we investigate synchronization phenomena changing the characteristics of the one memristor to do numerical simulations. Second, we analyze synchronization in dependence on the one memristor parameter by calculating the relative phase difference and the amplitude of oscillators. Finally, we research the power consumption of our proposed circuit model.

2. Proposed Model

2.1 Memristor Model

We use the charge-controlled memristor model [6]. Figure 1 (a) shows a schematic model and the Charge q - flux φ curve characteristics of the memristor is shown in Fig. 1 (b).

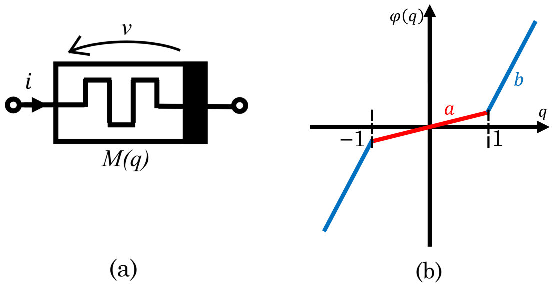


Figure 1: Memristor model. (a): Schematic model, (b): Charge q - flux φ characteristic curve.

Resistance value of the memristor $M(q)$ is called memristance. In this model, $M(q)$ is defined as the gradient of the piecewise linear function $\varphi(q)$ as Eqs. (1) and (2).

$$\varphi(q) = bq + 0.5(a - b)(|q + 1| - |q - 1|). \tag{1}$$

$$M(q) = \frac{d\varphi(q)}{dq} = \begin{cases} a & (|q| < 1) \\ b & (|q| > 1) \end{cases} \tag{2}$$

2.2 Coupled Circuit Model

Figure 2 shows the three coupled van der Pol oscillators with different memristor coupling strength.

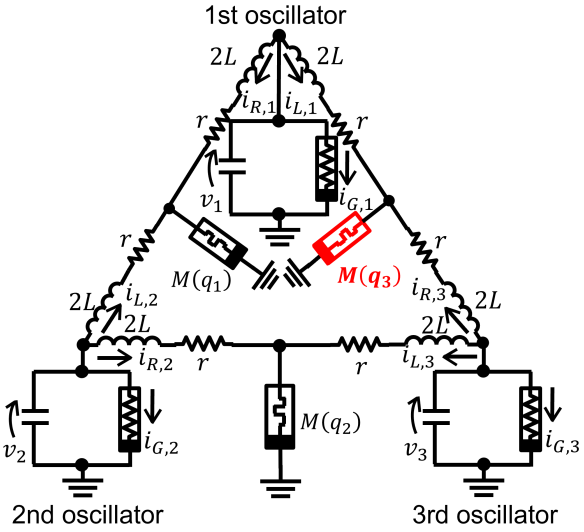


Figure 2: Three coupled van der Pol oscillators with different memristor coupling strength.

Where r is the tiny resistor to avoid L-loop in coupled oscillators as a ring structure. The $i_{G,k} - v_{G,k}$ characteristic curve of the nonlinear resistor is approximated by the following third-power polynomial equation.

$$i_{G,k} = -g_1 v_{G,k} + g_3 v_{G,k}^3 \quad (g_1, g_3 > 0, k = 1, 2, 3). \quad (3)$$

In addition, the $q - \varphi$ characteristic curve of the one memristor between first oscillator and third oscillator is different from the other memristors.

The circuit equation are given as Eq. (4) from Kirchhoff's circuit laws.

$$\begin{cases} C\dfrac{dv_k}{dt} = -i_{G,k} - i_{R,k} - i_{L,k} \\ 2L\dfrac{di_{R,k}}{dt} = v_k - r i_{R,k} - M(q)(i_{R,k} + i_{L,k+1}) \\ 2L\dfrac{di_{L,k}}{dt} = v_k - r i_{L,k} - M(q)(i_{R,k-1} + i_{L,k}) \\ \dfrac{dq_k}{dt} = i_{R,k} + i_{L,k+1} \\ M(q_k) = \dfrac{d\varphi(q_k)}{dq_k} = \begin{cases} a & (|q_k| < 1) \\ b & (|q_k| > 1) \end{cases} \\ M(q_3) = \dfrac{d\varphi(q_3)}{dq_3} = \begin{cases} c & (|q_3| < 1) \\ d & (|q_3| > 1) \end{cases} \\ \varphi(q_k) = bq_k + 0.5(a-b)(|q_k + 1| - |q_k - 1|) \\ \varphi(q_3) = dq_3 + 0.5(c-d)(|q_3 + 1| - |q_3 - 1|). \end{cases} \quad (4)$$

where a and b are the parameters of two memristors between second oscillator and the other oscillators. Similarly, c and d are the parameters of the memristor between first oscillator and third oscillator. Next, these circuit equation Eq. (4) need to be normalized in order to investigate synchronization phenomena calculating by Runge-Kutta method. We change the variable and parameters as follows.

$$v_k = \sqrt{\frac{g_1}{g_3}} x_k, i_{R,k} = i_{L,k} = \sqrt{\frac{g_1 C}{g_3 L}} y_{L,k}, q_k = z_k, \gamma = \sqrt{\frac{C}{L}}$$
$$t = \sqrt{LC}\tau, \varepsilon = g_1\sqrt{\frac{L}{C}}, \zeta = C\sqrt{\frac{g_1}{g_3}}, \eta = r\sqrt{\frac{C}{L}}. \quad (5)$$

Hence, the normalized circuit equation of the proposed model are given as follows.

$$\begin{cases} \dfrac{dx_k}{d\tau} = \varepsilon(1 - x_k^2)x_k - y_{R,k} - y_{L,k} \\ \dfrac{dy_{R,k}}{d\tau} = \dfrac{1}{2}(x_k - \eta y_{R,k} - M(z)(y_{R,k} + y_{L,k+1})) \\ \dfrac{dy_{L,k}}{d\tau} = \dfrac{1}{2}(x_k - \eta y_{L,k} - M(z)(y_{R,k-1} + y_{L,k})) \\ \dfrac{dz_k}{dt} = \zeta(y_{R,k} + y_{L,k+1}) \\ M(z_k) = \dfrac{d\varphi(z_k)}{dz_k} = \begin{cases} a & (|z_k| < 1) \\ b & (|z_k| > 1) \end{cases} \\ M(z_3) = \dfrac{d\varphi(z_3)}{dz_3} = \begin{cases} c & (|z_3| < 1) \\ d & (|z_3| > 1) \end{cases} \\ \varphi(z_k) = bz_k + 0.5(a-b)(|z_k + 1| - |z_k - 1|) \\ \varphi(z_3) = dz_3 + 0.5(c-d)(|z_3 + 1| - |z_3 - 1|). \end{cases} \quad (6)$$

The instantaneous power consumption of the memristor p is defind as Eq. (7).

$$p = M(q)i^2. \quad (7)$$

The average power consumption P is obtained by integrating the instantaneous power consumption p.

$$P = \int_{t_0}^{t} M(q)i^2 d\tau \quad (t_0 \leqq t). \quad (8)$$

Therefore, the total average power consumption of our proposed model P_{all} can be calculated by Eq. (9).

$$P_{all} = \frac{1}{T}\sum_{k=1}^{3} P_{k,k+1} = \frac{1}{T}\sum_{k=1}^{3} M(z_k)(i_{R_k} + i_{L_{k+1}})^2. \quad (9)$$

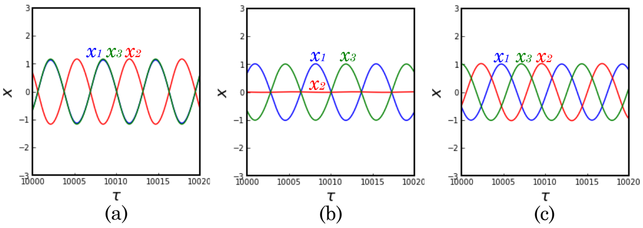
where T is a long period, $i_{L,4} = i_{L,1}$ and $P_{k,k+1}$ is the average power consumption of the memristor coupling between k-th oscillator and $(k+1)$-th oscillator.

3. Results

For the computer simulations, the normalized circuit equation Eq. (6) are calculated by Runge-Kutta method with step size $h = 0.01$. The parameters and variables are setting for $\tau = 20,000$, $\varepsilon = 0.1$, $\gamma = 1.0$, $\zeta = 0.1$, $\eta = 0.001$, $a = 0.1$, $b = 10$, $c = 0.1$, $x_1 = 1.0$, $x_2 = 1.1$, $x_3 = 1.2$, $y_1 = 2.0$, $y_2 = 2.1$, $y_3 = 2.2$, $z_1 = 1.0$, $z_2 = 1.1$ and $z_3 = 1.2$.

3.1 Synchronization Phenomena

This study analyzes synchronization phenomena by changing the one memristor parameter d. Figure 3 shows the three types of the time-series of x.



(a) (b) (c)

Figure 3: Time-series of x (a): In-phase and anti-phase synchronization state ($d = 0.01$), (b): Amplitude death ($s = 0.5$), (c): Three-phase synchronization state ($d = 10$).

In Fig. 3, in-phase and anti-phase synchronization occurs when $d = 0.01$. Amplitude death occurs when $d = 0.5$. Three-phase synchronization occurs when $d = 10$. Therefore we confirm that three synchronization states: three-phase synchronization state, in-phase and anti-phase synchronization state and amplitude death are obtained by changing d.

3.2 Synchronization in dependence on d

In this subsection, we analyze three synchronization states in dependence on d in detail. We investigate the relationship between the absolute value of the relative phase differences $|\Delta\theta_k|$ and the one memristor parameter d. The phase of first oscillator as a reference is denoted θ_1 and the relative phase difference of k-th oscillator $|\Delta\theta_k|$ is defined as Eq. (10).

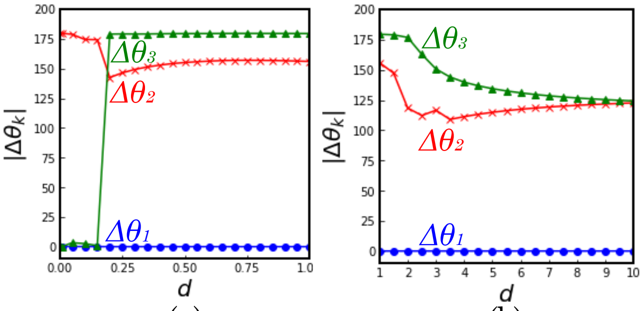
$$|\Delta\theta_k| = |\theta_k - \theta_1| = |\arctan\frac{y_k}{x_k} - \arctan\frac{y_1}{x_1}|. \qquad (10)$$

Here, $|\Delta\theta_k|$ is calculated by Poincaré maps. The methods of calculating $|\Delta\theta_k|$ are described as following steps.

1. A van der Pol oscillator with third-power has a stable limit cycle, so Poincaré section is defined as the plane that $x_k(\tau) > 0$ and $y_k(\tau) = 0$ in phase plane.

2. If x_1 and y_1 move from fourth quadrant to first quadrant ($x_1(\tau - 1) > 0$, $y_1(\tau - 1) < 0$ and $y_1(\tau) > 0$), $|\Delta\theta_k|$ is calculated by Eq. (10).

Figure 4 shows the relationship between $|\Delta\theta_k|$ and d.

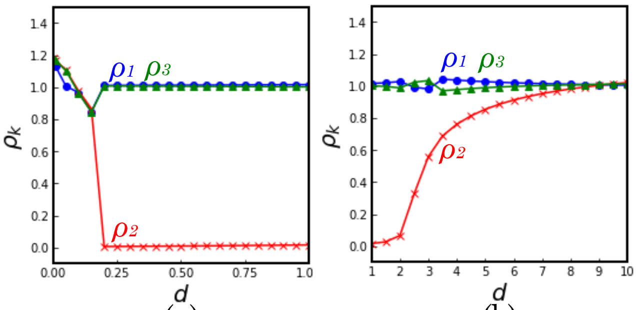


(a) (b)

Figure 4: Relationship between $|\Delta\theta_k|$ and d. (a): $0 \leqq d \leqq 1$, (b): $1 \leqq d \leqq 10$.

In Fig. 4, second oscillator anti-synchronized with first oscillator and third oscillator synchronized with first oscillator in the region $0.0 \leqq d \leqq 0.15$. In the region $0.2 \leqq d \leqq 2$, third oscillator anti-synchronized with first oscillator and $|\Delta\theta_2|$ converges to almost 150°. In the region $2.5 \leqq d \leqq 10$, three oscillators synchronized with each oscillator.

In order to analyze the behavior of second oscillator, we investigate the relationship between the amplitude of three oscillators ρ_k and the one memristor parameter d. Figure 5 shows the relationship between ρ_k and d.



(a) (b)

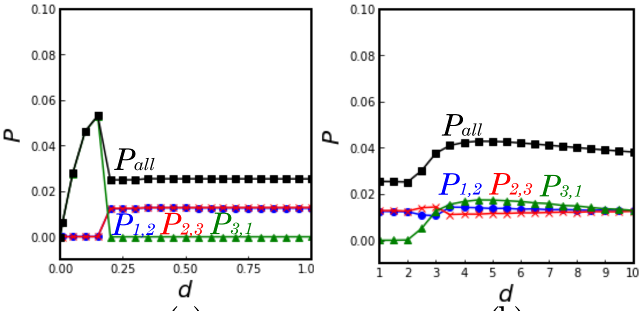
Figure 5: Relationship between ρ_k and d. (a): $0 \leqq d \leqq 1$, (b): $1 \leqq d \leqq 10$.

In Fig. 5, ρ_1, ρ_2 and ρ_3 converge to almost 1.0 in the regions $0 \leqq d \leqq 0.15$ and $2.5 \leqq d \leqq 10$. In the regions $0.2 \leqq d \leqq 2.5$, ρ_2 converges to almost zero. Therefore, first oscillator and third oscillator are anti-synchronized, and then the amplitude of second oscillator is dead in the regions $0.2 \leqq d \leqq 2.5$.

Here, we compare the simulation results of the relative phase difference and the amplitude. In-phase and anti-phase synchronization occur in the region $0.0 \leqq d \leqq 0.15$. In the region $0.2 \leqq d \leqq 2$, amplitude death occurs. In the region $2.5 \leqq d \leqq 10$, three-phase synchronization can be observed.

3.3 Power Consumption

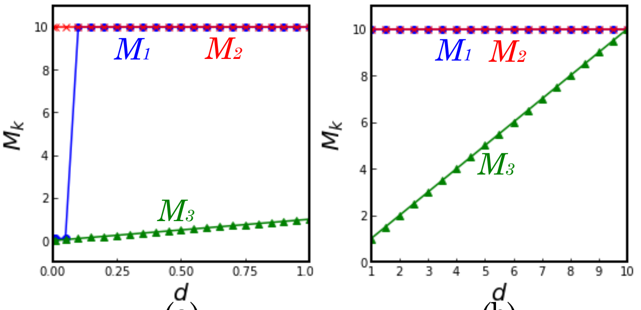
Generally, phase sift is caused by changes of the power consumption of coupling parts [3, 8]. We investigate the relationship between the power consumption of the memristor couplings P and the one memristor parameter d. Figure 6 shows the relationship between P and d.



(a) (b)

Figure 6: Relationship between P and d. (a): $0 \leqq d \leqq 1$, (b): $1 \leqq d \leqq 10$.

In the region $0.1 \leqq d \leqq 10$, P_{all} of in-phase and anti-phase synchronization state is larger than the one of three-phase synchronization state. Also, P_{all} of amplitude death is smaller than three-phase synchronization state. Up to this point, these results are consistent with those of [8]. However, in the region $0 \leqq d \leqq 0.05$, P_{all} of in-phase and anti-phase synchronization state is not larger than three-phase synchronization state. We focus on M_k of transient response time, so we investigate the relationship between memristances M_k and the one memristor parameter d. Figure 7 shows the relationship between M_k and d.



(a) (b)

Figure 7: Relationship between P and d. (a): $0 \leqq d \leqq 1$, (b): $1 \leqq d \leqq 10$.

In the region $0 \leqq d \leqq 0.05$, $M_1 = a = 0.1$. However, $M_1 = b = 10$ in the region $0.1 \leqq d \leqq 10$. We consider that the power consumption of in-phase and anti-phase synchronization state is smaller than three-phase synchronization state when the one memristance is larger than the other two memristances.

4. Conclusions

We have proposed three coupled van der Pol oscillators with different memristor coupling strength. By changing the charge-flux characteristic curve of the one memristor, we have obtained three synchronization states: three-phase synchronization state, in-phase and anti-phase synchronization state and amplitude death. Except for the case that the one memristor coupling is larger than the other couplings, three coupled van der Pol oscillators with different memristor coupling strength tends to synchronize to be minimum energy of the whole circuit model when two memristor couplings are larger than the one memristor coupling. For the future works, we would like to design this circuit model and then we provide the fundamental evidences to do circuit experiments.

References

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