Synchronizations in Three Coupled Oscillators with Memristor Synapses as Ring Structure

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Abstract— **Synchronizations are obtained in coupled oscillators via resistors. Recently, many researchers have focused on a memristor synapse. Few studies have studied synchronizations in coupled oscillators of electronic circuit models with memristor synapses. This study proposes the three coupled oscillators with memristor synapses, and researches various synchronizations changing the characteristics of the memristor.**

Keywords; Memristor; Synchronization; Oscillator

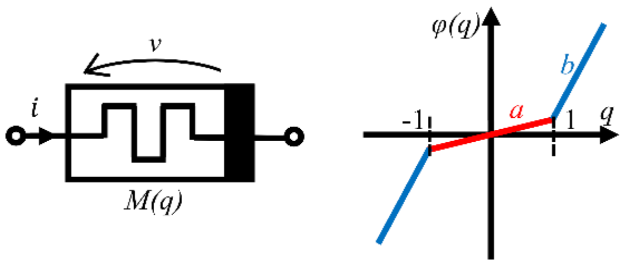
I. Introduction

Synchronization is the interested phenomenon and studied in the several fields. In electrical engineering, this phenomenon was observed in the circuit model [1]. In previous study, various synchronizations were investigated in many kinds of coupled oscillators via resistors [2]. A memristor was proposed, the characteristics of the charge and the flux was defined as the piecewise function by M. Itoh in 2008 [3]. It has dynamics for a resistor because a charge is defined as an integral of a current. The number of synchronization studies in coupled oscillators of electronic circuit models with memristor synapse was much little, so we focus on a memristor synapse.

This study proposed the three coupled oscillators with memristor synapses and investigated various synchronizations changing the characteristics of the memristor.

II. Circuit Model

Figure 1 shows a memristor model and the characteristics of the memristor. The characteristics of the memristor is defined as the gradient of the piecewise function. Resistance value of the memristor is called memristance. This study proposed three coupled oscillators with memristor synapses as shown in Fig. 2.



(a) Schematic symbol. (b) Charge-flux characteristics.

Figure 1. Memristor model.

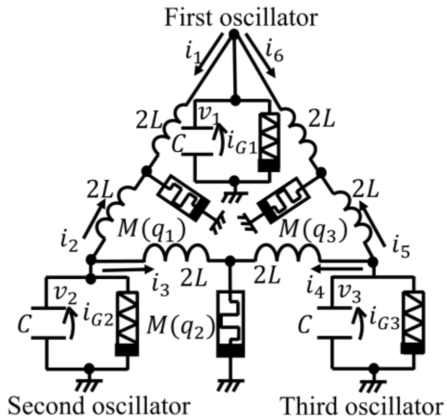


Figure 2. Three coupled oscillators with memristor synapses as ring.

We normalized the circuit equations of the coupled oscillators in Fig. 2 for the computer simulations. The normalized circuit equations describing first oscillators can be written by Eq. (1)

$$\begin{cases} \frac{dx_1}{d\tau} = \varepsilon x_1(1 - {x_1}^2) - y_1 - y_6 \\ \frac{dy_1}{d\tau} = \frac{1}{2}\{x_1 - M(z_1)\gamma(y_1 + y_2)\} \\ \frac{dy_6}{d\tau} = \frac{1}{2}\{x_1 - M(z_3)\gamma(y_5 + y_6)\} \\ \frac{dz_1}{d\tau} = \xi(y_1 + y_2) \end{cases} \quad (1)$$

where M is refined as Eq. (2).

$$M(z_1) = \frac{d\varphi(z_1)}{d\tau} = \begin{cases} a \ (|z_1| < 1) \\ b \ (|z_1| > 1) \end{cases} \quad (2)$$

z_1 is correspond to q_1 in change of the variables, so memristance depends on z_1. M is a when the absolute value of z_1 is smaller than one. On the other hand, M is b when the absolute value of z_1 is larger than one. Similarly, normalized circuit equations describing both second and third oscillators can be obtained.

In these simulations, the normalized circuit equations were calculated by using Runge-Kutta method. The parameters were set up to the nonlinearity $\varepsilon = 0.1$, the coupling strength $\gamma = 1$, the coupling factor $\xi = 1$, the scaling time $\tau = 20000$ and step size of Runge-Kutta method $h = 0.01$. In addition, the variables x, y and z are correspond to the voltage v, the current i and the charge q respectively.

III. RESULTS

A. Anti-phase and aynchronization

We chose the parameters a = 5.0 and b = 0.1 to strengthen one memristance. Three oscillators oscillated in Fig. 4 (a) ~ (c). First and second oscillators were anti-synchronized in Fig. 4 (d) and (g). On the other hand, third oscillator was asynchronized with the other oscillators in Fig. 4 (e) ~ (g). In Fig. 4 (h), M_1 was b, and M_2 and M_3 was a constantly. Coupling strength of the synapse between first and second oscillators was much larger than the other coupling parts. Therefore, first and second oscillators were anti-synchronized. In addition, third oscillator was not synchronized with first and second oscillators.

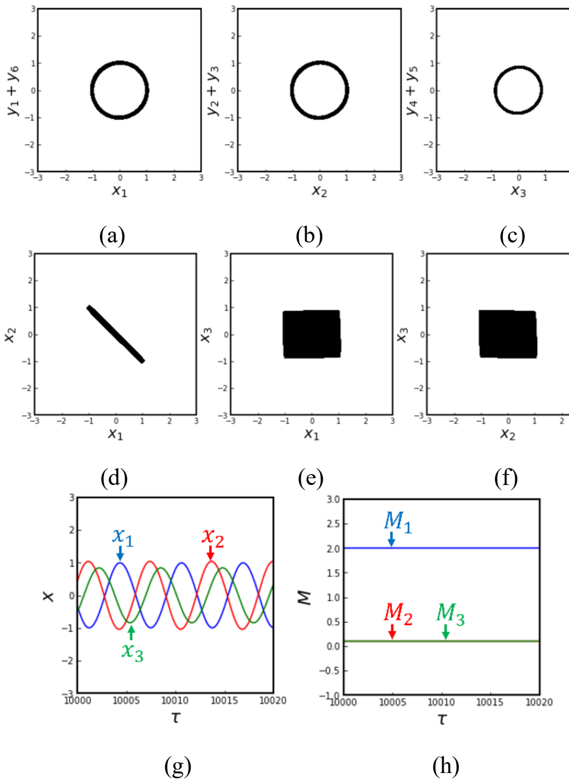


Figure 4. Anti-phase and asynchronization. (a) First oscillator attractor (x_1 vs $y_1 + y_6$). (b) Second oscillator attractor (x_2 vs $y_2 + y_3$). (c) Third oscillator attractor (x_3 vs $y_4 + y_5$). (d) Phase difference (x_1 vs x_2). (e) Phase difference (x_1 vs x_3). (f) Phase difference (x_2 vs x_3). (g) Time-series of x. (h) Time-series of M. $a = 5.0$, $b = 0.1$, initial values $x_1 = 1.0$, $x_2 = 1.1$, $x_3 = 1.2$, $y_1 = 2.0$, $y_2 = 2.1$, $y_3 = 2.2$, $y_4 = 2.3$, $y_5 = 2.4$, $y_6 = 2.5$, $z_1 = 1.0$, $z_2 = 1.1$, $z_3 = 1.2$.

B. Anti-phase and oscillation stop

We chose the parameters a = 0.1 and b = 10 to strengthen two memristances. First oscillator stopped oscillating in Fig. 5 (a). Second and third oscillators oscillated in Fig. 5 (b) and (c). Second and third oscillators were anti-synchronized in Fig. 5 (f). In Fig. 5 (h), M_1 and M_3 were b, and M_2 was a constantly. Coupling strength of the synapse between second and third oscillators was much smaller, so they were synchronized. Besides, first oscillator stopped oscillating.

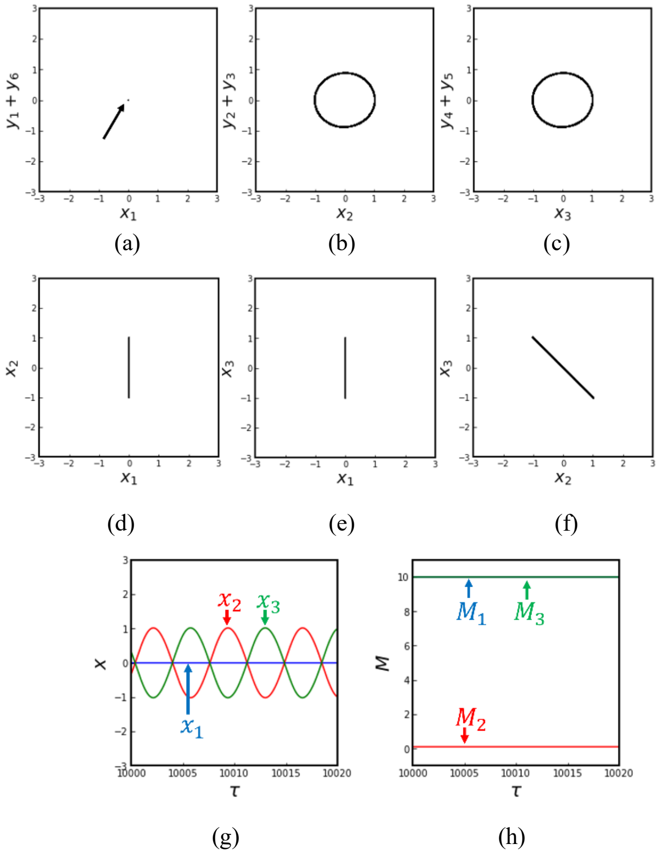


Figure 5. Anti-phase and oscillation stop. (a) First oscillator attractor (x_1 vs $y_1 + y_6$). (b) Second oscillator attractor (x_2 vs $y_2 + y_3$). (c) Third oscillator attractor (x_3 vs $y_4 + y_5$). (d) Phase difference (x_1 vs x_2). (e) Phase difference (x_1 vs x_3). (f) Phase difference (x_2 vs x_3). (g) Time-series of x. (h) Time-series of M. $a = 0.1$, $b = 10$, initial values $x_1 = -1.0$, $x_2 = -1.1$, $x_3 = -1.2$, $y_1 = 2.0$, $y_2 = 2.1$, $y_3 = 2.2$, $y_4 = 2.3$, $y_5 = 2.4$, $y_6 = 2.5$, $z_1 = 1.0$, $z_2 = 1.1$, $z_3 = 1.2$.

IV. CONCLUSONS

In this study, we proposed three coupled oscillators with memristor synapses and researched two types of synchronizations changing the characteristics of the memristor. As a result, anti-phase and asynchronization were obtained. Also, anti-phase and oscillation stop occurred by giving synapses dynamics.

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