Synchoronization Penomena of Coupled Oscillators in Weighted Three-Dimensional Complex Networks

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Abstract— **Complex networks have received a great deal of attention and have been studied in a variety of fields. However, most of these studies consider two-dimensional networks and analyze them by weighting edges. In this study, we extend the network to three dimensions and analyze the network from the viewpoint of synchronization phenomena for complex networks composed of van der Pol oscillators. Then, the coupling strength is set using the Euclidean distance in three-dimensional space and we compare synchronization rate with the two-dimensional network.**

Keywords; Oscillator; Synchronization; Conplex networks

I. INTRODUCTION

Complex networks have been a focal point in various disciplines, including sociology, biology, and engineering [1]-[3]. Within engineering, there's an evolving study of these networks using circuit models, and the phenomenon of synchronization among these circuits has become a captivating subject [4],[5]. While past research has demonstrated that a network's structure affects its synchronization, much of this work has been centered on two-dimensional networks [6].

In this study, we built a three-dimensional scale-free network model consisting of 100 van der Pol oscillators to delve into synchronization phenomena. In this configuration, we modulated the coupling strength of the network based on the nodes' degree and the edges' Euclidean distance. We anticipate this will enhance our comprehension of synchronization behaviors in a three-dimensional context.

II. SYSTEM MODEL

Figure 1 shows the van der Pol oscillator used in this study. This oscillator is a simple circuit consisting of only a capacitor, inductor, and nonlinear elements. Figure 2 shows the model of the three-dimensional scale-free network used Barabasi Albert model (BA model) in this study. The circuit in Fig. 1 corresponds to the nodes in Fig. 2. When these van der Pol oscillators are connected with resistors, a complex network is constructed as shown in Fig. 2.

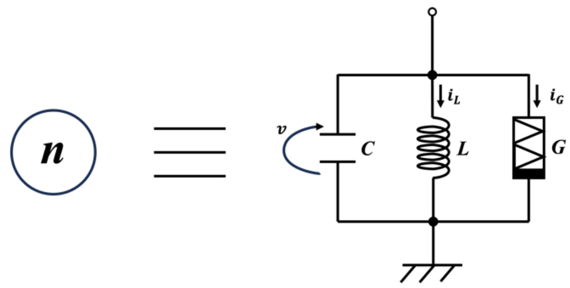

Figure 1. van der Pol oscillator.

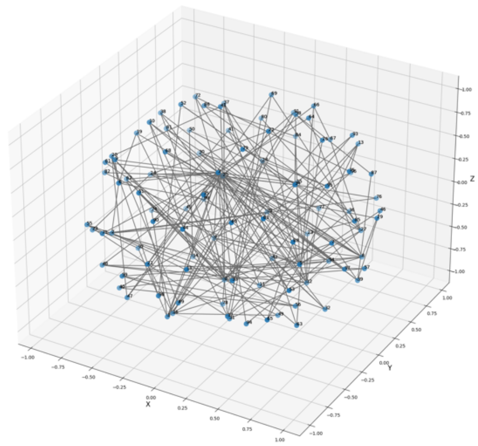

Figure 2. Barabasi Albert model (BA model) in three-dimensional space.

From the circuit depicted in Fig. 1, we derive the characteristic equation for the nonlinear element as well as the circuit equation. Utilizing the normalization parameter and variables, we can represent the circuit equation in its normalized form as follows:

$$\begin{cases} \dfrac{dx_n}{d\tau} = \alpha\left\{\varepsilon x_n(1-x_n^2) - y_n - \displaystyle\sum_{n,k=1}^{100} E_{nk}\gamma_{nk}(x_n - x_k)\right\} \\ \dfrac{dy_n}{d\tau} = x_n \qquad\qquad (n, k = 1, 2, 3, \cdots, 100). \end{cases} \tag{1}$$

The adjacency matrix of the network is denoted by E_{nk}. It's a matrix that confirms whether there's a connection between node n and node k. If nodes n and k are linked, E_{nk} is set to 1; otherwise, it's set to 0. The coupling intensity, γ_{nk} , is ascertained by applying the parameter q in the following manner:

$$\gamma_{nk} = \frac{1}{R_{nk}}\sqrt{\frac{L}{C}} = \frac{q \cdot w_n \omega_k}{(d_{nk})^2} . \qquad (2)$$

Here, d_{nk} is the Euclidean distance of the edge between node n and node k, and w is the degree of each node.

III. Result

In this study, the parameter of the van der Pol oscillator is set to $\varepsilon = 0.1$. Moreover, α represents the small error of the capacitor, in the range of [0.975:1.025] in increments of 0.0005. The parameter q, which determines the coupling strength, is set so that the coupling strength takes close to $\gamma = 0.5$ for each network.

Figure 3 shows the synchronization rates in two and three dimensions. The blue box shows the synchronization rate in two dimensions, and the green box shows the synchronization rate in three-dimensions. In two-dimensional networks, synchronization rates tend to be biased toward 0-10% and 90-100%. However, by changing from two dimensions to three dimensions, the frequency of nodes with 0-10% synchronization rate decreased and the frequency of nodes with 10% or more synchronization rate increased.

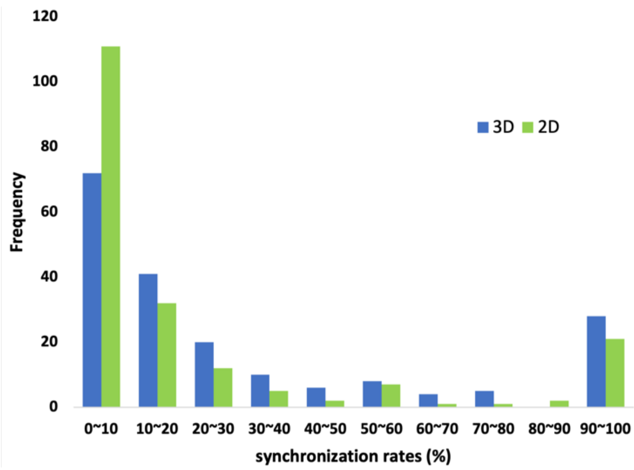

Figure 3. Synchronization rate distribution in two-dimensional and three-dimensional networks.

Table 1. Average coupling strength and synchronization rate in two-dimensional and three-dimensional networks.

	average coupling strength	*average synchronization rate*
2D	**0.510**	**24.83 [%]**
3D	**0.507**	**28.61 [%]**

Table 1 shows the average synchronization rates in two-dimensional and three-dimensional. We confirm that the average synchronization rate increased by about 4% by changing the network from two-dimensional to three-dimensional.

IV. Conclusion

In this study, we constructed two-dimensional and three-dimensional networks of the BA model with 100 van der Pol oscillators to investigate the synchronization phenomena between circuits. For the weighting of the network, we focused on the Euclidean distance of the edges and the degree of the node. As a result, there was a large tendency for synchronization rates to be biased toward 0-10% and 90-100% in the two-dimensional network, but this tendency was reduced when the network was extended to three dimensions.

In the future, we would like to approach the three-dimensional network closer to a brain model. Changing the network over time, the coupling strength of the edges changes with time. We can expect it to approach the synapses in the brain.

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