Rewiring Effect of High Synchronization Edges in Complex Oscillator Networks

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Abstract— **It is a random network that is constructed with 100 van der Pol oscillators. The synchronization phenomena between the circuits of the networks are analyzed. Then, focusing on the synchronization rate and changing the topology accordingly, the simulation is performed. It was confirmed that the synchronization rate changes when the topology is changed.**

Keywords; Switching; Complex networks; Synchronization; oscillator

I. Introduction

Complex networks are complex and huge networks that exist in reality. It has three properties that are scale-free, small-world, and clustered, and it is able to be reproduced using nodes and edges. This has attracted attention from various fields, including communications, engineering, economics, and others [1]-[3]. Especially in the engineering field, there have been interesting research such as synchronization phenomena between circuits [4]. This synchronization phenomena are a phenomenon in which the circuits become aligned with each other under the influence of surrounding objects [5].

In the previous study, it was investigated the synchronization phenomena which are caused by differences in the structure of the network. In this study, however, we focus on the synchronization rate and investigate changes in synchronization phenomena caused by differences in the synchronization rate. A random network is constructed using 100 van der Pol oscillators. By using arbitrary synchronization rates as thresholds, we investigate how the synchronization phenomena change when the network topology is changed by cutting edges. The network topology is changed by setting the threshold value to $70\sim100\%$ and cutting off edges with high synchronization rates above the threshold value. In addition, the coupling strength is changed from $\gamma = 0.2$ to $\gamma = 0.5$ for each threshold value, and how the synchronization phenomena change is investigated.

II. System Model

In this study, we use the Erdös Rényi model (ER model), which is one of the random networks. This network has 100 nodes, and the average degree is set to near 5.3. Also, the model is shown in Fig.1. It has the property that the cluster coefficients are small and the difference that the number of edges gathered at each node is small.

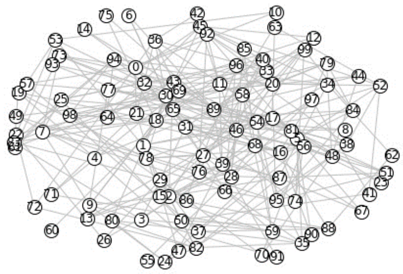

Figure 1. ER model

III. Circuit Model

The van der Pol oscillator is used in this study. It is shown in Fig.2. The circuit is a simple circuit that consists only of a capacitor, an inductor, and a nonlinear element. These van der Pol oscillators are connected using resistors to reproduce the network.

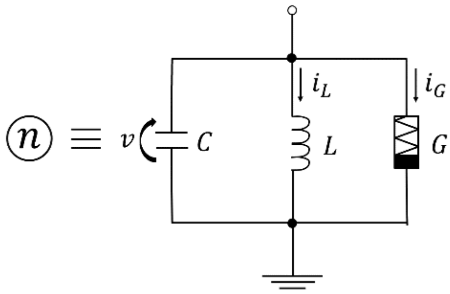

Figure 2. van der Pol oscillator

The normalized circuit equation is shown in Eq. (1).

$$\begin{cases} \dfrac{dx_n}{d\tau} = \alpha\left\{\varepsilon x_n(1-{x_n}^2) - y_n - \displaystyle\sum_{n,k=1}^{100} E_{nk}\gamma(x_n - x_k)\right\} \\ \dfrac{dy_n}{d\tau} = x_n \end{cases} \quad (1)$$

$$(n, k = 1,2,\cdots,100).$$

In this equation, α is the small error of each capacitor and E is the adjacency matrix of the network.

The network topology is changed by cutting edges and be investigated how the synchronization phenomena change.

IV. RESULTS

First, we define synchronization in this study. We calculate the synchronization rate when $\tau = 10000 \sim 20000$, which is the steady state of the voltage difference. We set the threshold value as shown in Eq. (2) and judge that the nodes are synchronized when they are within the range. In this equation, n and k are defined as the number of circuits.

$$|x_n - x_k| < 0.01. \quad (2)$$

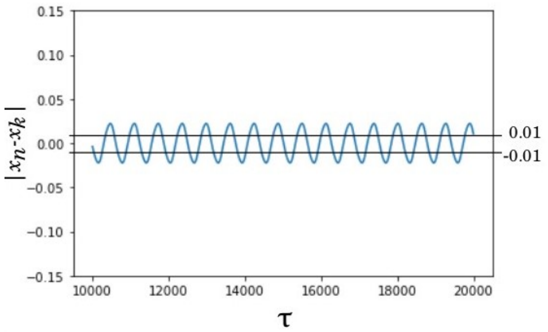

Figure 3. Voltage difference waveform

The changes of the average synchronization rates when the network topology is changed are shown in Fig.4. The simulation was conducted with the threshold set to 100% and the coupling strength changed from $\gamma = 0.2$ to $\gamma = 0.5$ in steps of 0.1. The results show that the synchronization rate is higher when the coupling strength is stronger. In addition, the synchronization rates tend to increase when the topology is changed.

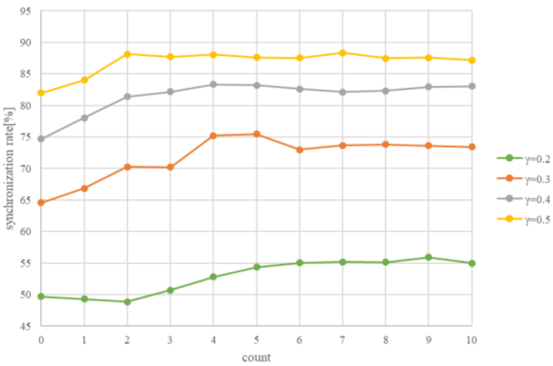

Figure 4. Changes of the average synchronization rate

Furthermore, the relationship between the average synchronization rate and total number of cutting edges is shown in Fig.5. It shows that the number of cutting edges increases when the average synchronization rate is higher.

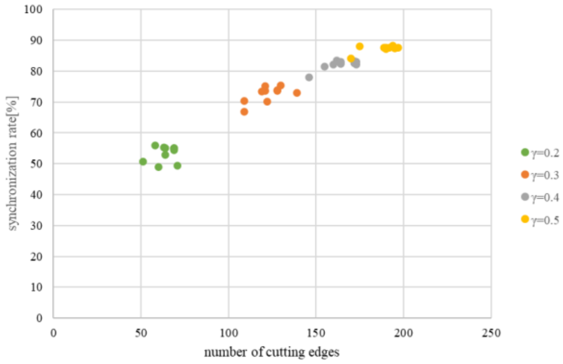

Figure 5. The relationship between the average synchronization rate and total number of cutting edges

V. CONCLUSION

In this study, we conducted research restructuring the network through cutting edges with high synchronization rates. Consequently, the average synchronization rate tends to be higher. It was also found an average synchronization rate became higher when the number of cutting edges was larger.

In the future, we would like to investigate the trends by conducting the simulation several times instead of just once. In addition, we would like to investigate dynamic simulations by moving the simulation time and change the conditions of connecting the edges.

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