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Frustrated Synchronization of Coupled Oscillators Using Polygonal Structures

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Abstract— In this study, we investigate an effect of frustration to triangular oscillatory network with strong coupling. Frustration as environmental factor is occurred by network topology which is composed from polygonal structure. We focus on the amplitude change of the proposed network using different frustration levels. By using computer simulations, the effect of frustration to triangular oscillatory networks with strong coupling is shown.

1. Introduction

Synchronization phenomena observed in coupled oscillatory systems are excellent for modeling high-dimensional nonlinear phenomena. In recent years, the synchronization phenomenon of oscillators and nonlinear circuits in related with complex networks has been the subject of significant research [1]-[10]. These research results are also expected to be used for engineering applications, such as the constructions of optimum communication transmission networks [11].

Our research group has been studying synchronization phenomena observed in polygonal networks using coupled oscillators [12]. The proposed model is that two odd polygons are shared by a single branch. Because two adjacent oscillators are coupled to be anti-phase states, the dilemma frustrates the oscillators in odd numbered polygonal networks. In our previous works, we have investigated the amplitude and phase changes in polygonal oscillatory networks when the coupling strength is changed. These amplitude and phase have also been solved by using numerical analysis of the averaging method, and the amplitude death has been observed.

In order to approach a more realistic network models, we extend the two coupled polygonal network model to two-dimensional space using 20 oscillators. The coupling topology is to be a triangulra network, and we investigate the ampliutde change with strong coupling. By using the computer simulations, we confrim that frastration changes depnding on the location of the oscillator.

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2. Network Model using van der Pol Oscillators

The conceptual network models with different frustration levels used in this study is shown in Fig. 1. In this figure, triangular oscillators are coupled with edges on 2-dimensional space and a circle denotes a van der Pol oscillator.

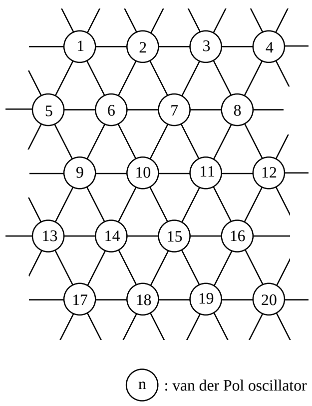

Figure 1: Network model with triangular oscillators.

Figure 2 shows a van der Pol oscillator. This oscillator is composed by an inductor, a negative resistance and a condenser. The oscillator component is very simple, however the oscillator could generate oscillation time wave. When the parameter of the nonlinearity is set to small value (e.g. ε=0.1), time wave form behaves similar to sin wave.

The circuit realization (target on 1st, 5th and 6th oscillators) of triangular oscillatory networks is shown in Fig. 3. In this circuit model, we use unique coupling method for two adjacent oscillators. Two adjacent oscillators are coupled by a resistor via a inductor which originally belongs to each van der Pol oscillators. The inductor of one van der Oscillator is divided to six to connect the next oscillators. If the oscillators are located at boundary position, the inductors connect to the earth resistance via resistor R.

By using this coupling scheme, two oscillators tend to synchronize at anti-phase state. However, in the triangular oscillatory network, two oscillators can not synchronized with anti-phase state because of the network structure. Then, the coupled oscillators synchronize with phase difference to minimize the enegy consumption. This is really original part compared with the other networks focusing on synchronization.

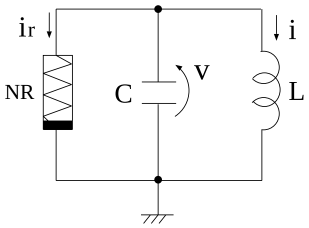

Figure 2: van der Pol oscillator.

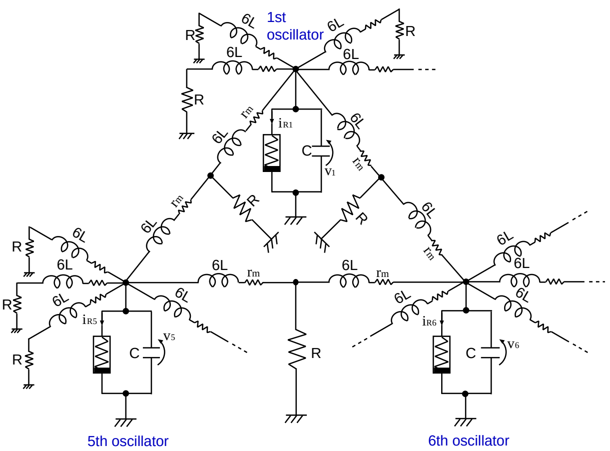

Figure 3: Circuit realization for 1st, 5th and 6th oscillators in Fig. 1.

We develop the expression of the circuit equations of this model. The $v_k - i_{Rk}$ characteristics of the nonlinear resistor are assumed to be the following third order polynomial equation;

$$i_{Rk} = -g_1 v_k + g_3 {v_k}^3 \quad (g_1, g_3 > 0),\ (k = 1, 2, 3, 4). \qquad (1)$$

By using the variables and the parameters,

$$t = \sqrt{LC}\tau, \quad v_k = \sqrt{\frac{g_1}{3g_3}} x_k,$$

$$i_n = \sqrt{\frac{g_1}{3g_3}} \sqrt{\frac{C}{L}} y_n,$$

$$\varepsilon = g_1 \sqrt{\frac{L}{C}}, \quad \gamma = R\sqrt{\frac{C}{L}}, \quad \eta = r_m \sqrt{\frac{C}{L}},$$

The normalized circuit equations governing the circuit are expressed as

$$\begin{cases} \dfrac{dx_k}{d\tau} = \varepsilon\left(1 - \dfrac{1}{3}{x_k}^2\right)x_k - (y_{ak} + y_{bk} + y_{ck}) \\ \dfrac{dy_{ak}}{d\tau} = \dfrac{1}{6}\left\{x_k - \eta y_{ak} - \gamma(y_{ak} + y_n)\right\} \\ \dfrac{dy_{bk}}{d\tau} = \dfrac{1}{6}\left\{x_k - \eta y_{bk} - \gamma(y_{bk} + y_n)\right\} \\ \dfrac{dy_{ck}}{d\tau} = \dfrac{1}{6}\left\{x_k - \eta y_{ck} - \gamma(y_{ck} + y_n)\right\} \\ \dfrac{dy_{dk}}{d\tau} = \dfrac{1}{6}\left\{x_k - \eta y_{dk} - \gamma(y_{dk} + y_n)\right\} \\ \dfrac{dy_{ek}}{d\tau} = \dfrac{1}{6}\left\{x_k - \eta y_{ek} - \gamma(y_{ek} + y_n)\right\} \\ \dfrac{dy_{fk}}{d\tau} = \dfrac{1}{6}\left\{x_k - \eta y_{fk} - \gamma(y_{fk} + y_n)\right\} \end{cases} \qquad (2)$$

In these equations, γ is the coupling strength, ε denotes the nonlinearity of the oscillators. For the computer simulations, we calculate Eq. (3) using the fourth-order Runge-Kutta method with the step size $h = 0.005$. The parameters of this circuit model are fixed as $\varepsilon = 0.1$ and $\eta = 0.0001$.

3. Simulation Results

For the computer simulations, 20 van der Pol oscillators are coupled in triangular oscillatory space like Fig. 1. By changing the coupling strength (γ) the network, the obtained amplitude of coupled oscillators are investigated.

Figures 4 to 8 show the simulation results of the obtained amplitude when the coupling strength is changed from 0.1 to 30.0. In the case of the weak coupling, the amplitude of all oscillators are the same (see. Fig. 4). By increasing the coupling strength, first, the amplitude of 6th, 11th and 14th oscillators decreases (see. Fig. 5). Then, the amplitude death of 11th oscillator can be confirmed when the coupling strength is set to $\gamma = 2.6$ (see. Fig. 6).

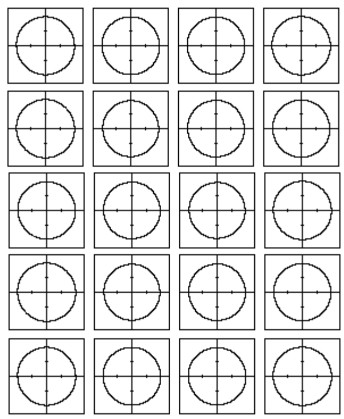

Figure 4: Attractors of 20 oscillators coupling network ($\gamma = 0.1$).

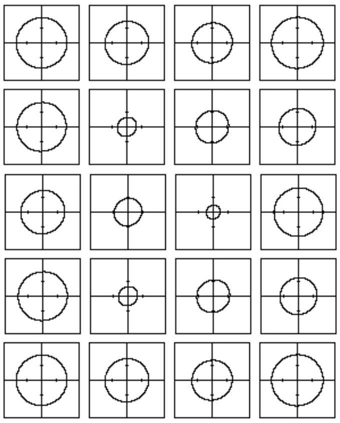

Figure 5: Attractors of 20 oscillators coupling network (γ = 1.0).

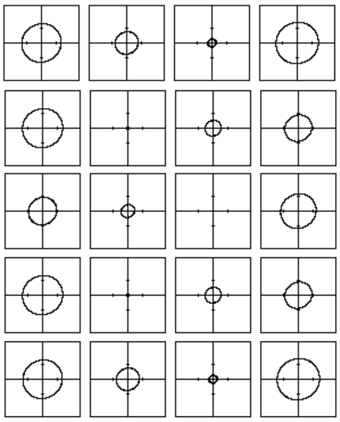

Figure 6: Attractors of 20 oscillators coupling network (γ = 2.6).

When the coupling strength is further increased, the amplitude of the oscillators located in the middle is reduced (see. Fig. 7). Finally, it is confirmed that the oscillators located on the outer side of the system produce torus-like oscillations (see. Fig. 8). In this network model, we confirm only the partial amplitude death and could not observe the amplitude death of whole network.

Next, we investigate the time wave forms of 20 oscillators when the coupling strength is set to very large as shown in Figs. 9 and 10. In the case of γ = 10.0, it responds to frustration with a decrease in the amplitude of the inner oscillators and the variation in the amplitude of the outer oscillators. In the case of γ = 30.0, it responds to frustration by producing like a torus attractor in the outer oscillators.

4. Conclusions

In this study, we have investigated the effect of frustration to triangular oscillatory network with strong coupling. Frustration as environmental factor is occurred by network topology which is composed from polygonal structure. We investigated on the amplitude change of the proposed network using different frustration levels. By using computer simulations, we confirm that the effect of frustration to triangular oscillatory networks with strong coupling.

For the future works, we need to investigate the phase states at various frustration levels. Increasing the number of oscillators and investigating the case of larger networks is also one of a future task.

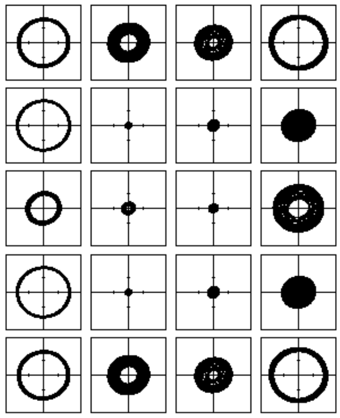

Figure 7: Attractors of 20 oscillators coupling network (γ = 10.0).

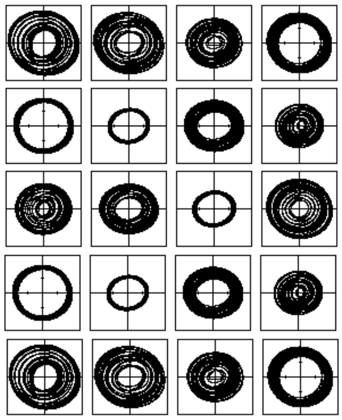

Figure 8: Attractors of 20 oscillators coupling network (γ = 30.0).

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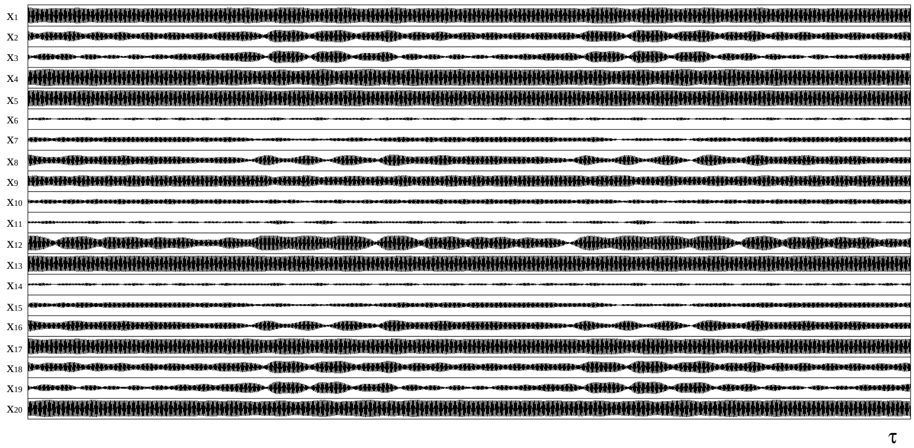

Figure 9: Time wave forms of 20 oscillators coupling network (γ = 10.0).

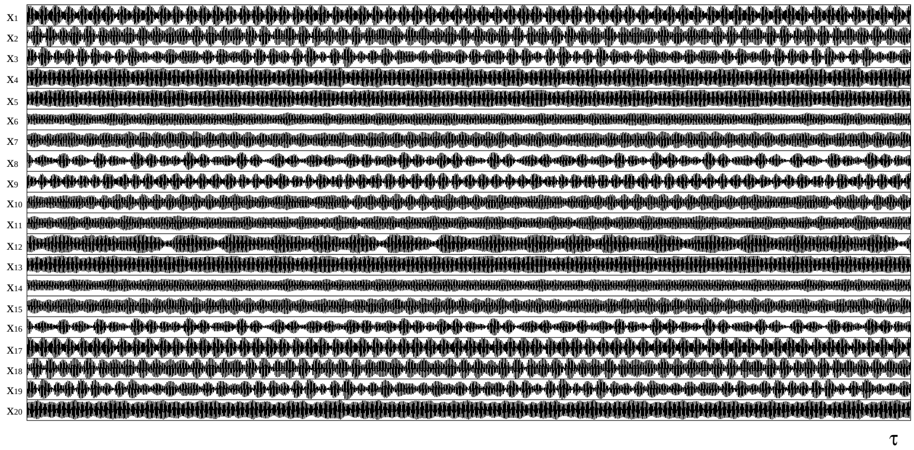

Figure 10: Time wave forms of 20 oscillators coupling network (γ = 30.0).

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