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Analysis of Hierarchical Hetero Associative Memory Using van der Pol Oscillator Synchronization

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Abstract– In this study, we proposed a new method of hetero-associative memory using the synchronization phenomenon of hierarchically connected van der Pol oscillators. As a result, we confirmed that the model can be configured with any number of oscillators for input and output. We also confirmed that this model has noise correction capability as well as an auto-associative memory model. In this study, we measured the noise correction capability of the proposed model and further examined its storage capacity.

1. Introduction

In nature, various synchronous phenomena exist, such as the luminescence of fireflies and the movement of pendulums. Similarly, synchronization phenomena based on phase differences have been observed in coupled oscillator systems [1]. Interestingly, it has been proven that pattern recognition and associative memory can be realized by utilizing the synchronization phenomena of oscillators [2]-[4]. This method differs from the currently widely used Neumann-type computers, which perform retrieval of stored information by inputting information similar to the stored information. Since the elements perform operations in parallel at this time, the robustness of the elements and savings in the number of operations are expected.

Therefore, our previous work proposed to realize pattern recognition and associative memory of images by adding a new memory matrix based on association to the coupling strength between van der Pol oscillators [5]-[6]. There are two patterns of associative memory depending on the pair of input and output patterns to be stored. One is when the input and output patterns are the same, this associative memory is called auto-associative memory. The other is when the input and output patterns are different, and this associative memory is called hetero-associative memory. Hetero-associative memory is realized by forming a memory matrix with different pattern pairs of input and output patterns. In our previous work, we proposed a hetero-associative memory model based on the synchronization phenomenon of van der Pol oscillators [7]. However, this model uses the same oscillator for input and output. As a result, there is a restriction that the same number of oscillators must be used for input and output. Therefore, we propose a new hierarchical mutual recall associative memory model consisting of an input layer and an output layer using van der Pol oscillators.

Furthermore, this model can correct noise as well as the auto-associative memory model. Therefore, we numerically evaluate the recall patterns when noise is inserted into the input patterns in Section IV.

We also evaluate the recall rate when the number of patterns to be memorized is increased, and investigate the relationship between the recall rate and the time to complete recall. This is discussed in Section V.

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2. Hierarchical Hetero Associative Memory by van der Pol Oscillators

In this section, we explain the proposed model.

First of all, Fig. 1 shows the previous study's hetero-associative memory model. Each cell consists of van der Pol oscillators, and all oscillators are coupled to each other via resistors. Proper setting of the coupling strength based on the input pattern and the pattern to be stored enables pattern recognition and recall. In this model, we confirmed that different patterns can be recalled at the input and output. However, it has the limitation that the same number of oscillators must be used for input and output because the same oscillator coupling system is used for input and output.

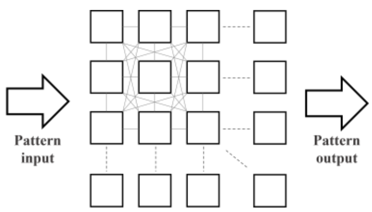

Figure 1: Associative memory model with mutually coupled oscillators.

Therefore, we propose a hetero-associative memory model with hierarchically connected van der Pol oscillators, as shown in Fig. 2 In this model, each cell consists of a van der Pol oscillator. In the input layer, each oscillator is mutually coupled as in previous studies. Furthermore, this model has an output layer, where each oscillator in the input layer is coupled to all oscillators in the output layer. This hierarchical structure allows the use of different numbers of patterns for inputs and outputs.

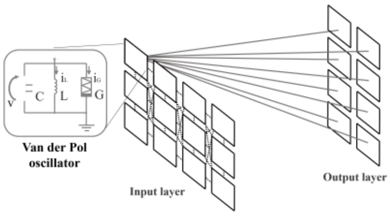

Figure 2: Hierarchical structure model consisting of input and output layers.

The normalized circuit equations are described as follows.

$$\begin{cases} \dfrac{dx_n}{d\tau} = \varepsilon x_n(1 - {x_n}^2) - y_n - \displaystyle\sum_{k=1}^{N} \boldsymbol{K}(x_n - x_k) \\ \dfrac{dy_n}{d\tau} = x_n \end{cases} \tag{1}$$

N is the number of oscillators to be connected to. Also, K is defined as

$$\boldsymbol{K} = \begin{cases} \boldsymbol{E_0} \times s \\ \boldsymbol{E} \times s \end{cases} \tag{2}$$

s is the coupling strength, and E_0 and E are the accumulation matrices determined based on the input and output sides of the storage pattern. Pattern creation was performed with black as -1 and white as 1. Let x_i denote the input pattern, a_i the input side of the storage pattern, and b_i the output side of the storage pattern.

$$\boldsymbol{x_i} = (\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3}, \cdots, \boldsymbol{x_N}) \tag{3}$$

$$\boldsymbol{a_i} = (\boldsymbol{a_1}, \boldsymbol{a_2}, \boldsymbol{a_3}, \cdots, \boldsymbol{a_N}) \tag{4}$$

$$\boldsymbol{b_i} = (\boldsymbol{b_1}, \boldsymbol{b_2}, \boldsymbol{b_3}, \cdots, \boldsymbol{b_N}) \tag{5}$$

Taking Figure 3 as an example, a_i corresponds to the inputs, “dog” and “flower”, and b_i corresponds to the outputs, “D” and “F”. Using these, a memory matrix can be created as shown in Eq. (7). Also, x_i represents the actual inputs. For example, in Section 4, noise is included in the inputs, and in this case, the noise pattern corresponds to x_i. If the number of pairs of patterns to be stored is P, the storage matrix is expressed as follows.

$$\boldsymbol{E_0} = \boldsymbol{x_i}{\boldsymbol{x_i}}^{\boldsymbol{T}} = \begin{pmatrix} \boldsymbol{x_1x_1} & \boldsymbol{x_1x_2} & \cdots & \boldsymbol{x_1x_N} \\ \boldsymbol{x_2x_1} & \boldsymbol{x_2x_2} & \cdots & \boldsymbol{x_2x_N} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{x_Nx_1} & \boldsymbol{x_Nx_2} & \cdots & \boldsymbol{x_Nx_N} \end{pmatrix} \tag{6}$$

$$\boldsymbol{E} = \sum_{k=1}^{P} \boldsymbol{a_i^k}\boldsymbol{b_i^k}^{\boldsymbol{T}} = \sum_{k=1}^{P} \begin{pmatrix} \boldsymbol{a_1^kb_1^k} & \boldsymbol{a_1^kb_2^k} & \cdots & \boldsymbol{a_1^kb_N^k} \\ \boldsymbol{a_2^kb_1^k} & \boldsymbol{a_2^kb_2^k} & \cdots & \boldsymbol{a_2^kb_N^k} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{a_N^kb_1^k} & \boldsymbol{a_N^kb_2^k} & \cdots & \boldsymbol{a_N^kb_N^k} \end{pmatrix} \tag{7}$$

Next, the input and output methods of the pattern are described.

Step 1: As shown in Fig. 2, the input layer is mutually coupled with each oscillator. From Eq. (2), the memory matrix E_0 of K is used to recognize the input pattern.

Step 2: Connect the input layer and the output layer. At this time, the oscillators of all input layers are interconnected with the oscillators of all output layers, and from Eq. (2), the output pattern is recalled by using the memory matrix of E for K.

Step 3: Check the phase of the voltage of each oscillator in the output layer. At this time, the difference between the phase of the voltage of each oscillator and that of the first oscillator is checked to determine whether they are synchronous or asynchronous. The output pattern is white if the oscillators are synchronized, and black if they are not. Note that the selection of the reference oscillator may result in the output of an inverted pattern, depending on the input pattern.

In addition, a threshold value is used to determine whether or not the synchronization is achieved. In this simulation, the threshold value was set to 1.0.

3. Simulation Results

We trained a 24×24 pattern of a dog and a flower in the input layer and a 14×12 pattern of the initials "D" and "F" in the output layer. Figure 3 shows the patterns of the learned pairs.

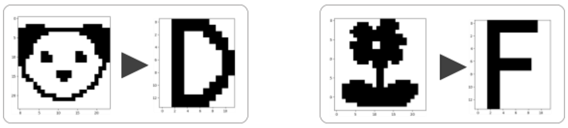

Figure 3: The patterns of the learned pairs (dog and flower).

In this study, simulations were performed using the Runge-Kutta method to analyze the oscillator behavior. Also, the step size is fixed with h=0.1. The variables in Eq. (1) are ε=0.1, s=0.002, $\tau \leqq 40$ in step1, $40 < \tau$ in step2.

Figure 4 shows the phase difference between the voltage of the reference first oscillator and the other oscillators in the output layer when the dog pattern is input.

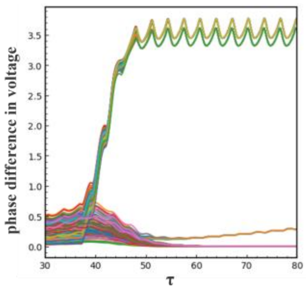

Figure 4: The phase difference in voltage.

Figure 5 shows the "D" recall process when inputting the dog pattern. It can be seen that "D" recalls the pattern normally.

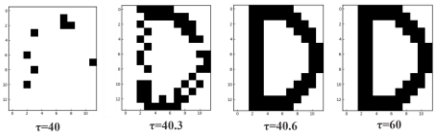

Figure 5: "D" recall process.

Figure 6 shows the recall process of "F" when the flower pattern is inputted without changing the learned memory matrix. It can be confirmed that "F" is successfully recalled as in the case of the dog pattern.

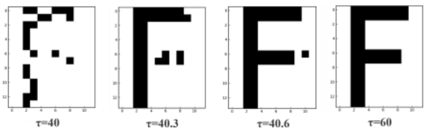

Figure 6: "F" recall process.

Figures 5 and 6 confirm that different numbers of inputs and recalls are possible by using input and output layers.

4. Numerical Evaluation of Noise Insertion

To numerically evaluate the characteristics of the output patterns when noise is inserted into the input patterns, we defined the degree of similarity ρ between the patterns.

If the two patterns at this time are Ai and Bi, the similarity ρ is expressed as follows.

$$\rho = \frac{1}{N}\sum_{i=1}^{N} A_i B_i \quad \textbf{(8)}$$

When two patterns are similar, the similarity ρ has a large value, and when they are the same, ρ=1.

Figure 7 shows the input noise patterns. 5 types of patterns with increasing noise in sequence. The percentage of noise inserted was quantified by the similarity between the noise pattern and the exact dog pattern. The noise was created by inverting the black-and-white pattern. The locations of the pattern to be inverted were selected where the pattern was not close to the floral pattern. As a result, when a large amount of noise was inserted, the pattern was inverted from the flower pattern.

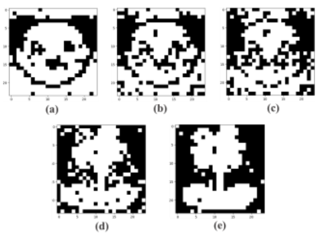

Figure 7: input patterns with noise. (a) ρ=0.90 (b) ρ=0.70 (c) ρ=0.50 (d) ρ=0.30 (e) ρ=0.11

The similarity between the pattern recalled when each noise pattern was input and the normal "D" pattern was calculated as the recall rate and shown in Fig. 8. It can be seen that the recall rate worsens as the noise increases. Finally, in Fig. 7(e), the recall was 4.5. At this point, we confirmed that the recall pattern was "F" because the input was close to the inverted pattern of the flower pattern. This is because, by definition, if "F" is recalled instead of "D," the recall rate is not 0, but 4.5, the similarity between "F" and "D."

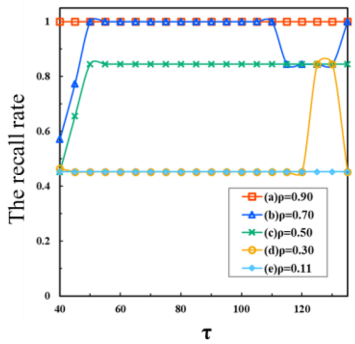

Figure 8: The recall rate for noise pattern input.

5. Evaluation of Recall Rate Change with Number of Learning Patterns

The third pairs of star and “S” patterns were learned to simulate recall. Figure 9 shows the patterns of the learned pairs.

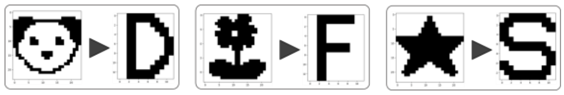
Figure 9: The patterns of the learned 3 pairs.

Figure 10 shows the phase difference between the voltage of the reference first oscillator and the other oscillators in the output layer when the dog pattern is input to the learning model in the three-pair pattern.

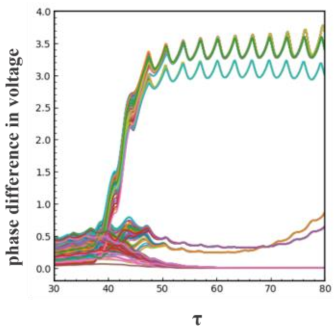

Figure 10: The phase difference between the voltage.

Figure 11 shows the "D" recall process when inputting the dog pattern. It can be seen that "D" recalls the pattern normally.

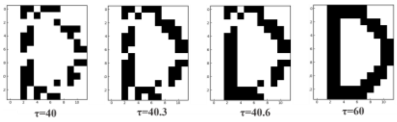

Figure 11: "D" recall process.

Figures 12 and 13 show the recall process of "F" and "S" when the flower pattern and flower pattern are input without changing the learned memory matrix. As in the case of the dog pattern, it can be seen that the "F" and "S" are successfully recalled.

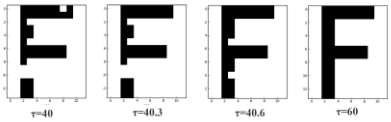

Figure 12: "F" recall process.

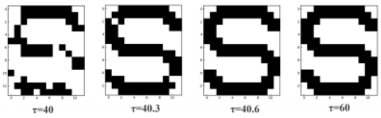

Figure 13: "S" recall process.

Next, we conducted a numerical analysis of recall patterns when the number of pairs of patterns to be learned was increased. In addition to the second set of dog and flower patterns proposed in section III, the third set of star patterns, the fourth set of apple patterns, and five sets of heart patterns were used to simulate an increased number of learning pattern pairs. The newly added apple and heart patterns are shown in Fig. 14.

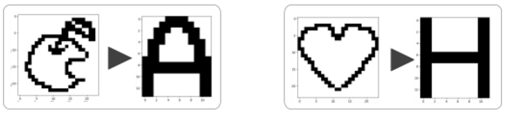

Figure 14: Additional learned patterns.

Figure 15 shows the respective recall rates for each of the dog patterns when the number of patterns to be learned was increased when the dog patterns were input. It can be seen that when the number of patterns to be learned was increased, the time to complete recall increased. Also, when the number of pairs was increased to five, complete recall became more difficult.

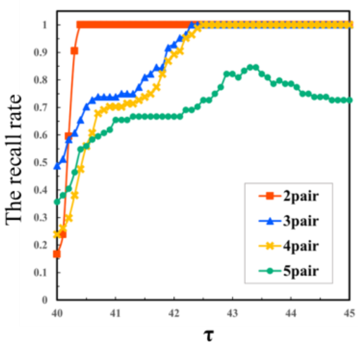

Figure 15: The respective recall when the number of patterns to be learned were increased.

6. Conclusions

In this study, we analyzed a newly proposed hierarchical hetero-associative memory model. We confirmed that the hierarchical model can handle different numbers of data by using different numbers of oscillators for input and output. In addition, since this model can correct noise, we evaluated the recall rate for noise patterns.

Furthermore, we investigated the effect of increasing the number of training patterns on recall. Increasing the number of patterns to be learned increased the time to recall, and increasing the number of patterns prevented complete recall. In future studies, we intend to investigate the relationship between the number of oscillators and memory capacity.

References

[1] S. K. Joshi, S. Sen, and I. N. Kar, "Synchronization of Coupled Oscillator Dynamics," IFAC-PapersOnLine, Vol. 49-1, pp. 320-325, 2016.

[2] P. Maffezzoni, B. Bahr, Z. Zhang and, L. Daniel, "Oscillator Array Models for Associative Memory and Pattern Recognition," IEEE Trans Circuits Syst, vol. 62, 6, pp. 1591 – 1598, June 2015.

[3] T. Zhang, M. R. Haider, I. D. Alexander, and Y. Massoud, "A Coupled Schmitt Trigger Oscillator Neural Network for Pattern Recognition Applications," 2018 IEEE 61st International Midwest Symposium on Circuits and Systems, pp. 238-241, August 2018.

[4] C. Frank, Hoppensteadt and M. Eugene, and Izhikevich, "Oscillatory neurocomputers with dynamic connectivity," Physical Review Letters, Vol. 82, 14, pp. 2983- 2986, April 1999.

[5] K. Yamamoto, Y. Nishio, and Y. Uwate, "Realization of Associative Memory using Synchronization of van der Pol Oscillators," Shikoku-section Joint Convention of the Institutes of Electrical and related Engineers, Vol.1, pp.22, 2021.

[6] K. Yamamoto, N. Yonemoto, Y. Uwate, and Y. Nishio, "Realization of Associative Memory by Oscillators Changing the Coupling Strength, "RISP International Workshop on Nonlinear Circuits, Communications and Signal Processing, pp. 125 – 128, March 2021.

[7] K. Yasufuku, K. Yamamoto, Y. Uwate, and Y. Nishio, "Analysis of Hetero Associative Memory Using Oscillator Synchronization, "Shikoku-section Joint Convention of the Institutes of Electrical and related Engineers, Vol.1, pp.14, 2022.