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Various Synchronizations on Coupled van der Pol Oscillators with Memristor Synapse

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Abstract— In this study, various synchronization phenomena are observed on the coupled circuit model of two van der Pol oscillators with the memristor synapse. In-phase single mode, double-mode oscillation and anti-phase single mode are confirmed by changing the memristor characteristics. In addition, this study investigates synchronization in dependence on the characteristics of the memristor and the coupling factor.

1. Introduction

The human brain has high information processing ability. All the details of the human brain is not clear, so many researchers have been studied using several methods. In electrical engineering, the memristor has been focused on because of dynamics. There is a possibility to realize the synape of the human brain.

The memristor is the forth basic circuit element: the resistor, the capacitor and the inductor. Basic physical variables are the voltage, the current, the charge and the flux. Ohm's law correlates the voltage and the current. Faraday's law of electromagnetic induction also correlates the voltage and the flux. The charge conservation is based on the current and the charge. The relationship between the voltage and the charge is described on Gauus's law of the charge. Ampére's circuital law associates the current with the flux. There are the basic circuit elements correspond to these laws. However, the law of the relationship between the charge and the flux had not been observed. L. O. Chua had expected that there are the law and the basic circuit element related the charge and the flux. So, the memristor was theoretically introduced from symmetry arguments by Chua in 1971 [1]. The most interesting properties of the memristor is resistance value depends on the charge or the flux. In 2008 [2], Hewlett-Packard fabricated the memristor, which is a nano-size solid-state device based on TiO_2 and has the hysteretic current-voltage characteristics. The memristor characteristics are modeled using the piecewise linear function of the charge and the flux [3]. Hence, many studies investigated synchronization phenomena on the several models with the memristor synapse [4-6].

Synchronization is a typical nonlinear phenomenon. It is one of the most interesting phenomena in nature, which is that many objects behave same motion such as the movement of the pendulums, fireflies luminescence, living things cry and so on. Many researchers have been studying synchronization phenomena in several fields. Heartbeat is that myocardial cells are synchronized with each other in the field of medical science. Belousov-Zhabotinsky reaction is popular as chemical oscillation In the human brain, each neuron produces a small current using the electric potential difference between the inside and outside of the cell membrane. A large number of neurons synchronize and release electric current at the same time to create a large electrical signal. It is one of the synchronization phenomena. Therefore, synchronization phenomena have been studied actively to analyze and realize the human brain behavior.

In previous study [7], synchronization phenomena were also observed on the coupled circuit model of two van der Pol oscillators via the nonlinear resistor with fifth-power. In-phase and anti-phase synchronizations were observed on the previous model and investigated time waves of phase difference, synchronization in dependence on the nonlinearity, synchronization in dependence on the coupling strength and amplitude of van der Pol oscillator. Also, various synchronization phenomena were investigated on the coupled circuit model of two van der Pol oscillators via the time-varying resistor [8]. As a result, various synchronization phenomena were confirmed by changing the characteristics of the time-varying resistor. However, other synchronization phenomena such as double-mode oscillation was not obtained on the coupled van der Pol oscillators via the resistor.

In this study, we proposed the coupled circuit model of two van der Pol oscillators with the memristor synapse. The most interesting point is that the memristor has dynamics for the resistor because the charge is defined as the current. Also, various synchronization phenomena such as in-phase single mode, double-mode oscillation and anti-phase single mode were researched on the proposed model. In addition, this study investigated synchronization in dependence on the characteristics of the memristor and the coupling factor.

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2. Proposed Models

Figure 1 shows the memristor model. From Ohm's law, the equation of between the current i and the voltage v is described as Eq. (1)

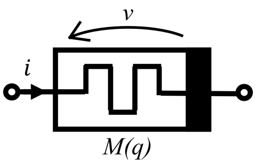

Figure 1: Memristor Model.

$$v = M(q)i \tag{1}$$

where memristance $M(q)$ is depend on the charge q. It is defined as Eq. (2) and characterized by piecewise linear function $\varphi(q)$ in Fig. 2. Horizontal axis is the charge q and vertical axis is the flux φ. The φ–q curve is given as Eq. (3).

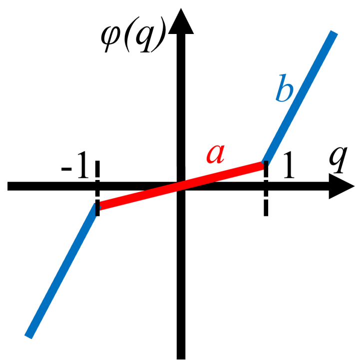

Figure 2: q-φ curve.

$$M(q) = \frac{d\varphi(q)}{dq} \tag{2}$$

$$\varphi(q) = bq + 0.5(a-b)(|q+1| - |q-1|) \tag{3}$$

where a is the slope of $\varphi(q)$ when the absolute value of the charge q is less than one. b is also the slope of $\varphi(q)$ when the absolute value of the charge q is greater than one.

Figure 3 shows the van der Pol oscillator. It is consist on a capacitor, an inductor and the nonlinear resistor with third-power.

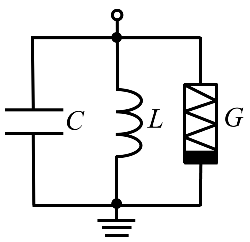

Figure 3: van der Pol Oscillator.

Figure 4 shows the circuit model. In this model, two van der Pol oscillators are coupled with the memristor.

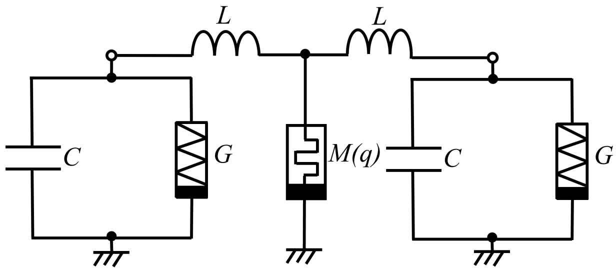

Figure 4: Circuit Model.

The circuit equations are given as Eq. (4).

$$\begin{cases} C\dfrac{dv_1}{dt} = -i_{G1} - i_1 \\ L\dfrac{di_1}{dt} = v_1 - M(q)(i_1 + i_2) \\ C\dfrac{dv_2}{dt} = -i_{G2} - i_2 \\ L\dfrac{di_2}{dt} = v_1 - M(q)(i_1 + i_2) \\ \dfrac{dq}{dt} = i_1 + i_2 \end{cases} \tag{4}$$

The i_{Gn} and v_n characteristics of the nonlinear resistor of van der Pol oscillator is defined as Eq. (5).

$$i_{Gn} = -g_1 v_n + g_3 v_n^3 \qquad (g_1,\ g_3 > 0) \tag{5}$$

By changing the variables and parameters,

$$v_k = \sqrt{\frac{g_1}{g_3}}x_k,\ i_k = \sqrt{\frac{g_1 C}{g_3 L}}y_k,\ q = z,\ t = \sqrt{LC}\tau$$
$$\varepsilon = g_1\sqrt{\frac{L}{C}},\ \gamma = \sqrt{\frac{C}{L}},\ \zeta = C\sqrt{\frac{g_1}{g_3}} \qquad (k = 1, 2).$$

where τ is the scaling time, ε is the strength nonlinearity, γ is the coupling strength and ζ is the coupling factor. The normalized circuit equations are obtained as Eq. (6).

$$\begin{cases} \dfrac{dx_1}{d\tau} = \varepsilon(1 - x_1^2)x_1 - y_1 \\ \dfrac{dy_1}{d\tau} = x_1 - \gamma M(z)(y_1 + y_2) \\ \dfrac{dx_2}{d\tau} = \varepsilon(1 - x_2^2)x_2 - y_2 \\ \dfrac{dy_2}{d\tau} = x_2 - \gamma M(z)(y_1 + y_2) \\ \dfrac{dz}{d\tau} = \zeta(y_1 + y_2) \end{cases} \tag{6}$$

where memristance $M(z)$ represents the slope of the function $\varphi(z)$, so $M(z)$ is redefined as Eq. (7)

$$M(z) = \frac{d\varphi(z)}{dz} = \begin{cases} a & (|z| < 1) \\ b & (|z| > 1) \end{cases} \tag{7}$$

3. Results

3.1. Various Synchronizations

In this section, this study investigated various synchronizations on the coupled circuit model with the memristor synapse. In this simulation, the parameters of the normalized circuit equations Eq. (6) were set up to $\varepsilon = 0.1$, $\gamma = 1.0$, $\zeta = 1.0$, $\tau = 20000$ and step size of Runge-Kuuta method $h = 0.01$.

This study confirmed that in-phase single-mode and anti-phase single-mode coexist at the same parameter values and were determined by the different initial values. The parameters of the characteristics of the memristor were set up to $a = 0.01$ and $b = -0.1$. In-phase single-mode synchronization state is shown in Fig. 5. Besides, Fig. 6 shows anti-single synchronization state.

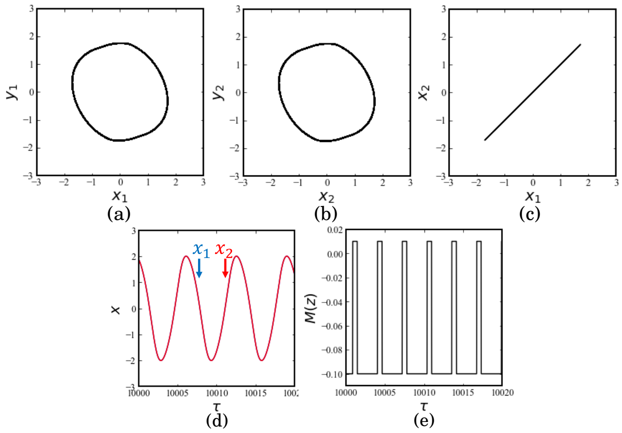

Figure 5: In-phase single-mode synchronization. (a) First circuit attractor (x_1 - y_1). (b) Second circuit attractor (x_2 - y_2). (c) Phase difference (x_1 - x_2). (d) Time wave forms of x. (e) Time wave form of M. Initial values $x_1 = 1.0$, $y_1 = 2.0$, $x_2 = 1.1$, $y_2 = 2.1$ and $z = 1.0$.

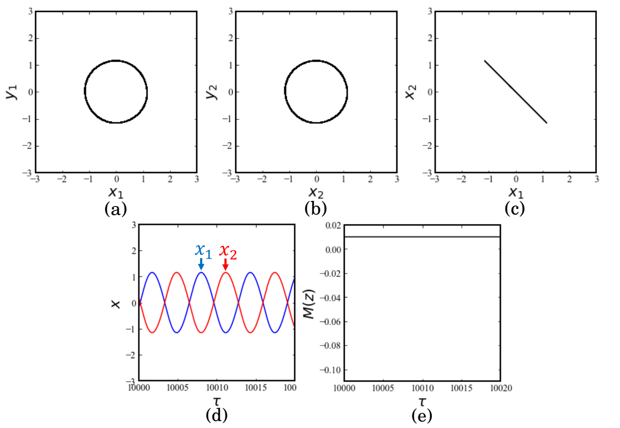

Figure 6: Anti-phase synchronization. (a) First circuit attractor (x_1 - y_1). (b) Second circuit attractor (x_2 - y_2). (c) Phase difference (x_1 - x_2). (d) Time wave forms of x. (e) Time wave form of M. Initial values $x_1 = 1.0$, $y_1 = -2.0$, $x_2 = -1.1$, $y_2 = 2.1$ and $z = 1.0$.

Also, this study confirmed that double-mode oscillation was obtained by changing the parameters of the memristor. The parameters of the characteristics of the memristor were set up to $a = -0.1$ and $b = 10$. Figure 7 shows the simulation results of double-mode oscillation.

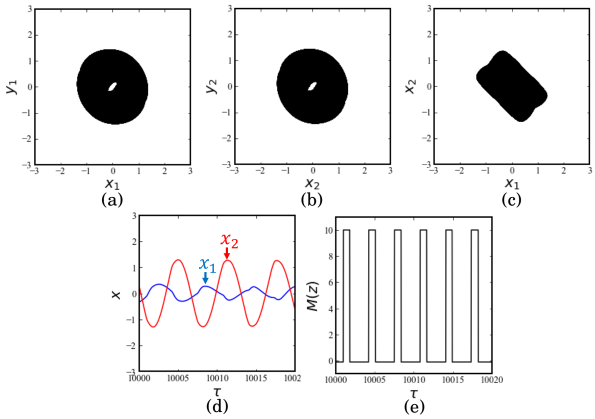

Figure 7: Double-mode oscillation. (a) First circuit attractor (x_1 - y_1). (b) Second circuit attractor (x_2 - y_2). (c) Phase difference (x_1 - x_2) (d) Time wave forms of x. (e) Time wave form of M. Initial values $x_1 = 1.0$, $y_1 = 2.0$, $x_2 = -1.1$, $y_2 = -2.1$ and $z = 1.0$.

3.2. Synchronization in Dependence on a and b

In this section, this study researched synchornization in dependence on the parameters of the memristor under the condition of double-mode oscillation in Fig. 7. In this simulation, the parameters of the normalized circuit equations Eq. (6) were set up to $\varepsilon = 0.1$, $\gamma = 1.0$, $\zeta = 1.0$, $\tau = 20000$ and step size of Runge-Kuuta method $h = 0.01$.

Figure 8 shows synchronization in dependence on the parameters of the memristor.

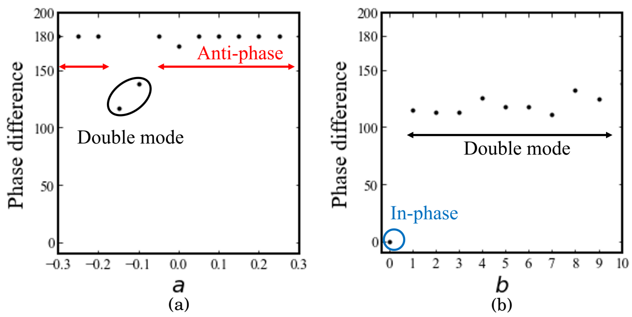

Figure 8: Synchronization in dependence on the parameters of the memristor. (a) Dependence on a ($b = 10$). (b) Dependence on b ($a = -0.1$). Initial values $x_1 = 1.0$, $y_1 = 1.1$, $x_2 = -2.0$, $y_2 = -2.1$ and $z = 1.0$.

In Fig. 8 (a), anti-phase single mode was obtained in a range from -0.3 to -0.2 and from -0.1 to 0.3. Doble-mode oscillation was also confimed in the regions of $-0.2 \leqq a \leqq -0.1$. In Fig. 8 (b), double-mode oscillation was constantly obtained in the regions of $1.0 \leqq b \leqq 10$. Besides, in-phase single mode was confirmed in the regions of $b \leqq 0.0$ because both of a and b were zero or negative.

3.3. Synchronization in Dependence on ζ

In this section, this study investigated synchronization in dependence on the coupling factor under the condition of double-mode oscilation. In this simulation, the parameters of the normalized circuit equations Eq. (6) were set up to $\varepsilon = 0.1$, $\gamma = 1.0$, $\tau = 20000$ and step size of Runge-Kuuta method $h = 0.01$. The parameters of the characteristics of the memristor were set up to $a = -0.1$ and $b = 10$.

Figure 9 shows synchronization in dependence on ζ.

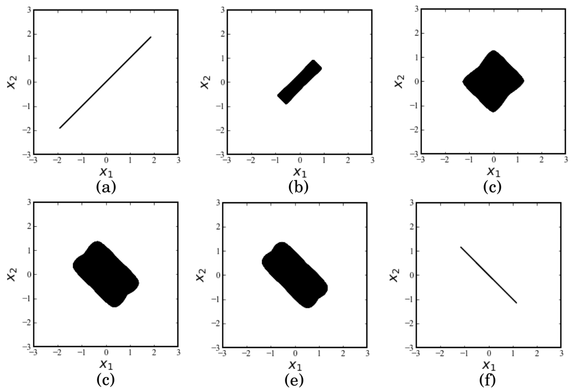

Figure 9: Phase difference (x_1 - x_2). (a) $\zeta = 0.1$. (b) $\zeta = 0.7$. (c) $\zeta = 0.8$. (d) $\zeta = 1.0$ (e) $\zeta = 1.2$. (f) $\zeta = 1.5$. Initial values $x_1 = 1.0$, $y_1 = 2.0$, $x_2 = -1.1$, $y_2 = -2.1$ and $z = 1.0$.

In-phase single mode was confirmed in Fig. 9 (a) when we chose $\zeta = 0.1$. Double-mode oscillation was obtained in when we chose $\zeta = 0.7$, 0.8, 1.0 and 1.2. Also, anti-phase single mode was obtained in whne we chose $\zeta = 1.2$. Next, we researched synchronization in dependence on the coupling factor ζ. Figure 10 shows synchronizations in dependence on ζ in the regions of $0.0 \leqq \zeta \leqq 2.0$.

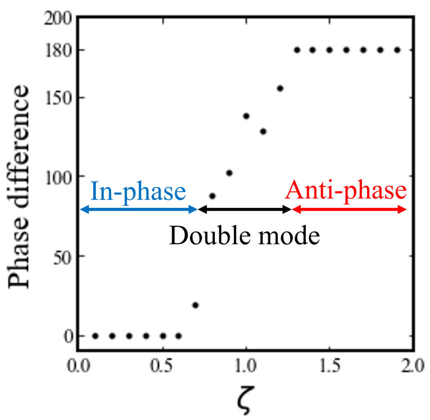

Figure 10: Synchronization in dependence on ζ. Initial values $x_1 = 1.0$, $y_1 = 1.1$, $x_2 = -2.0$, $y_2 = -2.1$ and $z = 1.0$.

In Fig. 10, In-phase single mode was confirmed in the regions of $0.0 \leqq \zeta \leqq 0.6$. Also, double-mode oscillation was obtained in the regions of $0.7 \leqq \zeta \leqq 1.2$. In the regions of $1.3 \leqq \zeta \leqq 2.0$, anti-phase synchronization was observed.

4. Conclusions

This study proposed the coupled circuit model of two van der Pol oscillators with the memristor synapse. First, in-phase and anti-phase synchronization states coexist at the same parameter values in the different initial values. Second, double-mode oscillation was confirmed by changing the characteristics of the memristor. Third, this study confirmed that synchronization depends on the characteristics of the memristor by analyzing the relationship between phase difference and the parameters of the memristor. Finally, we investigated synchronization in depencence on the coupling factor. As a result, in-phase single mode, double-mode oscilattion and anti-phase single mode were obtained by changing the coupling factor.

In the future, we would like to research the causes of double-mode oscillation on proposed model using theoretical analysis, and compare to the simulation results.

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