Clustering Feature Extraction of Chaotic Circuits with Learning on Coupling Weights

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***Abstract*— If clustering can be performed in a continuous system, there is potential usefulness in processing speed for larger scale clustering. We have proposed clustering method of coupled chaotic circuit networks with learning. In this study, we propose a method to extract the center of a cluster by stabilizing synchronization through learning in networks where synchronization is not stable due to chaos.**

I. Introduction

Memristors were proposed by Chua in 1971 [1]. Memristors are passive devices that store a passing charge and change resistance accordingly. Later, in 2008, Hewlett-Packard Laboratories developed a memristor using a thin film of titanium dioxide, which attracted attention as a fourth circuit element. In recent years, the development of brain computers using memristors has been actively studied in the field of neuromorphic engineering [2], [3]. Due to the characteristics of memristors, many computing methods have been proposed that use memristors as synapses [4], [5]. Synapses are connections between neurons that play an important role in signal transmission.

One of the basic rules of learning in the brain is the Hebbian rule. This rule has proposed by Hebb in 1949 [6]. This is a fundamental rule of learning and long-term memory based on the hypothesis that synapses become more efficient at transmitting electrical stimuli each time a neuron fires, and conversely, less efficient if they do not fire for a long time. The neurobiological mechanism of Hebb's rule has also been identified. The electrical signal enhances the coupling strength and strengthens the part of the system that communicates more.

Our research group has been investigating synchronization phenomena in coupled oscillators and coupled chaotic circuits. While many studies of synchronization in networks have used mathematical models for nodes, we have focused our research on electronic circuit models. The reasons for this are, first, that we would like to deal with synchronization observed as a physical phenomenon that actually exists, and second, that an electronic circuit model was necessary when we considered engineering applications in the future. Now that memristor devices are in practical use, replacing the coupling in our previously proposed coupled oscillator network with memristors is expected to have applications in modeling more complex, higher-dimensional nonlinear systems. Possible engineering applications include modeling of cyber-physical systems.

In recent years, much data analysis has been important in marketing and other areas. One of the data analysis is clustering. Clustering is a method of classifying similar data and is used in various fields such as data mining, image processing and biological data analysis [7]-[9].

In previous studies, we have proposed a method of clustering chaotic circuits on a two-dimensional plane using synchronization and confirmed its effectiveness [10]-[13]. As a further improvement, we are also investigating clustering when the coupling between circuits is changed as in Hebbian rule learning [14], [15].

However, the dynamics of the proposed coupled chaotic network with learning had not been investigated. Therefore, in this study, we focused on how the coupled chaotic circuit network with learning makes synchronization decisions, and investigated it in a network containing three clusters. By using the computer simulations, we confirm that the proposed method is a suitable system for extracting cluster centers by learning coupling weights.

II. Chaotic Circuits Networks

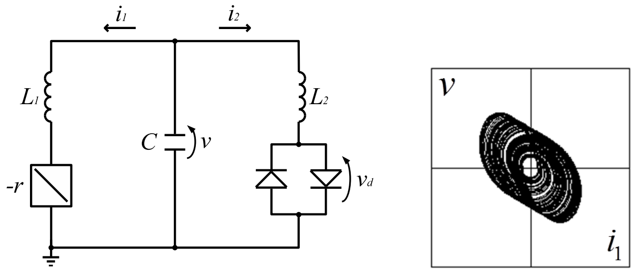
The nodes of the network are represented by chaotic circuits. The chaotic circuit is shown in Fig. 1. This chaotic circuit has been studied in the references [16]-[18]. This circuit composed of three memory elements, one linear negative resistance element, and one nonlinear resistance element consisting of two diodes.

The approximate $I - V$ characteristics of the nonlinear resistance element are indicated by the following equation, where the parameter r_d is the slope of the nonlinear resistance.

$$v_d(i_2) = \frac{r_d}{2}\left(\left|i_2 + \frac{V}{r_d}\right| - \left|i_2 - \frac{V}{r_d}\right|\right). \tag{1}$$

By changing the variables and using the parameters,

$$i_1 = \sqrt{\frac{C}{L_1}}Vx;\ i_2 = \frac{\sqrt{L_1 C}}{L_2}Vy;\ v = Vz;$$



(a) Chaotic circuit. (b) Chaotic attractor.

Fig. 1. Chaotic circuit model.

$$r\sqrt{\frac{C}{L_1}} = \alpha;\ \frac{L_1}{L_2} = \beta;\ r_d\frac{\sqrt{L_1 C}}{L_2} = \delta; t = \sqrt{L_1 C}\tau \tag{2}$$

The normalized circuit equations are obtained as follows.

$$\begin{cases} \dfrac{dx_i}{d\tau} = \alpha x_i + z_i \\ \dfrac{dy_i}{d\tau} = z_i + f(y) \\ \dfrac{dz_i}{d\tau} = -x_i - \beta y_i - \displaystyle\sum_{j=1}^{N} \gamma_{ij}(z_i - z_j) \\ \qquad (i, j = 1, 2, \cdots, N) \end{cases} \tag{3}$$

where $f(y)$ is described as follows:

$$f(y) = \frac{\delta}{2}\left(\left|y + \frac{1}{\delta}\right| - \left|y - \frac{1}{\delta}\right|\right). \tag{4}$$

In the computer simulations, we set the parameters to be $\alpha = 0.460$, $\beta = 3.0$ and $\delta = 470$. The characteristic function $f(y)$ can be described as a three-segment piecewise-linear function. In this study, the value of γ_{ij} reflects the distance between the circuits in an inverse manner, as described using the following equation:

$$\gamma_{ij} = \frac{g}{(d_{ij})^2}. \tag{5}$$

Here, d_{ij} denotes the Euclidean distance between the $i-th$ circuit and the $j-th$ circuit, while g is a scaling parameter that determines the coupling strengths.

III. Circuits Arrangement and Learning Process

A. Circuits Arrangement

Here, we consider the case of 100 chaotic circuits arranged in 2-dimensional space. The number of clusters is set to three, cluster-1, 2 and 3 consist of 50, 30, and 20 chaotic circuits, respectively. The chaotic circuits are randomly placed according to a normal distribution (σ) from the center of each cluster. The circuit arrangement with three clusters is shown in Fig. 2.

The purpose of clustering for this circuit layout is not to divide it into three clusters, but to extract as many nodes as possible from the central part of the cluster.

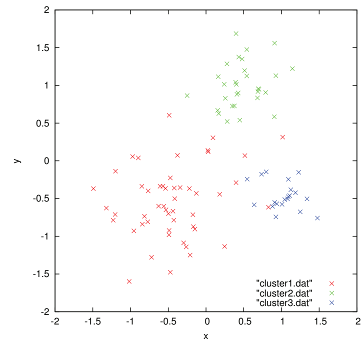


Fig. 2. Circuit arrangement with three clusters (N=100, the number of clusters: 3, red points: 50 (cluster-1), green points: 30 (cluster-2), blue points: 20 (cluster-3)).

B. Learning Process

In the previous study, we have proposed the clustering method that applies Hebbian rule as well as the determination of clusters by synchronization of chaotic circuit networks. The proposed clustering method is based on the concept of the Hebb rule. As the clustering algorithm, it makes the coupling strength of edges with high synchronization between circuits stronger and the coupling of edges with weak synchronization weaker. The clustering method with learning of coupled chaotic circuits is explained below. (The conceptual diagram of the computer simulation is shown in Fig. 3.)

[step-1] At the initial state, all nodes are fully connected with coupling strengths depending on distance. The scaling parameter g is set to $g = 0.00008$.

[step-2] After a transient phase, we apply two rules for a sequence of generations. Each generation has length $\tau_h = 10,000$.

- **(check synchronization:)** In order to check whether two nodes are alike, we calculate the synchronization ratio for every pair of oscillators. If the synchronization ratio is larger than 60%, the corresponding coupling strength becomes stronger with $\Delta\gamma = 0.00001$.

  In order to analyze the synchronization ratio, we define a synchronization state as

$$|x_k - x_n| < 0.3 \quad (k \in S_n)$$

[step-3] Step-2 is repeated until 100 iterations are reached ($H = 100$).

[step-4] At the final state ($H = 100$), we check the synchronization ratio for every pair of oscillators.

IV. Simulation Results

First, we investigated changes in the number of synchronized edges of coupled chaotic circuit networks. Figure 4 shows the change in the total number of synchronized edges between the clusters. From this figure, the proposed method with learning has less variation of synchronized edges with the simulation time. In contrast, the conventional method shows a large fluctuation of synchronized edges.

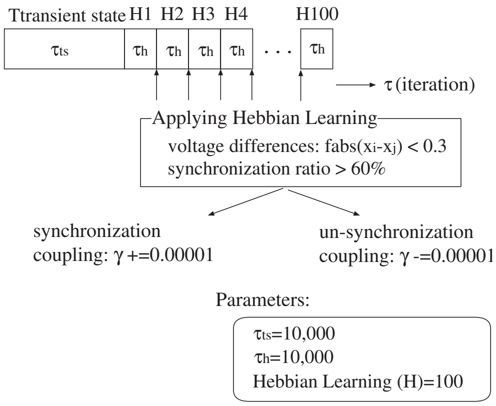


Fig. 3. Learning process for clustering.

The evolution of synchronized edges within the three clusters is shown in Fig. 5. It can be seen that for all clusters, the proposed method with learning has less variation in the number of edges, as well as the variation of synchronized edges for the whole cluster.

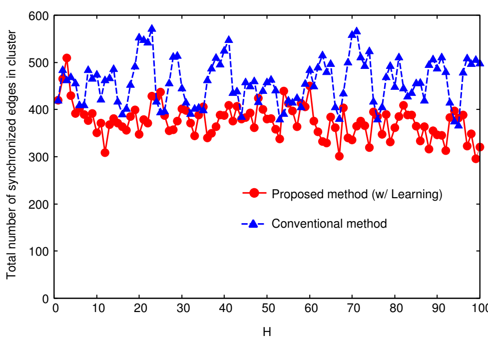
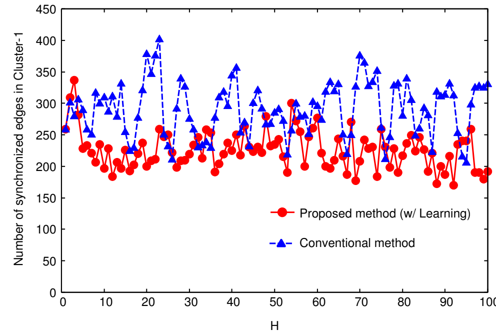


Fig. 4. Total number of synchronized edges in the whole cluster.

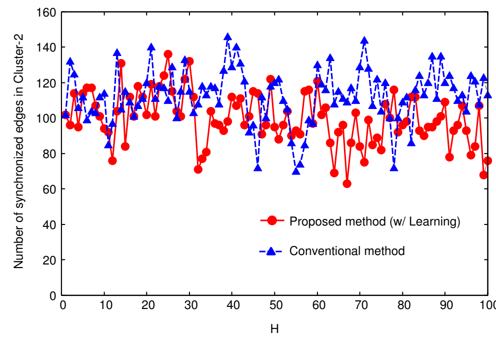
Figure 6 shows the change in synchronization edges across clusters. This edge is considered "miss edge" from the clustering concept. The large number of these missed edges is a cause of poor clustering. From the results, it can be said that both methods have a large variation of missed edges, however the proposed method clearly has a higher number of times with fewer missed edges. Therefore, we consider that learning of the coupling weights influences the control of missed edges.

Average of number of synchronized edges in each category is summarized in Tab. I. It can be seen that the number of synchronized edges within clusters is higher for the conventional method. On the other hand, the proposed method has fewer synchronized edges between clusters (miss edges) and is more effective for clustering.

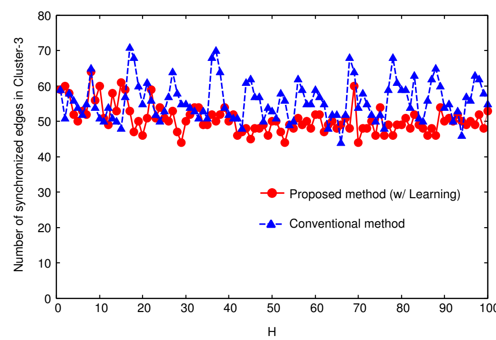
Figure 7 shows examples of the results of displaying synchronized edges for the proposed and conventional methods. In the proposed method, there are synchronized edges in the center of each cluster and the number of missed edges between clusters is small. In the conventional method, there are many



(a) cluster-1.



(b) cluster-2.



(c) cluster-3.

Fig. 5. Number of synchronized edges in the cluster.

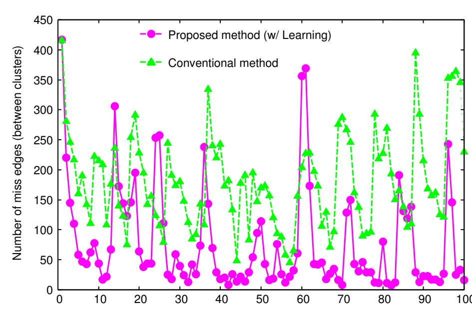


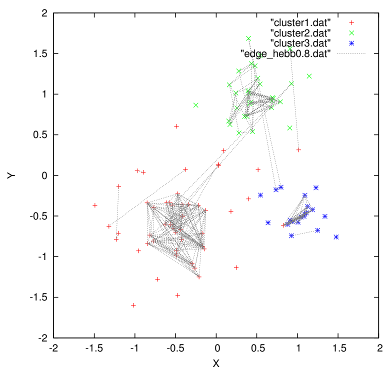
Fig. 6. Number of synchronized edges between clusters (miss edges).

TABLE I

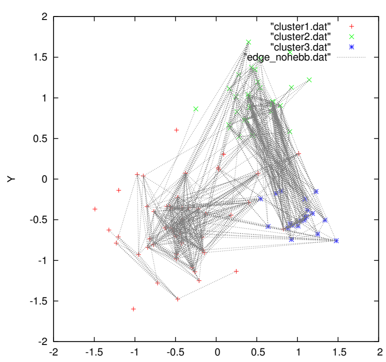
AVERAGE OF NUMBER OF SYNCHRONIZED EDGES IN EACH CATEGORY

Type	Proposed method	Conventional method
Total (in the cluster)	376.00	458.77
Cluster-1	225.63	290.38
Cluster-2	99.74	112.54
Cluster-3	50.63	55.85
Miss (between the clusters)	**76.75**	181.51

synchronized edges throughout, but the number of missed edges between clusters is also high. These results suggest that the proposed method with learning process is effective in extracting cluster features because synchronized edges are concentrated in the central part of the clusters.



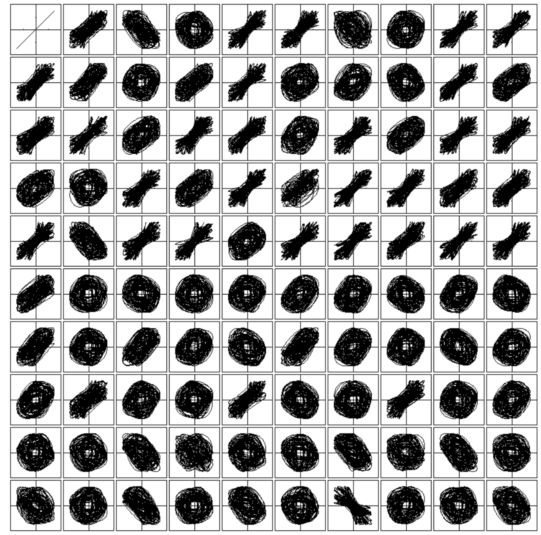
(a) proposed method with learning.



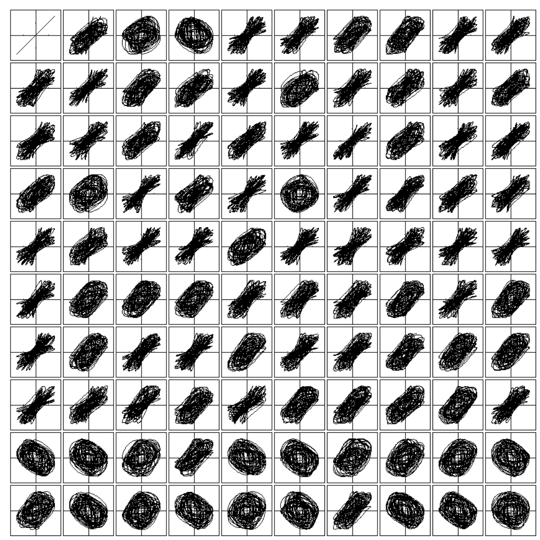
(b) conventional method without learning.

Fig. 7. One example of clustering result by displaying synchronized edges.

Finally, an example of a phase difference result is shown in Fig. 8. This figure shows the phase of the first circuit with reference to the other circuits in Lissajous. It can be seen that the proposed method synchronizes with only a few circuits, while the conventional method synchronizes with many circuits. Therefore, this is a supporting result that the proposed method is effective in cluster extraction.



(a) proposed method with learning.



(b) conventional method without learning.

Fig. 8. One example of phase difference between 1st circuit to the others.

V. CONCLUSION

In this study, we focused on the dynamics of the previously proposed clustering method using synchronization of coupled chaotic circuit networks with learning, and investigated changes in synchronized edges. Because of the chaotic circuit, synchronization in the network is not stable, but by learning the coupling weights, we were able to reduce the variation in the number of synchronization edges. We also found that the proposed method has fewer synchronized edges between clusters and is more likely to extract the center of the cluster.

Future work includes developing a method for accurate clustering and investigating the case where the number of clusters is increased. Clustering using memristor-coupled chaotic circuits is also one of our future research topics.

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