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Synchronization of Chaos Networks by Changing Layout Distribution

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Abstract

In this study, we investigate the synchronization rate of chaotic circuits for different parameters of the Gaussian distribution of the circuit arrangement when one clustering is used. We analyze the effectiveness of the proposed clustering by chaotic circuit network for any circuit layout distributions.

1. Introduction

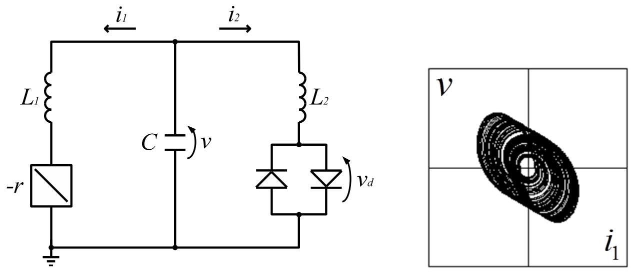
Chaotic systems are very simple, but they produce surprisingly complex signals. Therefore, it seems impossible for two chaotic systems to synchronize, but they can synchronize if the two systems exchange information in an appropriate way. The fundamentals of chaotic synchronization have been studied with regard to the geometry and stability of synchronization [1], [2]. Examples of applications using chaos synchronization include secure communication and image encryption [3], [4].

It is known that chaotic synchronization is also observed in coupled systems of chaotic circuits realized in electronic circuits. In the study of coupled chaotic circuit networks, chaotic circuits are in many cases mathematical models. We believe it is important to investigate the phenomena observed from chaos in electronic circuits for engineering applications. Our research group has proposed a clustering method using synchronization of chaotic circuit networks [5]. As an advanced version of this method, we have proposed a method that enables more difficult clustering in which the coupling between chaotic circuits varies with the synchronization rate, and we have confirmed the effectiveness of this method [6], [7]. By using computer simulations, we confirmed the effectiveness of the clustering accuracy of the proposed method. A discussion of the results shows that when the circuit arrangement is generated to follow a Gaussian distribution, the clustering accuracy depends on the parameters of the Gaussian distribution.

In this study, we focus on the synchronization performance of the proposed model (coupled chaotic circuits network with learning) by using the several types of circuit layout distributions. We compare the proposed model and the conventional model when it is applied to three different circuit distribution layouts. In the case of the proposed model, it was found that it is possible to extract the central part by using synchronization information of the circuit distribution even if the circuit distribution changes.

2. Chaotic Circuits Model

In this section, we explain the chaotic circuit model. Figure 1 shows the chaotic circuit model that has been investigated in the literature [8]-[10]. This circuit consists of three memory elements, one linear negative resistance element, and one nonlinear resistance element consisting of two diodes. The negative resistance is realized using the linear region of a negative impedance converter made from an operational amplifier.



(a) Chaotic circuit. (b) Chaotic attractor.

Figure 1: Chaotic circuit model.

The approximate $I - V$ characteristics of the nonlinear resistance element are indicated by the following equation, where the parameter r_d is the slope of the nonlinear resistance.

$$v_d(i_2) = \frac{r_d}{2}\left(\left|i_2 + \frac{V}{r_d}\right| - \left|i_2 - \frac{V}{r_d}\right|\right). \qquad (1)$$

By changing the variables, such that

$$i_1 = \sqrt{\frac{C}{L_1}}Vx;\ i_2 = \frac{\sqrt{L_1 C}}{L_2}Vy;\ v = Vz;$$

$$r\sqrt{\frac{C}{L_1}} = \alpha;\ \frac{L_1}{L_2} = \beta;\ r_d\frac{\sqrt{L_1 C}}{L_2} = \delta; t = \sqrt{L_1 C}\tau \quad (2)$$

The normalzied equations represent the circuit equations when all the chaotic circuits are coupled globally with each other (all-to-all coupling).

$$\begin{cases} \dfrac{dx_i}{d\tau} = \alpha x_i + z_i \\ \dfrac{dy_i}{d\tau} = z_i + f(y) \\ \dfrac{dz_i}{d\tau} = -x_i - \beta y_i - \displaystyle\sum_{j=1}^{N} \gamma_{ij}(z_i - z_j) \\ \qquad (i, j = 1, 2, \cdots, N) \end{cases} \quad (3)$$

where $f(y)$ is described as follows:

$$f(y) = \frac{\delta}{2}\left(\left|y + \frac{1}{\delta}\right| - \left|y - \frac{1}{\delta}\right|\right). \quad (4)$$

In the computer simulations, we set the parameters to be $\alpha = 0.460$, $\beta = 3.0$ and $\delta = 470$. The characteristic function $f(y)$ can be described as a three-segment piecewise-linear function. In this study, the value of γ_{ij} reflects the distance between the circuits in an inverse manner, as described using the following equation:

$$\gamma_{ij} = \frac{g}{(d_{ij})^2}. \quad (5)$$

Here, d_{ij} denotes the Euclidean distance between the $i-th$ circuit and the $j-th$ circuit, while g is a scaling parameter that determines the coupling strengths.

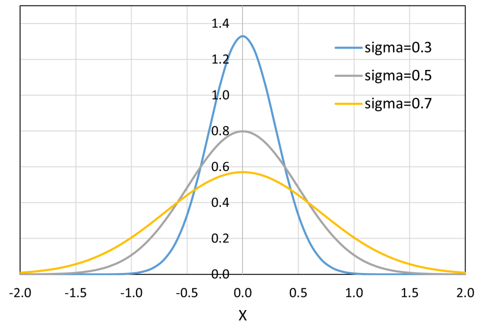
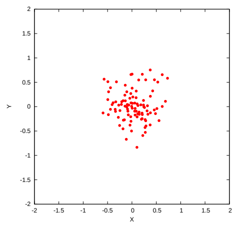
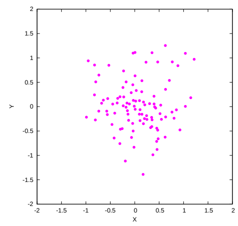


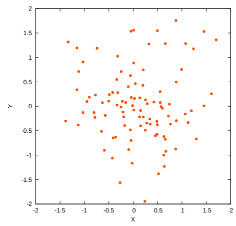
Figure 2: Gaussian distribution depending on σ.



(a) σ=0.3.



(b) σ=0.5.



(c) σ=0.7.

Figure 3: Circuit Arrangement.

3. Circuit Arrangement and Learning Process

In our previous studies, we have confirmed that the proposed method which is coupled chaotic network with learning has a high ability to extract cluster centers. Therefore, we consider three types of circuit arrangements with parameter σ generated by a Gaussian distribution. Figure 2 shows three types of Gaussian distributions when the parameter σ is set to 0.3, 0.5 and 0.7. The three types of circuit arrangements are shown in Fig. 3. As the value of σ increases, the circuits are spread out and arranged in a two-dimensional plane.

Next, we explain about learning process of coupled chaotic circuits networks. We have proposed a new clustering method that applies Hebbian rule by changing the coupling strength depending on the synchronization rate. The Hebbian rule states that synapses, which connect neurons become more efficient when neurons fire repeatedly, and less efficient when they do not fire for long periods of time. We apply this Hebbian rule to chaotic circuit synchronization. In other words, the coupling between the synchronized chaotic circuits is made stronger, and the coupling between the un-synchronized chaotic circuits is made weaker. The Hebbian rule is applied to the chaotic circuits network as following steps.

[step-1] At the initial state, all nodes are fully connected with coupling strengths depending on distance.

[step-2] After a transient phase, we apply two rules for a sequence of generations. Each generation has length $\tau_h = 10,000$.

- **(Determination of synchronization:)** In order to determine whether two nodes are alike, we calculate the synchronization ratio for every pair of oscillators. If the synchronization ratio is larger than 60%, the corresponding coupling strength becomes stronger with $\Delta\gamma = 0.0001$.

  In order to analyze the synchronization ratio, we define a synchronization state as

$$|z_k - z_n| < 0.3 \quad (k \in S_n)$$

[step-3] Step-2 is repeated until 100 iterations are reached ($G = 100$).
[step-4] At the final state ($G = 100$), we check the synchronization ratio for every pair of oscillators.

A flowchart of these steps is shown in Fig. 4.

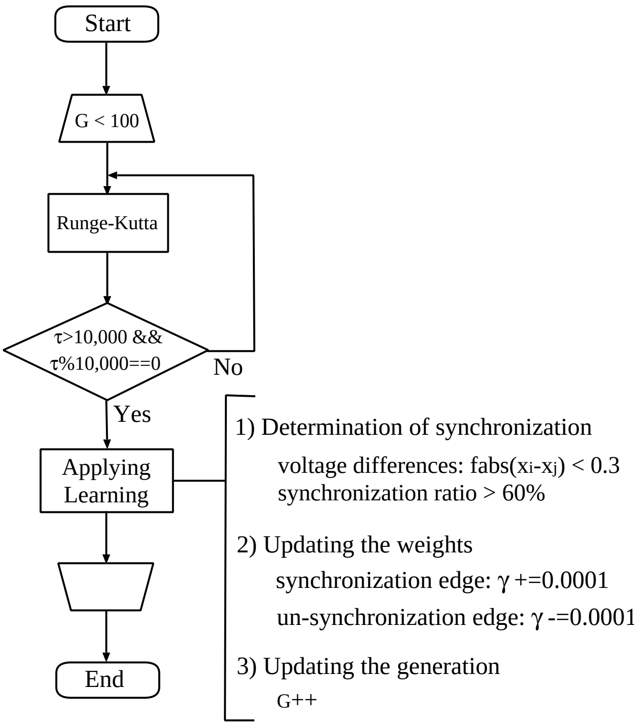
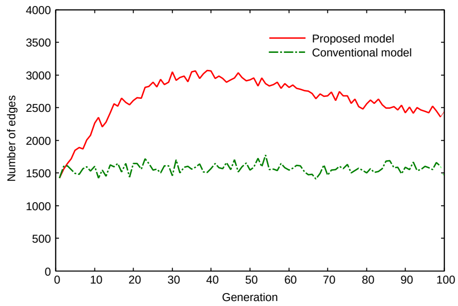
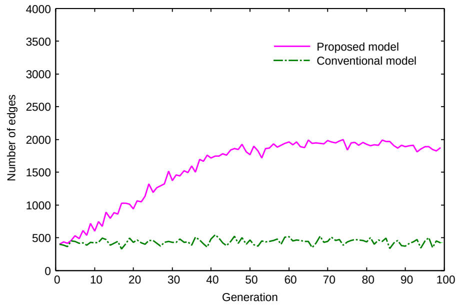


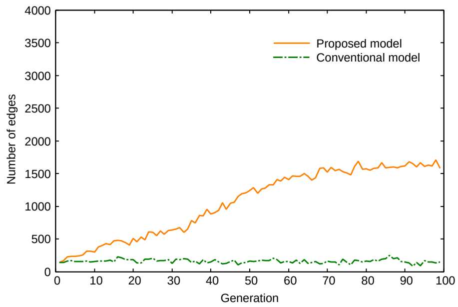
Figure 4: Flowchart of learning process.



(a) σ=0.3.



(b) σ=0.5.



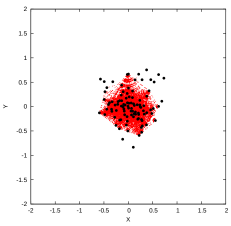
(c) σ=0.7.

Figure 5: Time evolution of number of synchronized edges.

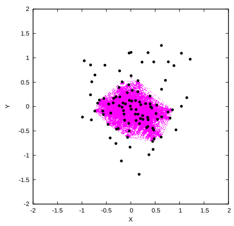
4. Simulation Results

Figure 5 shows the simulation results of number of synchronized edges with generation. First, in the conventional model, the number of synchronized edges is constant within a certain range of vibrations, even as the number of generators increases. In contrast, for $\sigma = 0.3$ in the proposed model, the number of synchronized edges increases with the generations, peaking once around $G = 40$. Although it decreases after the peak, the number of synchronized edges is significantly larger than in the conventional model. For $\sigma = 0.5$ and 0.7 of the proposed model, it can be seen that the number of synchronized edges also increases with the generators. In both models, the number of synchronized edges is higher when the value of σ is small. These results show that the number of synchronized edges is increased by updating the weights of the learning effects of the proposed model.

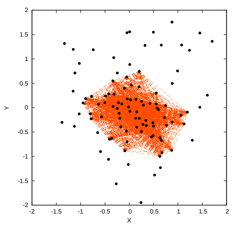
The results of displaying synchronized edges of the proposed and conventional models are shown in Figs. 6 and 7. The results show that the proposed model has more synchronized edges where the density of circuits is higher. Thus, in the case of σ=0.3, most nodes are synchronous. However, in the case of σ=0.7, nodes located away from the center are asynchronous. In the case of the proposed method, it can be seen that as σ increases, the range covered by the synchronization edges also expands. In the case of the conventional method, when σ is set to 0.3, the majority of the circuit is synchronized. As σ increases, the number of synchronous edges depends on the initial state and does not capture the characteristics of the cluster well.



(a) σ=0.3.

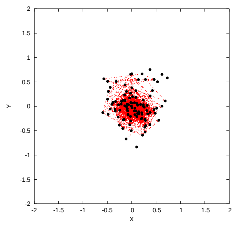


(b) σ=0.5.

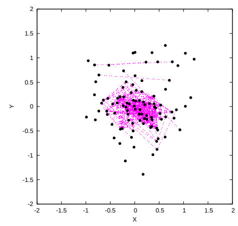


(c) σ=0.7.

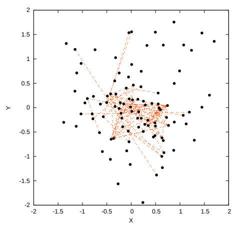
Figure 6: Clustering results of the proposed model with learning.



(a) σ=0.3.



(b) σ=0.5.



(c) σ=0.7.

Figure 7: Clustering results the conventional model without learning.

5. Conclusion

In this study, we investigated synchronized edges of coupled chaotic circuits network using learning process. By changing the parameter of Gaussian distribution of circuit layout, the proposed model with learning can extract the center of cluster. However, it was difficult to observe synchronization of the circuits far from the center. We would like to study in the future a learning algorithm that synchronizes even when the density of circuit layout is different. Furthermore, we hope to apply the clustering methods developed here to data mining, image processing and other applications in real life situations.

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