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Synchronization Phenomena in Complex Oscillatory Networks with Switching Coupling

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Abstract

Complex networks, which are related to real-world networks, have attracted much attention and have been studied in various fields. The complex networks have common characteristics and have been studied in terms of network topology and interactions between nodes. Complex networks with circuits have been studied, and synchronization phenomena between circuits have been confirmed.

In this study, a random network is constructed using 100 van der Pol oscillators. Then the synchronization phenomena between the circuits of the network are analyzed. The computer simulations are conducted by focusing on the synchronization rate and thereby changing the topology. As a result, it is confirmed that the synchronization rate decreases by changing the topology.

1. Introduction

Complex networks are huge, complex structures that exist in reality and have several properties. First, it is scale-free, in that it has hubs, which are nodes of large degree, but most of them are connected to only a few nodes and have small degree. Second, it is small world, in that any number of nodes relate to only a few nodes in the middle. The third is clustering, in which nodes are not isolated from each other, but rather several nodes are grouped together to form a small network of their own. These can be modeled by nodes and edges, and have attracted various fields of attention, such as engineering, sociology, and biology [1]-[3]. Furthermore, in the field of engineering, complex networks using circuits have been studied, and interesting phenomena such as synchronization have been investigated [4], [5]. In these studies, synchronization phenomena are investigated by focusing on the topology structure of the network, and the differences in the structure. Synchronization is phenomena in which things around us gradually come to be in step with each other according to the influence of other things around us. It is one of the most familiar phenomena in nature and has been the subject of various studies [6], [7].

In this study, we focus on the synchronization rate and investigate changes in the network by changing the topology. The network of Erdős Rényi model (ER model) using 100 van der Pol oscillators is constructed. Then the synchronization rate is investigated, edges with synchronization rates below a set threshold are cut off, and edges are added for that number of edges. Repeat this process and compare how the synchronization rate changes when the topology is changed.

2. Network model

In this study, we use the ER model. The number of nodes is 100, and the average degree is set to be about 5.1. First, Fig. 1 shows the model of the random network used in this study. This network has the property that the number of edges gathered at each node are small.

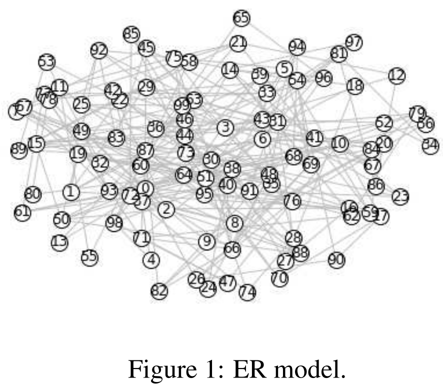

Figure 1: ER model.

3. Circuit model

Figure 2 shows the van der Pol oscillator used in this study. This is a simple circuit consisting only of a capacitor, an inductor, and a nonlinear element. These van der Pol oscillators are connected as resistors to reproduce the network.

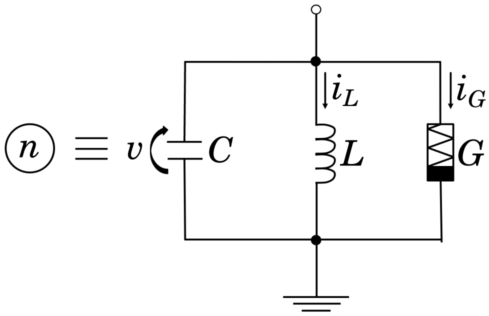

Figure 2: van der Pol oscillator.

The circuit equations are as follows:

$$\begin{cases} C\dfrac{dv_n}{dt} = -i_{L_n} - i_{g_n} - \displaystyle\sum_{n,k=1}^{100} \frac{1}{R}(v_n - v_k) \\ L\dfrac{di_n}{dt} = v_n \\ \qquad\qquad (n, k = 1, 2, \cdots, 100). \end{cases} \tag{1}$$

In addition, the nonlinear element equations are as follows:

$$i_g = -g_1 v + g_3 v^3. \tag{2}$$

To normalize the circuit equations, the following parameters are used:

$$v = \sqrt{\frac{g_1}{g_3}}x,\ i = \sqrt{\frac{g_1 C}{g_3 L}}y,\ t = \sqrt{LC}\tau$$

$$\varepsilon = g_1\sqrt{\frac{L}{C}},\ \gamma = \frac{1}{R}\sqrt{\frac{L}{C}}.$$

The normalized circuit equations derived from the above are shown as follows:

$$\begin{cases} \dfrac{dx_n}{d\tau} = \alpha\left\{ \varepsilon x_n(1-x_n^2) - y_n - \displaystyle\sum_{n,k=1}^{100} E_{nk}\gamma(x_n - x_k) \right\} \\ \dfrac{dy_n}{d\tau} = x_n \\ \qquad\qquad (n,\ k{=}1,\ 2,\ \cdots,\ 100). \end{cases} \tag{3}$$

In this case, α represents the small error of each capacitor and E_{nk} represents the adjacency matrix of the network. In this study, the synchronization rate is investigated, edges with synchronization rates below a set threshold are cut off, and edges are added for that number of edges. The threshold value of the synchronization rate that changes the topology is set to 50[%], and it is changed the connection 10 times. Then, the synchronization rates are compared when the coupling strength is changed from $\gamma = 0.3$ to $\gamma = 0.6$ by 0.1.

4. Simulation results

In order to investigate synchronization phenomena between circuits, we define synchronization as in Eq. (4).

$$|x_n - x_k| < 0.01. \tag{4}$$

In this study, the synchronization rates are calculated in the range of $\tau = 10{,}000$ to $\tau = 20{,}000$, where the voltage differences settle down to steady state. Where n, k are the number of circuits. Figure 3 shows the waveform of the differential voltage measured in this study. The two lines in the figure are the threshold values shown in Eq. (4), and when the waveform is within this range, we judge them to be synchronized and calculate the synchronization rate.

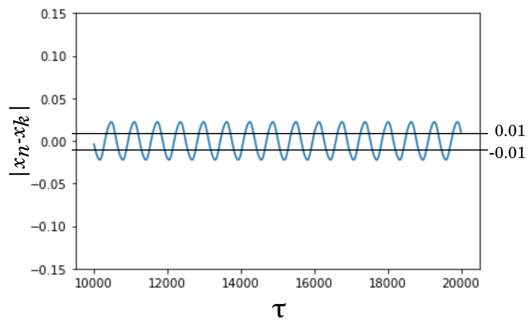

Figure 3: The differential voltage waveform for determining synchronization.

First, Table 1 shows the clustering coefficient (C), average path length (L), average synchronization rate ($\bar{SR}$), and number of cutting edges (CE) when we cut edges of synchronization rate below the threshold value and changed the topology 10 times. The coupling strength is set to $\gamma = 0.3$ for the simulation. As a result, it shows the clustering coefficient increases as the topology is changed when $\gamma = 0.3$ and the threshold is set to 50[%]. However, the average synchronization rate is decreased. Also, the topology is denser due to the increased clustering coefficients.

Table 1: The clustering coefficient (C), average path length (L), average synchronization rate ($\bar{SR}$), and number of cutting edges (CE) when we cut edges of synchronization rate below the threshold value and changed the topology 10 times(γ = 0.3).

	0th	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th
C	**0.04901**	0.03756	0.05835	0.07064	0.05984	0.04059	0.06277	0.04902	0.05695	0.05524	**0.08747**
L	2.954	2.934	2.993	3.035	3.058	3.005	2.970	2.977	2.987	3.018	3.012
SR	**60.58**	55.54	58.21	54.97	58.32	52.58	57.34	60.45	55.95	57.14	**50.69**
CE	-	122	145	130	135	129	150	138	125	135	**128**

Figure 4 shows the transition of the average synchronization rate at each coupling strength. It shows that the average synchronization rate becomes higher as the coupling strength increases. It can be seen that the synchronization rate decreases when the topology is changed by cutting the edges below the threshold value.

Also, Fig. 5 shows the relationship between the average synchronization rate and the number of cutting edges. It is shown that the synchronization rate is higher when the number of cutting edges is smaller.

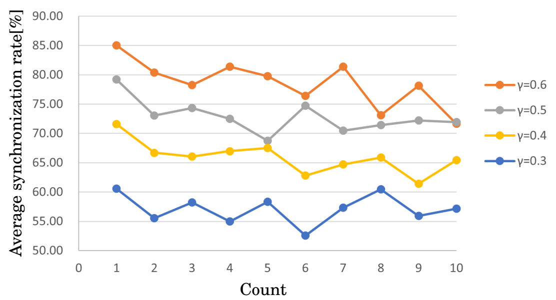

Figure 4: The transition of the average synchronization rate at each coupling strength.

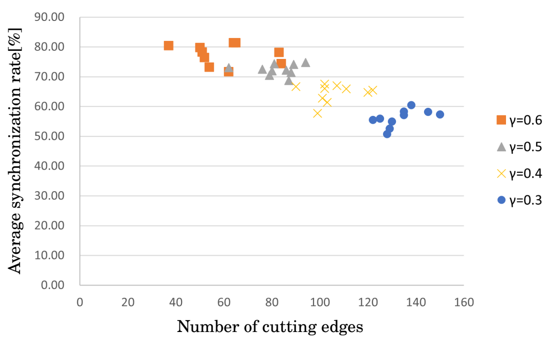

Figure 5: The relationship between the average synchronization rate and the number of cutting edges.

Finally, Figs. 6-7 show the relationship between the average synchronization rate and the number of pair nodes when the parameter γ is set to 0.3 and 0.6, respectively. They show that the number of pair nodes with 90-100[%] is the highest both for γ = 0.3 and γ = 0.6, and that the number is higher for γ = 0.6, which has stronger coupling strength. Also, we can see that the percentage of 90-100[%] is decreasing and the percentage around 0-30[%] is increasing by changing the topology. These are similar results were obtained for other coupling strengths.

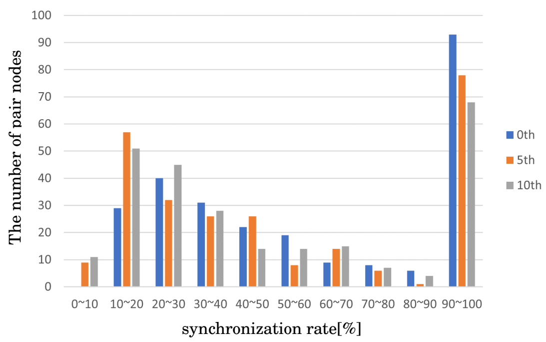

Figure 6: The relationship between the average synchronization rate and the number of paired nodes (γ = 0.3).

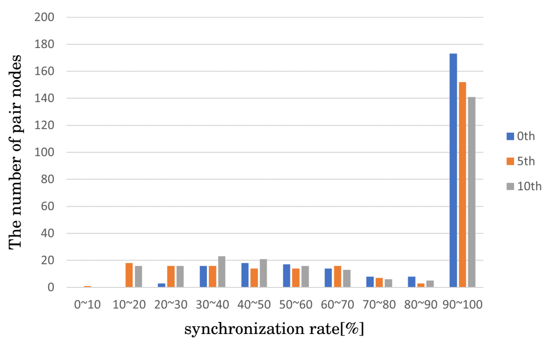

Figure 7: The relationship between the average synchronization rate and the number of paired nodes (γ = 0.6).

5. Conclusions

In this study, we investigated how the average synchronization rate changes when the topology is changed by cutting edges below a threshold value. The results showed that the clustering coefficient increased and the network became denser, but the average synchronization rate decreased. In addition, when the number of edges to be cut were smaller, the average synchronization rate became larger, and when the number of edges to be cut were larger, the average synchronization rate became smaller. This is considered to be due to the fact that the smaller the number of edges to be cut, the stronger the coupling strength and the higher the synchronization rate per edge.

In the future, the simulations were investigated with a loop at $\tau = 10{,}000$ to $\tau = 20{,}000$ in this study, but we would like to conduct dynamic simulations by running the simulation time. Also, the results of this study showed that the average synchronization rate decreased, but we would like to investigate a more detailed study and investigate how the average synchronization rate changes. In addition, we would like to compare how the average synchronization rate changes by different parameters apart from the coupling strength and by different network models.

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