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Chimera States in a Two-Group Network of Kuramoto model

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abstract

Synchronization phenomena can be seen in many aspects of our daily life and has applications in a variety of fields, such as pacemakers and telecommunications. In recent years, chimera states, where synchronous and asynchronous phenomena are mixed, have attracted much attention. The oscillators are phase-locked when synchronous, and incoherent when asynchronous.

In this study, we proposed a network of oscillators divided into two-group and coupled only in one part. We observed the time variation of the synchronization rate in each group and phase distribution of the oscillators. As a result, it was observed that the oscillators in two parts, a synchronous part and an asynchronous part.

1. Introduction

Networks of coupled system are very common in many scientific and engineering fields. One very important phenomenon about this kind of network is the collective synchronization, where all the oscillators in a large-scale network are locked to a common frequency or phase, although their native frequency are quite different and widely distributed. Therefore, synchronizing a network of coupled oscillators is a very important issue for various mathematically describe models, one of which is the famous Kuramoto model used to study such collective synchronous phenomena in 1975. In the Kuramoto model, oscillators are generally all-to-all coupled. Normally, if the oscillator is all-to-all coupled with a certain coupling strength, it is expected that it synchronise completely with time. This study presents about a chimera state in which an array of identical oscillators split into two dominants. Namely, one coherent and phase locked, the other incoherent and asynchronized.

In this study, a system with two-group is proposed and synchronization state is investigated. By using computer simulations, we observe the phase state of the oscillators by changing coupling strength K.

2. Kuramoto model

The Kuramoto model is used in this study. We consider a network of N coupled limit-cycle oscillators whose phase are θ_i, i = 1, 2, ..., N. The equation of the Kuramoto model is shown in Eq. (1).

$$\frac{\partial \theta_i}{\partial t} = \omega_i + \frac{K}{N}\sum_{j=1}^{N}\sin(\theta_j - \theta_i). \tag{1}$$

At this time, the native frequencies ω_i of the oscillators are randomly distributed. Where K is the global coupling strength.

These oscillators are located around a cycle and rotate at their own frequencies to define a complex order parameter R.

$$Re^{i\varphi} = \frac{1}{N}\sum_{j=1}^{N} e^{i\theta_j}. \tag{2}$$

Where φ indicates the average phase of the coupled oscillators, and the order parameter R $(0 \le R \le 1)$ indicates a measure of phase coherence. In the case of R = 1, it indicates that all oscillators come to a single tight clump. In the case of R = 0, it indicates that oscillators are scattered uniformly around the cycle. From Eq. (2), Eq. (3) is as follows.

$$\frac{\partial \theta_i}{\partial t} = \omega_i - KR\sin(\theta_j - \varphi). \tag{3}$$

3. Proposed system

In conventional studies, all-to-all oscillators are coupled. We proposed that oscillators are divided two parts. Oscillators are all-to-all coupled in the same group. Figure 1 shows a simplified version of coupling.

In this paper, ω_i follows the Cauchy distribution. The number of oscillators N = 100 and the coupling strength K = 5.0 or 9.0. K is constant everywhere.

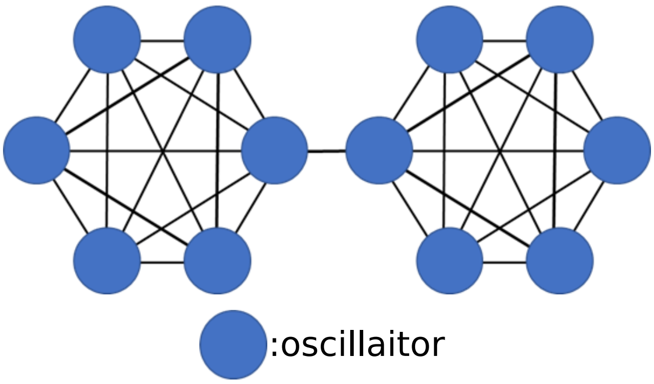

Figure 1: Simplified diagram of the coupling of the proposed system.

4. Simulation results

4.1 For Two-group case K = 5.0

Figure 2 shows time evolution of R. R shows the overall synchronization rate. $R1$ and $R2$ indicate the synchronization rate of each group. $R3$ shows the synchronization rate of the oscillators connecting each group. $R1$ and $R2$ differ in the way they are synchronized. $R2$ is more slowly synchronized than $R1$.

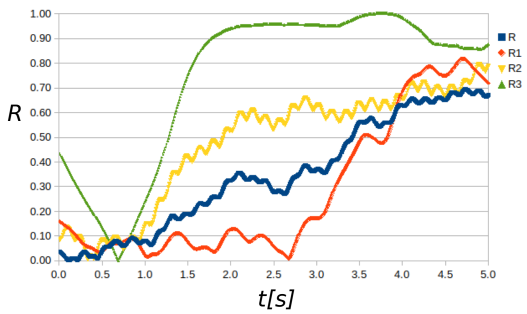

Figure 2: Time evolution of R in two-group.

Figure 3 shows phase histogram. When t=0.0, the oscillators are falling apart. It can be seen that the phase is trying to remain constant over time. However, two peaks are seen. This could be due to the division of the oscillators into two-group.

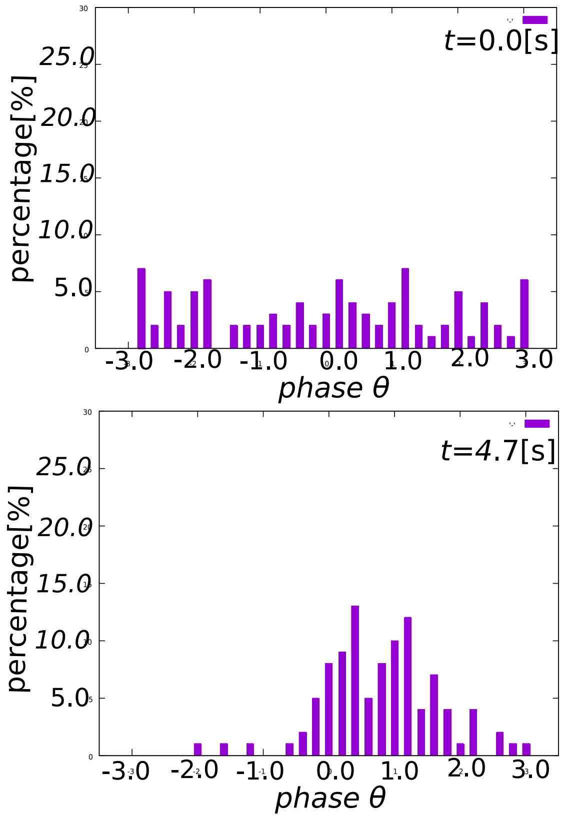

Figure 3: Phase histogram in two-group.

Similar to Fig. 3 and 4 shows the distribution of phase. One group is made up of 50 oscillators, each of which is found to be locked phase. On the other hand, oscillators locked another group of phases are also seen.

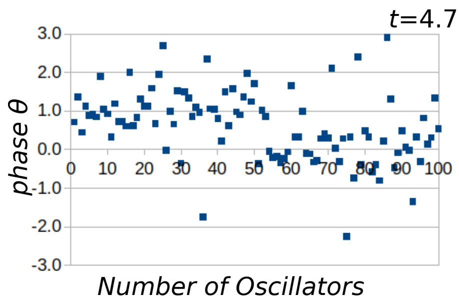

Figure 4: Phase diagram in two-group.

4.2 For original Kuramoto model K = 5.0

The fact that the network is one-group is the same as in the original Kuramoto model. Figure 5 shows time evolution of R. The synchronization rate has increased compared to Fig. 2.

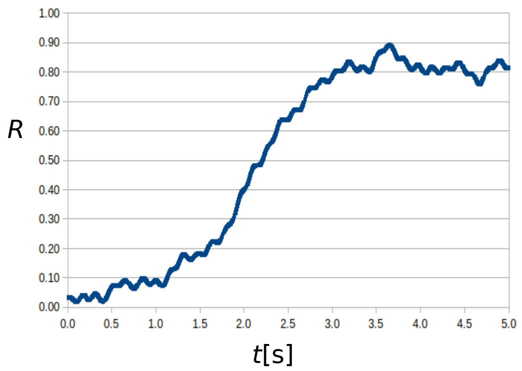

Figure 5: Time evolution of R in one-group.

Figure 6 shows the frequency distribution of phases as in Fig. 3. When t = 0.0, the phases of all oscillators are scattered. However, unlike the two-group case, only one peak are seen.

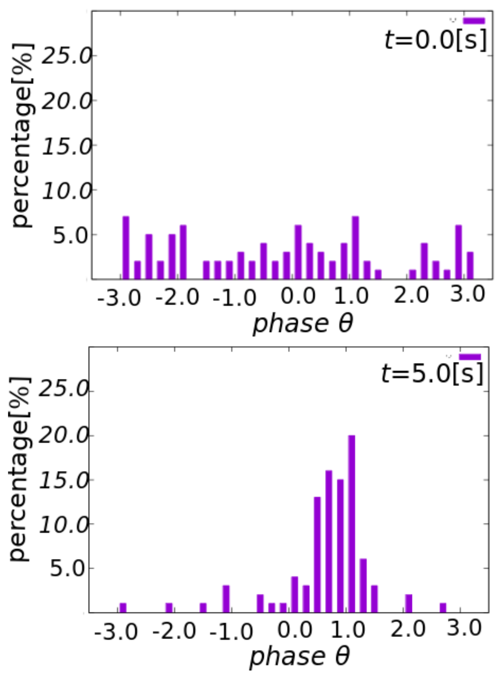

Figure 6: Phase histogram in one-group

Figure 7 shows the phase of each oscillator as in Fig. 4. It shows that the phases are trying to become constant. Phase is locked some extent, but asynchronous phase changing can be seen.

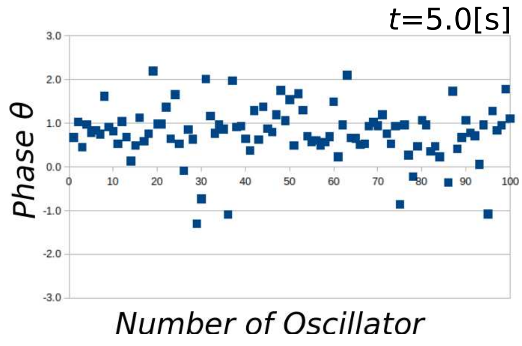

Figure 7: Phase diagram in one-group.

4.3 For Two-group case K = 9.0

The case of increased bond strength is shown below Here, coupling strength K = 9.0. Figure 8 shows that both groups are synchronised and that the groups are synchronised with each other.

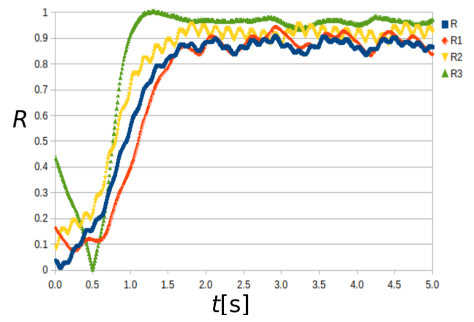

Figure 8: Time evolution of R in two-group K = 9.0.

Figure 9 shows the temporal variation of the frequency distribution. Unlike Fig. 3 and Fig. 6, Fig. 9 shows the time variation of the frequency distribution in 0.1 second increments. Several oscillators are found to be freely phase-shifting. Most phases are clustered in one place, but some oscillators are asynchronous.

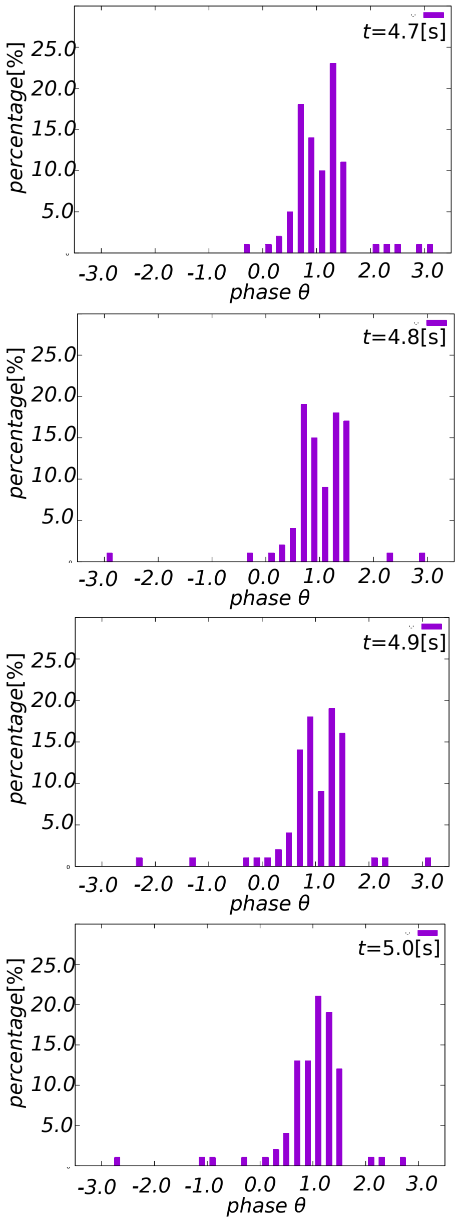

Figure 9: Phase histogram in two-group $K = 9.0$.

Figure 10 shows phase diagram as Fig. 4 and Fig. 7. The phase is reached a more constant value than in Fig. 4. As in Fig. 4, many phases are found to be locked two values.

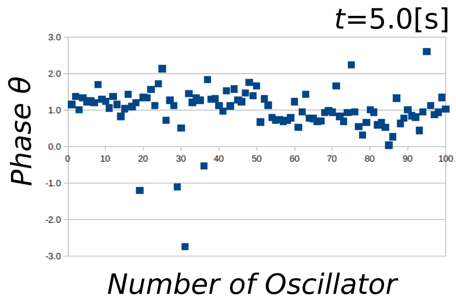

Figure 10: Phase diagram in two-group $K = 9.0$.

5. Conclusions

In this study, we proposed two-group Kuramoto model. The phase was seen to be fixed and freely changing. In other words, a mixture of synchronised and asynchronous oscillators states was observed. However, it was difficult to observe even the specific behaviour of each oscillator. In the future, We would like to investigate the time variation of the synchronised percentage and the phase of each oscillators. These make the observation of the chimera states even clearer.

References

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